A

SYSTEM

OF

NATURAL PHILOSOPHY.

KALOKAIR PHILOSOPHY

OR

MENTAL SYSTEM

A SYSTEM OF NATURAL PHILOSOPHY,

BEING

A COURSE of LECTURES

IN

Mechanics, Optics, Hydroſtatics, and Aſtronomy;

Which are read in St Johns College Cambridge,

By T. RUTHERFORTH D.D. F.R.S.

Chaplain to his Royal Highness the Prince of Wales.

IN TWO VOLUMES.

VOL. II.

CAMBRIDGE,

Printed by J. BENTHAM, Printer to the Univerſity,

For W. THURLBOURN, Bookſeller in Cambridge; and ſold by J. Beecroft, at the Bible and Crown in Lombard-Street, London.

MDCCXLVIII.

A SYSTEM OF NATURAL PHILOSOPHY

MVSEVM BRITAN NICVM

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MDCCLXXII.

A SYSTEM OF NATURAL PHILOSOPHY.

HYDROSTATICS.

CHAP. I.
Of the properties of fluids.

FLUIDS are divided into two ſorts, incompreſſible or unelaſtic ones, ſuch as water, and compreſſible or elaſtic ones, ſuch as air. In that part of philoſophy, which, in the proper ſenſe of the word, is called hydroſtatics, we explane the preſſure of water and of other unelaſtic fluids upon the bottoms and ſides of veſſels, that contain them, and upon bodies immerſed in them; more particularly in caſes where this preſſure is balanced by ſome other force, ſo that no motion is produced by it in the fluid itſelf. In that part of philoſophy, which is called hydraulics, we explane the motion of unelaſtic fluids through pipes, ſpouts, tubes, or any holes in the bottoms or ſides of veſſels. In that part of philoſophy which is called pneumatics, we explane the properties, the preſſure, and the effects ariſing from the preſſure or from the elaſticity of the air.

But in the general title I have uſed the word *hydroſtatics* in a more extenſive ſenſe than the etymology of the word will juſtify, and have made it ſtand for every part of philoſophy, that treats of the nature, the preſſure, and the motion of fluids whether unelaſtic or elaſtic.

1. *The denſity of a body is the quantity of matter, which it contains, when the bulk of it is given.*

2. *The ſpecific gravity of a body is the weight of it, when the bulk is given.*

It is called the ſpecific gravity, becauſe it is the comparative weight of different ſpecies or ſorts of bodies. Thus if the ſpecific gravity of gold is ſaid to be to that of water as 19 to 1, the meaning is that bulk for bulk, or under equal dimenſions, the weight of gold is to that of water as 19 to 1,

or that a cubic inch of gold for inſtance will weigh nineteen times as much as a cubic inch of water. In like manner if the ſpecific gravity of gold is ſaid to be to that of ſilver as 19 to 11, the meaning is that bulk for bulk, or in equal maſſes of theſe two different ſorts of metal, the weights are as 19 to 11.

3. *The ſpecific gravity of bodies is as their denſity.*

For the ſpecific gravity is the weight of a given bulk. And, by propoſition 27 of mechanics, the weight of bodies is as the quantity of matter contained in them. Therefore the ſpecific gravity of a body is as the quantity of matter contained in a given bulk, that is, by propoſition 1, as its denſity.

4. *The denſity of bodies, when their weight is given, is inverſely as their bulk.*

If the weight is given, ſo likewiſe is the quantity of matter, by propoſition 27 of mechanics. But in bodies, where the ſame quantity of matter fills a leſs ſpace or leſs dimenſions, it muſt lie cloſer together or be more denſe; and where it fills a greater ſpace or greater dimenſions, the ſeveral particles muſt be farther aſunder and the maſs conſequently muſt be leſs denſe; that is, when the quantity of matter or weight is given, the denſity is greater as the bulk is leſs, and the denſity is leſs as the bulk is greater; or the denſity is inverſely as the bulk.

5. *The ſpecific gravity of bodies is inverſely as their bulk, when their weights are equal.*

The ſpecific gravity of bodies is as their denſity, by propoſition 3. And the denſity of bodies is inverſely as their bulk, when their weights are equal, by propoſition 4. Therefore the ſpecific gravity is likewiſe inverſely as their bulk, when their weights are equal. Thus if the ſpecific gravity of gold is to that of ſilver as 19 to 11; and a cylinder of gold 11 inches high weighs a pound; a cylinder of ſilver having an equal baſe and weighing a pound muſt be 19 inches high. For ſince the ſpecific gravities are as 19 to 11, the bulks, that is, the heights, muſt be as theſe gravities inverted or as 11 to 19. If the ſpecific gravity of mercury is to that of water as 14 to 1, and a cylinder of mercury of a certain weight is 30 inches high; then a cylinder of water of an equal baſe and having the ſame weight muſt be 14 times as big, that is, 14 times as high. So that the height of the cylinder of water will be $14 \times 30 = 420$ inches, or 35 feet.

6.

6. *Fluids are divided into two ſorts; elaſtic or compreſſible, and unelaſtic or incompreſſible.*

All fluids are expanded by heat, and contract themſelves again as they cool. If a ſmall quantity of air is tyed up in a bladder; though there is too little of it to fill the bladder, yet by being laid before a fire and warmed it may be made to expand itſelf, till it fills the bladder and ſtretches it out as much as if more air had been blown into it. But this property is not peculiar to air. Spirits of wine are capable of being rarified or expanded by heat; ſo likewiſe water, mercury, and oyl, when they are heated ſwell to a greater bulk. And all of them, as they cool, will be condenſed and be reduced again to the ſame dimenſions that they had, before they were heated. It is not therefore a rarefaction by heat and condenſation by cold, by which elaſtic fluids are diſtinguiſhed from unelaſtic ones. Since this property belongs not only to the air, which is elaſtic; but likewiſe to ſpirits, water, mercury, oyl, &c. which are not ſo. But air is called an elaſtic fluid, becauſe it is compreſſible, that is, becauſe by an external force any quantity of it may be ſqueezed cloſer together ſo as to fill a leſs ſpace than uſual; and when this compreſſing force is removed, the air will dilate itſelf to its former dimenſions and fill the ſame ſpace that it did, before it was compreſſed. This is a property not to be found in water, as is evident by the experiment cited from Mr. Locke in propoſition 1 of mechanics.

7. *A fluid is a body, whoſe parts yield to any force impreſſed upon them, and in yielding are eaſily moved one amongſt another.*

I have defined a fluid already in mechanics, and repeat the definition in this place only to defend it againſt an objection, that has been made to it. The definition is ſaid to be imperfect, becauſe it does not expreſs an inequality of preſſure, for it is well known that the parts of an incompreſſible fluid will not yield to any force ſo as to move, unleſs the force is greater on one ſide than on the other. But there ſeems no occaſion to expreſs this reſtriction in the propoſition. For when we ſay that the parts of a fluid will yield to any force and may be put in motion by it; we may take it for granted, that our readers will underſtand we mean a force ſo applyed as to be capable of producing motion: there is no occaſion in this caſe to tell them particularly, that the force muſt not be oppoſed and its effect be deſtroyed by another equal force acting in a contrary direction: unleſs, if we ſhould ſay in another caſe that a horſe can draw a weight of 1000 pounds, there would be occaſion to add this reſtriction of juſt the ſame ſort, provided there is not another horſe of equal ſtrength to pull againſt him in the contrary direction.

I ſhould think indeed, that any one who reads theſe words, *the parts of a fluid will yield to any force that is impreſſed upon them*, muſt deſign and endeavour to miſunderſtand them, if he was to aſk whether the writer meant that the parts of a fluid will yield to two equal forces in oppoſite directions. The writer ſpeaks planely enough concerning one force; and where equal forces act in contrary directions, there muſt be two of them at leaſt: a circumſtance, which the author of the definition, if we may judge from the words of it, never once ſuppoſed. He ſpeaks only of one force; and it would have been nonſenſe to ſay, that any one force will produce motion in the parts of a fluid, unleſs that one force acts equally in contrary directions: for this is in effect to ſay, that one force will produce ſuch a motion, unleſs this one force ſhould be two. Though indeed even two equal and contrary forces would produce motion in the parts of a fluid; for, if there was nothing elſe to ſtop them, they would ſlip from between theſe forces by a ſide motion: and if they are acted upon by any force at all, nothing leſs than innumerable equal forces in contrary directions can prevent them from moving.

8. *The weight of fluids is as their quantity of matter.*

This may be proved concerning fluids in particular in the ſame manner, that it has been proved already, in propoſition 27 of mechanics, concerning bodies in general. For if ſeveral drops of different fluids were let fall from the top of an exhauſted receiver, that is, in a ſpace where they meet with no reſiſtance, they will all come to the bottom of it together. Therefore all of them deſcend with the ſame velocity, and conſequently their moments are as their reſpective quantities of matter. But the moments in all caſes are as the moving forces, by the ſecond law of motion, and the moving forces, by which theſe drops are made to deſcend, are the weights of the drops. From whence it follows that their weights are as their moments, that is, as the reſpective quantities of matter contained in them.

We need not indeed have been at the trouble of repeating this demonſtration. For as the truth of the propoſition has been proved already of bodies in general, this was in effect proving it not only of hard bodies but of fluid ones too.

9. *Fluids gravitate or have weight in their proper place, that is, in fluids of the ſame ſort.*

The meaning of this propoſition will be beſt underſtood by conſidering two of the principal objections to the truth of it. When you are drawing

a bucket of water from a well, you feel no weight of water in the bucket, whilſt it is below the ſurface of the well, that is, whilſt it is in its proper place, or in a fluid of the ſame ſort. But, as ſoon as you have drawn the bucket above the ſurface you immediately feel the weight. So that the water in the bucket ſeems to have no weight whilſt it is in other water, but upon being drawn up into the air, which is a fluid of a different ſort, its weight is reſtored to it.

Now the weight of the water contained in the bucket is not felt, whilſt it is below the ſurface, as the objection ſuppoſes; yet it is not becauſe that weight is deſtroyed, but becauſe it is ſupported. For we ſhall ſhew hereafter, that, when the bucket is below water, there is a preſſure upwards againſt it, which is at leaſt equal to the weight of the water contained in it. Therefore this weight, as it is ſuſtained by the preſſure upwards and not by the rope, to which the bucket hangs, cannot be felt by the perſon who has hold of the rope and is to draw the water up. But this preſſure ceaſes, as ſoon as the bucket is drawn above the ſurface and gets out of the water into the air. And then the weight, that is in the bucket, not being ſupported by any contrary force, is perceived. It is not my buſineſs at preſent to explane what this preſſure upwards is, or to ſhew from whence it proceeds. This ſhall be explaned hereafter.

Another objection to the weight of fluids, whilſt they are in their proper place, is that we do not feel the weight of the air, in which we live. But this objection will be eaſily anſwered, when we ſhall prove hereafter that the air preſſes equally in all directions: ſo that if, for inſtance, we ſtretch out our hand with the palm upwards, we ought not to perceive the weight of the incumbent air; becauſe the preſſure upwards againſt the back of it will be equal to that weight, and this latter force will ſupport our hand as much as the other depreſſes it; that is, theſe two equal and oppoſite forces will deſtroy each other, and we ſhall not be ſenſible of any effect produced by either of them.

And now the principal objections are removed, there will be no great difficulty in proving the truth of the propoſition. It cannot be denyed that the weight of a veſſel, in which water or any other fluid is contained, is always proportional to the quantity of matter contained in it. Now the weight of the whole could not be as the quantity of matter, or proportional to the weight of all its parts, unleſs all the parts of the fluid, every drop contained in the veſſel, had weight and concurred in producing the gravity of the whole. But though the whole may be conſidered as water in air or in a fluid of a different ſort, yet each drop of water in

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the veſſel is in water or in a fluid of the ſame ſort. And ſince we have ſhewn that every drop gravitates and concurs in producing the gravity of the whole, it follows, that fluids gravitate in their proper place.

The truth of this propoſition may likewiſe be made evident to ſenſe. For if a veſſel, which holds juſt as much water as weighs a pound, is weighed in water, when it is full it will weigh a pound more, than if it is weighed in the ſame manner, when it is empty and cloſe ſtopped to prevent any water from getting into it. Therefore the water, which weighs a pound when the veſſel is out of water, weighs a pound likewiſe, when it is in water.

I had ſome ſcruple about mentioning this experiment; becauſe there is more difficulty to explane it, than there is to prove the propoſition without it. For might not one, who is learning philoſophy, aſk me how this experiment is conſiſtent with what has been already ſaid in anſwer to the firſt objection againſt fluids gravitating in fluids of the ſame ſort? You told me, ſays he, that the weight of water in a bucket is not felt, whilſt it is below water, becauſe there is a preſſure upwards, which is equal to the weight of it and ſupports it. But now you ſay, that the pound of water in the veſſel weighs a pound, even whilſt it is below water, notwithſtanding the preſſure upwards muſt be equal to the weight and ſupport it in this caſe as well as in the other. If therefore the preſſure upwards is the reaſon why the weight of water in the bucket is not perceived, ſhould not it equally be a reaſon why the weight of water in this veſſel ſhould not be perceived?—Thoſe, who underſtand philoſophy will think this a very trifling difficulty: but thoſe, who are to teach it, know that it is ſuch an one as a beginner cannot eaſily get over. The preſſure upwards againſt the veſſel does indeed ſupport a weight equal to that of the water contained it: juſt as a preſſure of the ſame ſort was ſaid to ſupport the weight of water in the bucket, as mentioned in the anſwer to the firſt objection. I am therefore to ſhew how the weight of water in the veſſel comes to be perceived, when it is ſupported by a contrary force. The preſſure upwards againſt the veſſel is the ſame, whether it is full or empty, and is equivalent to a pound of water: this, when the veſſel is weighed in water, is to be ſubſtracted from the weight of the veſſel alone, if it is empty: and ſo it muſt likewiſe, though the veſſel is full, ſuppoſing the water contained in it to loſe its gravity by being in a fluid of the ſame ſort: ſo that the weight of the veſſel in water would upon this ſuppoſition be the ſame whether it is full or empty; for it would always be equal to the weight of the veſſel out of water, after one pound is ſubſtracted from it. But if the water contained in the veſſel does not loſe its

weight

weight by being in a fluid of the ſame ſort; then the weight of the empty veſſel in water will, as before, be equal to its weight out of water, after one pound is ſubſtracted from it: but its weight when full will be equal to the weight of the veſſel and water together after one pound is ſubſtracted from it. That is ſince the water in the veſſel is ſuppoſed to weigh a pound, the veſſel, if weighed in water, will be a pound heavier when it is full than when it is empty.

Suppoſe the empty veſſel weighs 6 pounds in air, and that by the preſſure upwards when it is in the water 1 pound of its weight is ſupported, then it will weigh 6—1=5 pounds in water. If a pound of water fills the veſſel, it will weigh, when full, 6+1=7 pounds in air. But upon ſuppoſition that water had no weight in water; then as the preſſure upwards ſupports 1 pound weight, and the water, which fills the veſſel, has no weight, the veſſel when full ought to weigh no more than 7—2=5 pound in water. For of the 7 pounds which the veſſel and water together weigh in air, 1 pound is ſubſtracted on account of the waters loſing its weight, and 1 pound on account of what the preſſure upwards ſupports. Whereas the veſſel, when full, will weigh 6 pounds in water, ſo that of the whole weight no more than 1 pound is loſt: and as the preſſure upwards ſupports thus much of its weight; we may conclude that there is no loſs of weight on any other account, and conſequently that water in water, or in its proper place, retains its weight.

From hence we may obſerve, that ſince the weight, which any veſſel loſes in water by the preſſure upwards, is at leaſt as much as the water weighs which fills the veſſel; a bucket ought likewiſe to loſe the weight of as much water as will fill it; and conſequently when it is full of water, the weight of that water cannot be perceived, whilſt it is below the ſurface of the well, not becauſe that weight is deſtroyed, but becauſe it is ſupported.

CHAP. II.

Of the preſſure of fluids upon the bottoms and ſides of the veſſels that contain them.

10. *When a fluid is forced through a tube, which is wider in ſome parts than it is in others, the velocity of the fluid in every ſection of it will be inverſely as the area of the ſection.*

ADMN, Plat. XVIII. fig. 1, is a bended tube, which is larger at IL than it is at FG. If this tube was filled with water to the height ADFG, by forcing the water downwards in the leg ADBP it would be made

made to riſe in the other leg KHMN. And in this caſe the water, as it paſſes through the circular area or ſection FG of the leſſer part, will move faſter than it does, as it paſſes through the ſection IL of the greater part: and the velocity in theſe two ſections will be as their areas inverſely; that is, the velocity at IL will be as much leſs than the velocity at FG, as the bore of the tube at FG is leſs, than the bore of it at IL.

The water, that is forced out of one part of the tube, is driven into the other; ſo that equal quantities paſs through every ſection of it in the ſame time. There cannot paſs leſs water through FG than paſſes through IL in the ſame time: for if there did, the quantity of water between FG and IL muſt be encreaſed; and this cannot happen, unleſs it was condenſed; which is impoſſible. Neither can there paſs more water through FG than paſſes through IL in the ſame time; for if there did the quantity of water between FG and IL muſt be diminiſhed, and this cannot happen unleſs it was either to be expanded, or to part aſunder ſo as to leave an empty ſpace between IL and FG: the former of theſe is impoſſible, as water is an unelaſtic fluid; and it will be eaſy to underſtand that the latter will not happen, for we cannot ſuppoſe that any force which drives the water down at IL can make it riſe in ſuch a manner at FG as to part the ſtream between theſe two ſections. But if the quantity, which paſſes through FG in a given time, is neither leſs nor greater than what paſſes through IL, then it follows, as before, that equal quantities paſs through each ſection in the ſame time. But equal quantities cannot paſs through unequal parts of the tube, in the ſame time, unleſs the water runs faſter in proportion, where the tube is narrower, and ſlower where it is wider. There cannot, in a given time, as much water run through the ſection at FG as runs through the ſection at IL, unleſs the water moves as much faſter at FG than it does at IL as the tube is narrower at FG than it is at IL, that is, unleſs the velocity in each ſection is inverſely as the area of the ſection. This demonſtration might be applyed in the ſame manner, though the tube was ſtrait; or though, inſtead of being made of two cylindrical parts, the figure of it had been ever ſo irregular.

11. *When a fluid is forced through a tube, which is wider in ſome parts than it is in others, the moment of it will be the ſame in every ſection of the tube.*

The quantity of water at IL is juſt as much greater than at FG as IL is greater than FG; that is, the quantity of water, which fills the ſection at IL is to the quantity of water at FG, as IL is to FG; or the quantity of water in each ſection is directly as the area of the ſection. But, by the laſt

laſt propoſition, the velocity is inverſely as the area; therefore the velocity is every where inverſely as the quantity of matter, and conſequently, by propoſition 14 of mechanics, the moment is every where the ſame.

12. *When two tubes, that contain the ſame ſort of fluid, communicate with each other; the fluid will always ſtand at the ſame perpendicular height in both of them.*

If two tubes ADBP and FGKH, Plat. XVIII. fig. 1, communicate with each other, and water is contained in both of them; this water will not be in equilibrio, unleſs the perpendicular height of it is the ſame in both. The water cannot deſcend in either leg without riſing in the other: and therefore the water in ADBP and in FGKH may be looked upon as two forces acting in contrary directions; and conſequently theſe forces will be in equilibrio, or will reſt, only when their moments are equal. The moments of theſe two cylinders of water will be equal, when their quantities of matter are inverſely as their velocities, by propoſition 14 of mechanics. And ſince their velocities are inverſely as their baſes at BP and KH, by propoſition 10, their moments will be equal when their quantities of matter are directly as their baſes. But the quantity of matter in cylinders is as their baſes, only when their perpendicular heights are equal. Euc. b. XII. prop. 11. Therefore the quantity of matter in theſe two cylinders will not be directly as the baſes, or inverſely as the velocities, that is, their moments will not be equal, unleſs their perpendicular heights are the ſame.

If it ſhould here be objected, that it is abſurd to talk of the velocity of the water either in ADBP or in FGKH, when by the ſuppoſition theſe two columns are in equilibrio, and conſequently are at reſt or have no velocity at all; I muſt deſire the reader to look back to propoſition 49 of mechanics, where he will find this objection obviated in the caſe of the lever; and then he may apply what has already been ſaid in that place to the inſtance now before us. For we do not here conſider the velocity, with which the two columns of water ADBP are moving, but the velocity, with which they muſt begin to move. And ſince, if their perpendicular height is the ſame, the velocity, with which they muſt begin to move, will be inverſely as their reſpective quantities of matter, it follows, that they cannot begin to move but with equal moments. Now their motions muſt be in contrary directions, becauſe one column cannot deſcend without making the other aſcend. Therefore theſe equal moments will deſtroy each other, and conſequently the two columns will continue at reſt, or will be in equilibrio.

If any water was poured into the leg MNKH, ſo as to encreaſe the perpendicular height in this leg, it would encreaſe the quantity of matter and conſequently the moment in it. By this means the equilibrium would be deſtroyed and the water would deſcend in the leg FGKH, where the moment is made greater, and aſcend in the other leg till it was level in both, that is, till the equilibrium was reſtored.

But ſuppoſe that the leg MNKH was made of leather or any other pliable ſubſtance, and that there was a pack-thread tyed round it any where between FG and KH: then by drawing this thread very ſtrait, the ſection of the tube, where it was tyed, would be made narrower, and conſequently ſome of the water, that was in it before, would be ſqueezed out of it. By this means the water in the leg FGKH would riſe higher than the level, without the addition of any new water poured into it. But though the quantity of matter on this ſide is not encreaſed, yet the moment will be made greater, and the equilibrium will be deſtroyed. For ſince one part of the leg is made narrower, the velocity, by propoſition 10, will be encreaſed in that part; and this will make the moment in this leg greater than it was before: till the water by ſinking again to the ſame level, that it ſtands at in the other leg, has diminiſhed the quantity of matter on this ſide, as much as the velocity is encreaſed. And ſince, when one of the tubes, by being made narrower in one part than it was, ceaſes to be cylindrical, the perpendicular height continues equal in both; we may conclude that in communicating tubes of all figures, and in veſſels and their ſpouts, whether they are cylindrical or not, the water in different parts of them will not be in equilibrio, unleſs its perpendicular height in all the parts is the ſame. We may extend this propoſition in another manner to communicating tubes or to veſſels and their ſpouts, let their figure be ever ſo complex and irregular. For by the laſt propoſition the moment of water in every ſection of a veſſel and its ſpout is the ſame, that is, every infinitely thin lamina, ſurface, or ſheet of water, in any part of the communicating tubes, has the ſame quantity of motion. From hence it follows, that the quantity of motion in either of the tubes will be equal, when the number of infinitely thin ſurfaces in each is equal; that is, when the perpendicular heights AB and FK are equal: for an equal number of infinitely thin ſurfaces of water lying one upon another, between BP and AD, and between KH and FK, will make two columns of water ADBP and FGKH equal in height to each other. But if the perpendicular height in one of theſe tubes is greater than in the other; then the number of infinitely thin ſurfaces, and conſequently the moment, will be the greateſt

greateſt in that tube, where the perpendicular height is the greateſt: and therefore the water in the two tubes not being in equilibrio will ſwing backwards and forwards, till it ſtands at laſt at the ſame height in both of them.

From hence likewiſe we may underſtand why water cannot be conveyed by means of a pipe, that is laid into a reſervoir, or baſon, to any place higher than the level of the water in the baſon. For, ſince the pipe and baſon are two communicating veſſels, the water will be in equilibrio, and conſequently will reſt, only when it ſtands at the ſame height in both.

13. *When two tubes, each of which contains a different ſort of fluid, communicate with each other; the heights of theſe fluids will be inverſely as their ſpecific gravities.*

The weight of a fluid in two communicating tubes is always inverſely as the velocities, that is, inverſely as the bores of the tubes, when the moments in them are equal, by propoſition 12. Therefore, when the bores of each tube are given, the weights of fluid in each are given likewiſe. But where the weights are given the bulks will be inverſely as the ſpecific gravities: and the bulks of fluid contained in two tubes, whoſe bores are given, will be as the height of the fluid. Therefore the heights, when the moments are equal, will be inverſely as the ſpecific gravities.

If one tube contains water and the other oil of turpentine, then, ſuppoſing the ſpecific gravities of water and oil of turpentine to be to each other as 10 to 8, and the water to ſtand at the height of 8 inches in one tube, the oil will ſtand at the height of 10 inches in the other: that is, when the ſpecific gravities are as 10 to 8, the heights, to cauſe an equilibrium, muſt be as 8 to 10, or as thoſe gravities inverted. For if the tube, where the oil is, had contained water, as the other does, this water would have ſtood at the height of 8 inches, by propoſition 12: and when there is oil in it inſtead of water, there muſt be oil enough to have the ſame weight, that the water would have had; that is, oil enough to weigh as much as 8 inches of water would weigh. But, where the bulks are equal, the weight of oil is to that of water as 8 to 10, and conſequently 8 inches of oil does not weigh ſo much as 8 inches of water would have done. But 10 inches of oil will have juſt the ſame weight that 8 inches of water would have had; ſince, by propoſition 5, the weights are equal, if the bulks are inverſely as the ſpecific gravities. Therefore either 8 inches of water or 10 of oil in one tube will be in equilibrio with 8 inches of water in the other.

14. *The ſurface of a fluid, which is contained in an open veſſel, will be level, if it is either preſſed equally every where, or not preſſed at all.*

No part of the fluid can ſtand higher than the reſt: for if any part is thus raiſed, ſince all the particles, which make up the heap, have gravity, and ſince they yield to every impreſſion and are eaſily moved, they will yield to the force of gravity, and will ſlip down, till they come to a level. From the nature of a fluid this muſt certainly be the caſe, where the ſurface of it is not preſſed by any external force. And a preſſure, which is equal upon all parts of the ſurface, cannot poſſibly raiſe one part higher than another: therefore it will make itſelf level when it is preſſed by ſuch a force, in the ſame manner as if it was preſſed by no force at all.

The ſurface of water contained in an open veſſel will be level, when the veſſel ſtands in a vacuum, that is, in a receiver from whence the air is exhauſted: becauſe then it is not preſſed at all by any external force. And this ſurface will likewiſe be level, when the veſſel ſtands in the open air: becauſe then the ſurface is preſſed equally every where.

15. *The lower parts of a fluid, when it cannot be condenſed, will not be moved out of their places by the weight or preſſure of the parts above them.*

A drop of water at EI, in the veſſel ABCD, Plat. XVIII. fig. 2. will not be moved out of its place by the preſſure or weight of the column of water FGEI that is above it. This drop certainly cannot be made to move upwards by any weight which reſts upon it. And it cannot move downwards, unleſs the water EIOR is either condenſed to make room for it, which is contrary to the ſuppoſition, or elſe is removed out of the way. But the water EIOR cannot be removed out of the way, unleſs it goes into the ſpace INRD, or into the ſpace MECO. Now the ſpace INRD is full of water already; and therefore it cannot move this way, unleſs it raiſes the lamina or ſurface of water IN. But the preſſure downwards upon EI cannot make any part of this ſurface riſe; particularly it cannot make IH riſe; for IH is preſſed down by the weight of a column of water GLIH equal to FGEI. Therefore, unleſs the weight of one column preſſing downwards could raiſe the weight of another equal column, neither IH nor any part of the ſurface IN will riſe: and conſequently the water EIOR cannot move out of its place towards ND or into the ſpace INRD. And in the ſame manner we may prove, that it cannot move towards MC or into the ſpace MECO.

Therefore

Therefore ſince EIOR cannot move out of its place at all by the preſſure, which a drop at EI ſuſtains, this drop cannot move downwards. And laſtly the drop cannot move ſideways. For it cannot be made to move ſideways, unleſs the preſſure upon it can remove ſome drop as IH, at the ſame depth with itſelf, out of its place. But every drop, as IH, at the ſame depth with EI, is kept in its place by a weight GIHL equal to the weight FGEI, which EI ſuſtains. Therefore the preſſure upon EI cannot remove IH, unleſs one force could overcome another which is equal to it. But, if EI can move neither upwards, downwards, nor ſideways, the preſſure from above cannot produce any motion at all in it.

All the lower drops are preſſed downwards by the weight of thoſe, which are above them: but then all the drops, which are at the ſame depth, bear an equal weight: and for this reaſon not any one of them can move; becauſe there is no room for it to move, unleſs it was to put ſome other drop out of the way to make room for itſelf. But each drop, at the ſame depth, is kept in its place by an equal force: and conſequently, unleſs equal forces in contrary directions could produce motion, no one drop can be moved by the preſſure downwards upon it, or by the weight it ſuſtains.

16. *All the parts of a fluid, at equal depths below the ſurface, are preſſed equally in all directions.*

There are two particulars expreſſed in this propoſition, which ſhould be proved diſtinctly. Firſt, the drop EI, Plat. XVIII. fig. 2, is preſſed as much upwards and ſideways, as it is preſſed downwards by the weight of the incumbent fluid FGEI. And ſecondly, two equal drops one at EI and the other at IH, or at the ſame depth any where below the ſurface, are preſſed downwards upwards and ſideways equally.

Firſt, the drop EI is preſſed equally in all directions. This follows from the foregoing propoſition. For the drop EI is preſſed downwards by the incumbent fluid; and, becauſe it is a fluid itſelf, it will yield to every impreſſion and will begin to move, unleſs it is acted upon by equal forces in all poſſible directions. But, by propoſition 15, it cannot move at all. Therefore there muſt be equal forces acting upon it in all poſſible directions. So that it is ſupported or preſſed upwards by a force equal to the weight of the incumbent column FGEI. And an equal preſſure acts upon it from all ſides, by which means alone it can be kept from moving out of its place ſome way or other.

The

The effect of this preſſure upwards, when all the parts of the fluid are in equilibrio, is to keep the drop EI from deſcending, as it would neceſſarily do on account of the weight which preſſes it downwards, unleſs it was prevented by a contrary force equal to that weight. But the equilibrium of the fluid may be ſo deſtroyed, that the preſſure upwards will produce a different effect, and inſtead of keeping the drop EI at reſt in its place will make it aſcend. This preſſure upwards is a counterpoiſe to the incumbent weight: and the drop EI, acted upon by both forces at the ſame time, will not move either upwards or downwards. It cannot deſcend becauſe this preſſure prevents it, and it cannot aſcend becauſe the incumbent weight prevents it. Therefore, if the incumbent weight FGEI is removed, the drop ought to aſcend. Now ſuppoſe there was an empty tube of a ſufficient length, which was open at both ends: if the lower end of it was to be cloſed with ones finger, till that end was ſunk into the water down to EI, and the other end was above the ſurface AB; then it is plain that as no water could get into the tube, whilſt the end, which was cloſed with the finger, was ſinking, there will be no water above EI, till the finger is taken away; that is, the incumbent weight is by this means removed. And I ſuppoſe my reader does not want to be informed, that upon removing the finger and opening the lower end of the tube, the water will aſcend in it, or that the preſſure upwards, when there is no force acting in a contrary direction, will cauſe the drop EI to move.

If the veſſel ABCD was filled with a hard body inſtead of a fluid; as it would be, if, for inſtance, we ſuppoſe the water to freeze and to be changed into ice; then no part of it as EI would be preſſed upwards at all. If a hole FGEI was to be bored in it, I imagine the reader knows, that, notwithſtanding the incumbent ice is by this means taken away, yet there can be no preſſure upwards againſt EI; becauſe it will continue in its place and not be made to aſcend. The parts of a hard body cohere together, and this force keeps them at reſt in their places. But the parts of a fluid yield to every impreſſion, and a force acting in any one direction will put it in motion, unleſs it is kept from moving by an equal force acting upon it in the oppoſite direction.

Secondly, the drop IH equal to EI, and at the ſame depth with it, is preſſed in all directions, as much as EI is. For if IH was not kept in its place by as great a force as preſſes downwards; then the preſſure on EI would move it: becauſe the force, which acts upon it in one direction, would be greater upon this ſuppoſition, than the force which ſuſtains it on the ſide IH. But, by the laſt propoſition, EI cannot move. Therefore the force

force which ſuſtains it on the ſide IH, that is, the force, which keeps IH in its place, is equal to the weight of fluid above EI. Now ſince IH, by the former part of this propoſition, is preſſed equally in all directions, and ſince the forces which preſs it are what keep it in its place, it follows that IH is preſſed as much as EI, or that theſe two equal drops at the ſame depth from the ſurface are preſſed with equal forces in all directions.

17. *The preſſure of a fluid upon the bottom or any part of the bottom of a cylindrical veſſel, the ſides of which are perpendicular to the horizon, is as the ſurface preſſed and as the perpendicular height of the fluid above it.*

The truth of this will be evident almoſt as ſoon as it is explaned. If a cylindrical veſſel ABCD, Plat. XVIII. fig. 2, the ſides of which are perpendicular to the horizon, is filled with water, the whole weight of the water contained in it, and no other force beſides that weight, preſſes upon the bottom CD. Therefore the preſſure upon the bottom is proportional to the weight or to the quantity of matter contained in the veſſel. But the contents of cylinders are as their baſes and perpendicular heights. Euc. b. XII. prop. 11, 14. Therefore the quantity of matter contained in the veſſel, and conſequently the preſſure againſt the bottom of it, will be as the baſe, that is, as the ſurface of the bottom, and as the perpendicular height of the fluid above it.

So likewiſe any part of the bottom, as OR, bears the whole weight, of the water above it and no more. Therefore the preſſure upon OR will be proportional to the quantity of matter contained in the column, that reſts upon it. And, ſince this quantity of matter is as OR the baſe of the column and as OF the height of it, the preſſure upon this part of the bottom will be as OR the ſurface preſſed and as OF the perpendicular height of the fluid above it.

18. *The preſſure of a fluid upon the bottom of any veſſel is equal to the preſſure upon the bottom of a cylindrical one, that has an equal baſe, and is filled with the ſame ſort of fluid to the ſame height.*

As a cylindrical veſſel is of the ſame breadth at the top and the bottom; the principal differences between ſuch an one and any other veſſel are, that the other may be either narrower or wider at the top than at the bottom. Therefore, to prove the propoſition, it will be neceſſary to conſider the preſſure on the bottom, firſt, when the veſſel is narrower at the top than at the bottom, and ſecondly, when it is wider at the top than at the bottom.

Firſt,

First, if the veſſel is narrower at the top than at the bottom, like the conical veſſel MNFT, Plat. XVIII. fig. 3. the preſſure upon FT the bottom, when the veſſel is full, as high as MN, is as great as the preſſure upon the bottom of a cylindrical veſſel, whoſe baſe is equal to FT, and whoſe height is FC. The veſſel ABCD, Plat. XVIII. fig. 2, is ſuch a cylinder. Therefore the propoſition requires us to prove, that the preſſure upon FT the bottom of the cone is equal to the preſſure upon CD the bottom of the cylinder, when they are both filled with water or any other fluid, notwithſtanding there will be more water in the cylinder than there is in the cone. If OR in the middle of FT is taken equal to OR in the bottom of the cylinder, then as there is an equal column of water over each of them, MNOR over one, and FGOR over the other, it is evident that both of them ſuſtain an equal weight, and conſequently the preſſure upon both is the ſame. It is likewiſe evident, from the laſt propoſition, that every part equal to OR at the bottom of the cylinder, is preſſed juſt as much OR is. And farther, I ſay, that every part equal to OR at the bottom of the cone is preſſed as much as OR is. In particular the preſſure upon FI is equal to the preſſure upon OR. For, by propoſition 16, all equal parts of a fluid at equal depths from the ſurface are preſſed equally. But the drops contiguous to FI and OR are at equal depths from the ſurfaces. Therefore theſe drops and conſequently the parts FI and OR, on which thoſe drops reſt, are preſſed equally. Since therefore every part equal to OR in the bottom both of the cone and of the cylinder is preſſed as much as OR is, and ſince one bottom is equal to the other, ſo that there will be juſt as many parts equal to OR in one bottom as there is in the other; it follows that the whole preſſure upon FT is equal to the whole preſſure upon CD.

It may ſtill be a difficulty with the reader, after he has ſeen that the propoſition is true, to find out the reaſon why it is true. For it certainly does not ſeem likely at firſt view that FI, with no more water over it than what fills the ſpace FEI, ſhould be preſſed as much as OR, which ſuſtains the whole column MNOR. But it muſt be remembered that the water FEI preſſes upwards, againſt FE, as well as downwards againſt FI, by propoſition 16. And the preſſure upwards may be eſtimated by the help of propoſition 12. For if there was a hole made at FE and a tube ſoldered into the hole, the water by the preſſure upwards againſt the tube would be ſupported at the ſame height that it ſtands at in the veſſel. Therefore this preſſure is equal to the weight of as much water as would fill the tube CAFE. Now, ſince the ſame preſſure,

which

which would ſupport the water in ſuch a tube, acts upon FE; and ſince CAFE is the difference between FEI and MNOR, the preſſure upwards againſt FE muſt be equal to the difference between the weights of water, which reſt upon FI and upon OR. But the reaction of FE downwards is equal to the action upwards againſt it, by the third law of motion, that is, FE keeps the water down with a force equal to that, with which it endeavours to riſe: and this preſſure or force, as has been proved, is equal to the difference of weight between FEI and MNOR. From whence it follows, that the weight of FEI and the reaction of FE together will be equal to the weight of MNOR. And ſince FI ſuſtains both the weight of water FEI and likewiſe the reaction or force with which that water is kept from riſing, but OR ſuſtains only the weight of water MNOR, the preſſure upon FI will be equal to the preſſure upon OR. In the ſame manner it might be proved that the reaction of ST, or force with which the water STV is kept down againſt the bottom, is equal to the weight of as much water as would fill the ſpace DBST. Therefore VT is as much preſſed by this reaction and by the weight of water SVT together, as OR is preſſed by the weight MNOR alone. And the reaction of the whole conical ſurface MFNT is equal to the weight of as much water as would fill the whole ſpace CMFBNT, or to the weight of as much water as makes the difference between the cone MNFT and the cylinder CBFT. Therefore the bottom of the cone is as much preſſed by the weight of water which fills the cone and by this reaction together, as the ſame bottom FT would be preſſed by the weight of as much water as would fill up the whole cylindrical ſpace CBFT; that is, the preſſure upon the bottom of a conical veſſel is equal to the preſſure upon the bottom of a cylindrical one of the ſame baſe and height.

The ſame ſort of reaſoning may be applyed to the veſſel DBLP, Plat. XVIII. fig. 4, which conſiſts of two cylindrical parts. NMLP a greater cylinder at the bottom, and DBIV a leſſer one at the top. For the preſſure upon LP, when the veſſel is full of water, will be as great, as if the veſſel was as wide at the top as it is at the bottom; that is, as great as it would be upon the ſame bottom LP ſuppoſing the veſſel was one uniform cylinder whoſe baſe was LP and height LF. LA and OR two equal drops at the ſame depth are preſſed equally, by propoſition 16. And OR having as much water to ſuſtain is as much preſſed, as if the cylinder was uniform or as wide at the top as it is at the bottom. Therefore LA or CP or any other equal part of the bottom, and conſequently the whole bottom, is as much preſſed in one caſe as it would be in the other. Indeed LA or CP have leſs water to ſuſtain than OR has: but the column NTLA preſſes up-

wards againſt NT with a force, which is equal to the difference between this column and DBOR, or to the weight of as much water as would fill FENT. For, by propoſition 12, if there was to be a hole made at NT and a tube FENT ſoldered into it, the preſſure againſt the bottom of the tube would ſupport water in it to the height NF, or to the ſame height it ſtands at in DBIV. Now, by the third law of motion, the reaction of NT downwards is equal to the preſſure upwards againſt it, that is, the force with which NT keeps the water below it down againſt LA is equal to the force with which this water preſſes againſt NT. Therefore LA is preſſed not only by the weight of water NTLA, but likewiſe by the reaction of NT, which is equal to the weight of as much water as would fill FENT, or would make NTLA equal to DBRO. From whence it follows, that the weight and reaction together, by which LA is preſſed, are equal to the weight DBRO, by which OR is preſſed. The ſame may be proved of CP and the reaction of SM; and of every other equal part of the whole bottom LP and of the whole cover NIVM. Therefore LP, by the weight of water above it and by the reaction of the cover together, is as much preſſed as if it was the bottom of a cylindrical veſſel FHLP having the ſame dimenſions at the top as at the bottom, and filled with water to the height LF.

Though the preſſure upon FT, Plat. XVIII. fig. 3, is equal to the preſſure upon CD, fig. 2, when both veſſels are filled with water to the ſame perpendicular height; yet, if they were filled with ice or any other hard body inſtead of water, CD would be more preſſed than FT. For CD whether the veſſel is filled with ice or water ſuſtains the whole weight of the body which reſts upon it, and no more. But FT, which beſides the weight MNFT ſuſtains the reaction of the ſides MFNT, when the veſſel is filled with water, will ſuſtain only the weight, when it is filled with ice. Becauſe that part of the preſſure, which was owing to the reaction, muſt neceſſarily ceaſe, when there is no reaction: and the ſides will not react, when there is no preſſure upwards againſt them. But ice or any other hard body does not preſs upwards, this is a property, which, as it ariſes from the nature of a fluid, belongs to fluids only. Therefore FT, when the veſſel is filled with ice, will be preſſed only by the weight, which it ſuſtains, and conſequently will be leſs preſſed than CD, in proportion as the cone is leſs than the cylinder, when their baſes and heights are equal; that is, the preſſure upon FT will be to the preſſure upon CD as 1 to 3. Euc. b. XII. prop. 10. For the ſame reaſon LP, Plat. XVIII. fig. 4, if it was full of ice, would be as much leſs preſſed than when it is full of water, as the quantity of matter contained in the compound veſſel, or veſſel made of

two cylinders DBIV and NMLP, is leſs than the quantity of matter that might be contained in a ſimple veſſel, or veſſel conſiſting of only one cylinder, whoſe baſe is LP and height LF.

The reader muſt remember that I here ſuppoſe the veſſel to be juſt filled with ice, when I affirm that the preſſure of ice in a veſſel is leſs than the preſſure of water would be. If he does not obſerve this caution, perhaps, upon recollecting that when the water freezes in a bottle, it frequently burſts the bottle, he might imagine that the preſſure of ice is greater than that of water. Now we affirm the preſſure of water to be greater than that of ice, only when a veſſel is juſt full of either: whereas if the water, which juſt fills a bottle, freezes, it ſwells in freezing, ſo that the dimenſions of it, when it is turned into ice, will be greater than they were before, and the ice burſts the bottle, becauſe it is too big to be contained in it.

The ſecond caſe of this propoſition is, when the veſſel is wider at the top than at the bottom. For in this caſe likewiſe the preſſure of any fluid upon the bottom of it is the ſame as in a cylindrical veſſel of an equal baſe and filled with the ſame ſort of fluid to the ſame height. The preſſure upon the bottom OR of the veſſel ABOR, Plat. VII. fig. 5, is the ſame, as if the veſſel had been a cylindrical one STOR having an equal baſe OR and the ſame perpendicular height ST with the veſſel ABOR. For the bottom OR in either caſe ſuſtains juſt the ſame quantity of fluid and conſequently the ſame quantity of matter. If it is the bottom of a cylinder then it ſuſtains no more than the column STOR, becauſe the veſſel holds no more. And if it is the bottom of an inverted cone, as ABOR, then it ſuſtains only the ſame column: for though the veſſel holds more than this, yet all the reſt of the fluid is ſupported by the ſides AO and BR, and therefore does not preſs the bottom at all.

Thus we find upon the whole, that whether a veſſel is narrower or wider at the top than at the bottom, the preſſure upon the bottom is the ſame as in a cylindrical veſſel of the ſame baſe and height. For when it is narrower at the top than at the bottom, though it holds leſs water than the cylindrical one would, yet the preſſure is not leſs, becauſe the reaction of the ſides ſupplies the defect: and when it is wider at the top than at the bottom, though it holds more water than the cylindrical one would, yet the preſſure is not greater, becauſe the ſides ſupport the exceſs.

44. *Though the preſſure upon the bottom of two veſſels, which contain different quantities of the ſame fluid, may be equal, yet their weights will be proportional to the quantities contained in them.*

The bottoms CD and FT, Plat. XVIII. fig. 2, 3, are equally preſſed, when both the veſſels are full of water, notwithſtanding ABCD contains three times as much water as MNFT, by propoſition 18. But ABCD will weigh three times as much as MNFT. This ſeems to be an objection to the truth of the foregoing propoſition. For, if the bottoms CD and FT are preſſed equally themſelves, ought not they, when the veſſels are ſet in a ſcale, each of them to preſs the ſcale equally? But if they were to do this, ſhould not they have the ſame weight? Therefore as they have not the ſame weight, how can they preſs the ſcale equally? And if their reſpective bottoms do not preſs the ſcale equally, how can they be preſſed equally themſelves? And thus at firſt ſight this propoſition ſeems to be ſo inconſiſtent with the former, that either one or the other muſt be falſe. To remove this objection and ſhew the truth of this propoſition, it will be neceſſary to recollect what the preſſure on the bottom FT ariſes from. It has been already obſerved that FT is preſſed by the weight of water in the veſſel and by the reaction of its ſides together. Now the reaction downwards being equal to the action upwards, in weighing the veſſel theſe two forces deſtroy each other, the veſſel is as much ſuſtained by one of them, as it is preſſed down by the other. And, as theſe two forces deſtroy each other, all that preſſes the ſcale, in which the veſſel ſtands, will be the weight of the water, which is contained in it. It may here perhaps be aſked, why we conſider both the reaction downwards and the action upwards, when we come to weigh the veſſel; but take notice of the reaction only, when we are eſtimating the preſſure upon the bottom? For does not there ſeem to be the ſame reaſon for taking both of them into the account in one caſe, that there is in the other? And if the reaction and action deſtroy each other, then the preſſure upon the bottom, as well as the weight of the veſſel, ſhould be proportional to the quantity of water contained in it. The anſwer to this is very obvious. In eſtimating the preſſure upon the bottom, we ought to take only the reaction downwards into the account, for the action upwards is againſt the ſides and does not affect the bottom at all. But in eſtimating the weight of the whole veſſel, we muſt take notice of all forces, that act upon any part of it, as well of the action upwards againſt the ſides, as of the reaction downwards upon the bottom.

20. *A given quantity of water, may be made to preſs with a force ſufficient to lift any weight whatever.*

As much water as will fill the tube DBIV, Plat. XVIII, fig. 4. preſſes upwards againſt VM with a force equal to the weight of as much water as would

would fill the whole ſpace BHVM, by propoſition 18. But if we conſider VM the baſe as invariable, then the ſpace BHVM will be as its height VB: and the weight of the water required to fill it, and conſequently the preſſure upwards againſt VM, which is equal to this weight, will be as its height VB; that is, the preſſure of as much water as will fill the tube DBIV, will always be proportional to the perpendicular height of the tube. But if we can contrive that IV the baſe of the tube ſhall diminiſh in the ſame proportion that BV the height of it is encreaſed, then the tube may be encreaſed to any length whatever, and yet the ſame quantity of water will fill it. But the preſſure againſt VM is as the perpendicular height of the tube. Therefore the preſſure of as much water as will fill the tube DBIV, or of any given quantity of water, may be made equal to any weight whatever.

If VM was made of leather, and the preſſure againſt it, when the tube BV is of a certain length, was juſt ſufficient to ſupport a pound weight laid upon VM ſo as to keep it from ſinking down the leather; then this preſſure would not ſupport a weight of 10 pounds laid upon VM. But if the tube was to be made ten times longer, and the bore of it IV was to be made ten times leſs, then the ſame quantity of water would fill it; and thus the preſſure of the ſame quantity of water, ſince it is as the perpendicular height, would be made ten times greater. It might be encreaſed in the ſame manner to any degree and be made equal to 100 or 1000 pounds or to any weight whatever.

21. *The preſſure upon the bottom of an inclined cylindrical veſſel is equal to the preſſure upon the bottom of one whoſe ſides are perpendicular to the horizon, if their bottoms are equal and the veſſels are filled with the ſame ſort of fluid to the ſame height.*

The preſſure upon NI, Plat. XVIII. fig. 6. the bottom of the inclined veſſel ABNI is equal to the preſſure upon IO the bottom of the perpendicular one EDIO, provided theſe bottoms are equal to each other, and the veſſels are filled to the ſame level AD with the ſame ſort of fluid. The only reaſon there can be to think otherwiſe is, becauſe AN the lower ſide of the inclined veſſel may ſeem to ſupport the water, juſt as an inclined plane ſupports any other heavy body. And if this was the caſe, ſince the two veſſels contain equal quantities Eucl. b. XII. prop. 11, 14. the bottom IN would be leſs preſſed, where part of the weight of water is ſupported by the ſide, than IO, where the whole has nothing to reſt upon but the bottom. But as the water is ſupported by the lower ſide AN, ſo it preſſes upwards againſt the ſide BI; and the reaction ariſing from

from hence adds as much preſſure to the water as AN takes off by ſupporting a part of its weight. Therefore, ſince the preſſure, which is made leſs than in proportion to the waters weight by one cauſe, is made juſt as much greater by another; it will upon the whole be as that weight or as the quantity of water contained in the veſſel ABNI. But ABNI and EDIO contain equal quantities: and the preſſure upon the bottom of each of them is proportional to their contents. Therefore the preſſure upon the bottom of each will be equal.

The nature of the veſſel ABNI is ſuch as to be in a manner compounded of the two veſſels ABOR and MNFT. Plat. XVIII. fig. 5, 3. In proof of the laſt propoſition it was obſerved, that AO fig. 5. ſupports as much water as reſts upon it, and ſo likewiſe does the ſide AN fig. 6. But though this takes off a part of the preſſure from the bottom OR and from the bottom NI. Yet it was farther obſerved in proof of the laſt propoſition that NT fig. 3. by its reaction adds to the preſſure of the water, and ſo likewiſe does BI. Therefore ſince the preſſure of the water is diminiſhed by the ſide AN, and juſt as much encreaſed by the equal ſide BI; this preſſure upon the whole will be the ſame, as if no effect had been produced by either ſide; that is, it will be as the baſe and height of the veſſel, which are equal to the baſe and height of the veſſel EDIO by the ſuppoſition.

If the veſſels ABNI and EDIO were filled with ice or any other hard body inſtead of water, the preſſures upon the bottoms would not be equal. For EDIO preſſes the bottom EO with its whole weight. But ABNI = EDIO preſſes NI with only part of its weight: for the reſt is ſupported by the inclined plane AN. And becauſe ice, as it is a hard body, does not preſs upwards, there is no reaction from the ſide BI to add as much preſſure as the inclined plane takes off.

22. *The preſſure upon the bottom of all veſſels whatever is proportional to their baſe and perpendicular heights.*

For the preſſure upon the bottoms of veſſels, either wider or narrower at the top than at the bottom, and likewiſe the preſſure upon the bottoms of inclined veſſels is equal to the preſſure upon the bottom of a cylindrical veſſel of the ſame baſe and height, the ſides of which are perpendicular to the horizon, by propoſitions 18, 21. And the preſſure upon the bottom of ſuch a cylindrical veſſel is proportional to its baſe and height, by propoſition 17. Therefore the preſſure upon the bottoms of all veſſels whatever is likewiſe proportional to their baſe and perpendicular height.

23. *The*

23. *The preſſure of a fluid upon every infinitely ſmall part in the ſide of the veſſel, that contains it, is equal to the weight of a column of the ſame fluid, whoſe baſe is the part preſſed and whoſe height is the diſtance of that part from the ſurface of the fluid.*

We have hitherto conſidered the preſſure of fluids upon the bottoms only of the veſſels, which contain them. Their preſſure upon the ſides of the veſſel is next to be eſtimated. But this cannot be done at once. We muſt firſt eſtimate the preſſure upon each infinitely ſmall part of the ſide; and then we muſt add the preſſures upon all the parts into one ſum to find out the preſſure upon the whole. At preſent we are concerned only about the firſt of theſe, about eſtimating the preſſure upon each infinitely ſmall part of the ſide. If L Plat. XVIII. fig. 7. is a point in the ſide of the veſſel ABCD, and if AB is the ſurface of the fluid contained in it; ſuppoſe that fluid to be water, then the preſſure upon L will be equal to the weight of a column of water whoſe baſe is L the part preſſed, and whoſe height is LA the diſtance of that part from the ſurface. For it is evident that the infinitely ſmall drop, which lies next to L is preſſed downwards by the weight of ſuch a column. And this drop is preſſed ſideways againſt L, as much as it is preſſed downwards, as has been proved in propoſition 16. Therefore L is preſſed with a force equal to the weight of ſuch a column. In like manner the infinitely ſmall drop contiguous to M is preſſed downwards by the weight of the incumbent fluid, that is, by a column whoſe baſe is the drop, and whoſe height is MA the diſtance of M from the ſurface. But as much as this drop is preſſed downwards, ſo much likewiſe it is preſſed ſideways againſt M. Therefore the point or infinitely ſmall part M in the ſide of the veſſel is preſſed by a force equal to the weight of a column, whoſe baſe is the drop, that is contiguous to it, or, which comes to the ſame thing, whoſe baſe is the part M, and whoſe height is the diſtance of that part from the ſurface. The preſſures upon N or C or any other point in the ſide of the veſſel may be eſtimated in the ſame manner.

Now, ſince the parts L, M, N, and C are equal to each other, becauſe they are infinitely ſmall, it follows that the columns, whoſe weights are equal to the preſſures againſt L, M, N, and C, will have equal baſes: and therefore the quantities, that they contain, or their weights, will be proportional to their heights. But their heights are the reſpective depths of L, M, N, and C below the ſurface. Therefore their weights, and conſequently the preſſures againſt L, M, N, and C, are proportional to the reſpective diſtances of theſe ſeveral parts from the ſurface. From hence

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we ſee the reaſon, why ciſterns and any other veſſels, which are deſigned to hold water, ſhould have their ſides made ſtronger towards the bottom than is neceſſary towards the top. For thoſe parts ſhould be the ſtrongeſt, which are to be preſſed by the greateſt force: and the lower parts of the ſide are always preſſed the moſt, becauſe they are at the greateſt diſtance from the ſurface.

If AC the ſide of the veſſel is inclined Plat. XVIII. fig. 8. the preſſure upon each infinitely ſmall part of it is to be eſtimated in the ſame manner, as if it had been perpendicular. And here indeed the truth of the propoſition is more obvious. For if AB is the ſurface, it is evident what force preſſes the ſeveral points L, M, N, and C: they are each of them preſſed by the weight of the water, that they ſuſtain. Thus L is preſſed by the column LS, M by the column MT, N by the column NV, and C by the column CX; that is, each of theſe infinitely ſmall parts is preſſed by the weight of a column, whoſe baſe is the part preſſed and whoſe height is the diſtance of that part from the ſurface.

24. *The preſſure of a fluid upon any plane whatever is equal to the weight of a body, which has the ſame denſity with the fluid, and is formed by raiſing perpendiculars upon each infinitely ſmall part of the plane equal in height to the diſtance of that part from the ſurface of the fluid.*

To determine the preſſure upon the ſide of a veſſel, it is neceſſary to collect the preſſures upon each infinitely ſmall part of it into one ſum: and here we are to explane the method of doing this. Let ABCD Plat. XVIII. fig. 7. be a cubical veſſel filled with water to the top. It has been proved already that the preſſure upon each infinitely ſmall part in the ſide AC is equal to the weight of a column of water whoſe baſe is the part preſſed and whoſe height is the diſtance of that part from the ſurface AB. From whence it follows, that if upon the point L a perpendicular LO be raiſed, whoſe baſe is L and whoſe height LO is equal to LA the diſtance of L from the ſurface, then, if this perpendicular conſiſted of matter of the ſame denſity with water, the weight of this perpendicular or column would be equal to the preſſure upon the point L. In the ſame manner let a perpendicular MP be raiſed upon the point M, and let the length of that perpendicular be equal to MA the diſtance of M from the ſurface; then, if this perpendicular was a column of ice or any other matter having the ſame denſity with water, its weight would be equal to the preſſure upon the point M. If perpendiculars were raiſed in the ſame manner upon every point between A and C; all of them together would fill up the area of the triangle ACD. And if all

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theſe perpendiculars were colums of ice, the ſheet of ice ACD would be the ſum of them all added together; and conſequently the preſſure upon the whole line AC, becauſe it is equal to the ſum of the preſſures upon all its parts, muſt be equal to the weight of this triangle ACD conſiſting of ice or of any other matter having the ſame denſity with water. But AC is not the ſide of the veſſel, it is only one line in the ſide: for the ſide of a cubical veſſel is a ſquare ſurface, which will indeed appear like a line, when the edge of it is towards the eye, but will appear in the form AECF, Plat.XVIII.fig. 9, when it is oblique to the eye. Now the preſſure upon the line AC is equal to the weight of the triangle ACD as has been proved already. In the ſame manner it may be proved that the preſſure upon the line HI is equal to the weight of the triangle HIK; and the preſſure upon EF to the weight of the triangle EFG. From hence it follows, that the preſſure upon the whole ſide will be equal to the weight of as many ſuch triangles, as there can be lines drawn upon it in the ſame manner that AC, HI, and EF are drawn. But all theſe triangles together would fill up the whole ſpace, or make up the mathematical ſolid, CFGDAE. Therefore the preſſure upon the ſide will be equal to the weight of this ſolid, ſuppoſing it to conſiſt of matter which has the ſame denſity with water. And this ſolid is formed, as the propoſition directs, by raiſing perpendiculars upon each point in each line of the ſide reſpectively equal to the diſtance of that point from the ſurface of the water. The ſum of all the perpendiculars upon one line make a triangle, and the ſum of all the triangles or of all the perpendiculars upon the whole ſide make the ſolid.

In like manner, if AC, Plat.XVIII. fig. 8, is a line drawn in the inclined ſide of a veſſel, in which the water reaches to the level AB; though the veſſel is not repreſented in the figure, but only a line in the ſide of it and the level of the water, yet the preſſure upon this line may be eſtimated as before. SL is the diſtance of L from the ſurface. Let therefore a perpendicular LO equal in length to LS be raiſed upon the point L; then, if this perpendicular was a column of ice or other matter of the ſame denſity with water, the weight of it would be equal to the preſſure upon L, by the laſt propoſition. For the ſame reaſon, if a perpendicular MP is raiſed upon the point M, and is made equal in length to MT the diſtance of M from the ſurface; ſuch a perpendicular conſiſting of matter of the ſame denſity with water, would have the ſame ſize and the ſame weight as the column of water MT. And ſince, by the laſt propoſition, the preſſure upon M equals the weight of the incumbent water MT, it likewiſe equals the weight of the perpendicular MP. The ſame may likewiſe

wiſe be ſaid of the points N and C. They are preſſed by the weight of the incumbent columns NV and XC, which is equal to the weight of the perpendiculars NQ, CR, ſuppoſing thoſe perpendiculars to be equal in height to NV and RC, and to conſiſt of matter whoſe denſity is the ſame with that of the columns NV and XC. Thus the preſſure upon the whole line, being made up of the preſſures upon all its parts, will be equal to the weight of as many perpendiculars, as can be raiſed in this manner between A and C. The ſum of all thoſe perpendiculars is the triangle ACR, whoſe weight therefore is equal to the preſſure upon the line AC. But if as many ſuch triangles were added together, as there are lines parallel to AC in the whole ſide of the veſſel, all theſe triangles together would form a ſolid. And ſince this ſolid is the ſum of all the preſſures upon each point of the ſide, the weight of it, ſuppoſing it to conſiſt of matter that has the ſame denſity as water, would be equal to the preſſure upon the whole ſide.

From hence we may prove, that when a cubical veſſel is filled with water the preſſure upon any one ſide of it is but half the preſſure upon the bottom of it. For the bottom ſuſtains a preſſure equal to the whole weight of water in the veſſel, as will eaſily appear from propoſition 17. And the preſſure, which the ſide ſuſtains is equal to the weight of the priſm CFGDAE, Plat. XVIII. fig. 9. But this priſm ſince it has the ſame baſe CFGD and the ſame height CA, that the cube has, is but half the cube. Euc. b. XI, prop. 28, and conſequently will weigh but half as much as the cube weighs. Therefore the preſſure againſt the ſide, which is equal to the weight of the priſm, will be but half the preſſure upon the bottom, which is equal to the weight of the cube.

This may likewiſe be proved in another manner, from propoſition 23. Let 5 points as A, L, M, N, and C be taken in a line AC, which is drawn down the ſide of a cubical veſſel and is perpendicular to the bottom of it. Then, if theſe points are at equal diſtances from each other, the preſſures upon them, reckoning from A downwards, will be reſpectively as 0, 1, 2, 3, 4. For the preſſure upon any point is proportional to its diſtance from the ſurface; and as A is cloſe to the ſurface, its diſtance from thence and conſequently the preſſure upon it=0. Call the depth LA 1, and the preſſure upon L=1. M is twice as deep below the ſurface as L is. Therefore the preſſure upon M will be twice the preſſure upon A, or will=2. N is three times as deep. Therefore the preſſure upon N=3. And for a like reaſon the preſſure upon C=4. From hence it follows that the preſſure upon the whole line AC being equal to the preſſures upon all its parts=0+1+2+3+4=10, ſuppoſing the whole

ſide

ſide conſiſted of no more than theſe 5 points. But if ſuch another line CD is drawn along the bottom and is perpendicular to the ſide, then this line CD will be equal to the line CA. For the bottom and ſide of a cube being equal ſquares, the reſpective heights CD and CA are equal. And if 5 points are taken in the line CD at equal diſtances from each with thoſe taken in the line AC, it is evident that each point in the bottom, being as far from the ſurface as C is, muſt be preſſed as much as C, or with a weight=4. Therefore the ſum of the preſſures upon all the 5 points, that is, upon the whole line CD, if it conſiſts of no more than 5 points, will be equal to 4+4+4+4+4=4×5=20. But the preſſure upon an equal line in the ſide =10 as has been proved. Therefore the preſſure upon a point in the line in the ſide being only=10 is but half the preſſure upon an equal line at the bottom, for this=20. Or if each of theſe lines conſiſts of ever ſo many points the proof will be the ſame. For the preſſure upon the line in the ſide will always be as the ſum of a number of terms in arithmetical progreſſion beginning from 0 and ending with the greateſt preſſure: and the preſſure upon an equal line at the bottom will always be as the ſum of the ſame number of terms each of which is equal to the laſt in that progreſſion. But the former ſum will be but half the latter. Therefore the preſſure upon a line in the ſide of a cubical veſſel filled with water will be only half the preſſure upon an equal line at the bottom. Now the ſide of a cubical veſſel being equal to the bottom of it, whatever number of lines ſuch as AC the ſide conſiſts of, the ſame number of ſuch lines as CD the bottom conſiſts of. And, ſince there are juſt as many lines in the ſide as there are in the bottom, the preſſure upon all the lines contained in the whole ſide will be to the preſſure upon all the lines contained in the whole bottom, as the preſſure upon one line in the ſide is to the preſſure upon one line in the bottom. Thus, ſuppoſe the ſide conſiſts of 5 ſuch lines AC, then the preſſure upon each of theſe lines being equal to that upon AC or to 10, the preſſure upon the whole ſide, will be 5×10=50. And ſince the bottom muſt be equal to the ſide, it will conſiſt of 5 ſuch lines as CD, each of which ſuſtains a preſſure=20=what CD ſuſtains, conſequently the whole bottom or all the 5 lines together are preſſed by a force=5×20=100. But 50 is to 100, as 10 is to 20. Therefore the preſſure upon the whole ſide being to the preſſure upon the whole bottom, as the preſſure upon one line in the ſide is to the preſſure upon one line in the bottom, it follows, from what has been proved already, that the preſſure upon any one ſide is half the preſſure upon the bottom.

A cubical veſſel has four ſides, and the preſſure upon one of them is half the preſſure upon the bottom. Therefore the preſſure upon two of

them is equal to that upon the bottom; and the preſſure upon all four of them is double that upon the bottom; and the preſſure upon the ſides and bottom together is triple that upon the bottom alone. But ſince the preſſure upon the bottom of ſuch a veſſel is equal to the gravity of the fluid contained in it, and ſince all the preſſure upon the ſides and bottom is produced by the fluids gravity, it follows that this gravity produces a preſſure three times as great as itſelf.

This propoſition is not confined to the preſſure upon the ſides of veſſels: it delivers a general rule for eſtimating the preſſure upon all planes whatever, upon the bottoms of veſſels as well as upon the ſides of them. And if we apply it to this purpoſe, we ſhall find that the preſſure will be the ſame, when determined by this rule, as it has already been proved to be. Thus, in propoſition 18, it has been ſhewn that when the veſſel MNTF is full of water, the preſſure upon FT, Plat.XVIII. fig. 3, is equal to the preſſure upon the bottom of a cylindrical veſſel full of water, or to the weight of a cylinder of water, whoſe baſe is equal to FT and its height equal to FC. But by the rule now delivered the preſſure upon FT will be equal to the weight of a ſolid formed out of matter having the ſame denſity with water, by raiſing perpendiculars upon every point in the ſurface FT equal in length to the diſtance of each point from the level of the water. The level of the water is CMNB, and ſome of theſe perpendiculars are FC, AI, OM, RN, VD and TB. Innumerable ſuch perpendiculars as theſe raiſed upon the circular ſurface FT would form a cylinder, whoſe baſe would be FT and height FC: and, if the matter of this cylinder had the ſame denſity with water, its weight would be equal to that of an equal cylinder of water. Therefore the preſſure eſtimated by this rule comes out equal to what we have already ſhewn it to be in propoſition 18. I need not be particular in applying this rule to any other caſes of preſſure upon the bottoms of veſſels: this inſtance is ſufficient to let the reader ſee the manner in which it is to be done.

Since then the preſſure of fluids upon any part either of the bottom or the ſide of a veſſel is equal to the weight of a column of the ſame fluid, which is formed by perpendiculars upon every point in the part that is preſſed equal in length to the diſtance of that point from the ſurface of the fluid in the veſſel, and which has for its baſe the part acted upon by the preſſure; it follows, firſt, that where the area of the part preſſed is given, the preſſure will be as the perpendicular diſtance of that part from the ſurface of the fluid: becauſe the area of the part preſſed is the baſe of the column, whoſe weight is equivalent to the preſſure, and this weight, when the baſe is given, is as the perpendicular height. And ſecondly,

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where the depth of the part preſſed is given, the preſſure will be as the area of that part: becauſe, the area of the part preſſed is the baſe of the column, whoſe weight is equivalent to the preſſure; and where the height or diſtance from the ſurface is given, the weight, and conſequently the preſſure equal to that weight, is as the baſe. Thus a ſquare inch, taken in the ſide of a veſſel 9 inches below the ſurface of the water contained in it, is preſſed nine times as much as a ſquare inch in the ſide taken only 1 inch below the ſurface. Or, if a ſquare inch is taken in the ſide of a veſſel 9 inches below the ſurface of the water and a ſquare inch 4 inches below the ſurface, the preſſure upon theſe two equal areas or parts will be as the perpendicular height of the water above them or as 9 to 4. But if 1 ſquare inch is taken in one ſide of a veſſel and 4 ſquare inches are taken at the ſame depth with the other any where elſe in the ſame or in any other of its ſides, then the preſſure upon theſe unequal areas or parts will be as 1 to 4 or as the areas themſelves, that is as the baſes of the two columns, whoſe weights are equivalent to the forces, that preſs them.

CHAP. III.

Of the action of fluids upon bodies immerſed in them, and the methods of comparing the ſpecific gravity of bodies.

25. *The weight, which a body retains, when it is immerſed in a fluid, is called its reſpective gravity.*

26. *When a hard body is immerſed in a fluid, the preſſures of the fluid againſt the oppoſite ſides of it are equal to each other.*

FOR a body immerſed in a fluid is preſſed in the ſame manner as the containing veſſel is. And therefore the oppoſite ſides of the body will be preſſed equally, becauſe they are at the ſame depth from the ſurface of the fluid, by propoſition 24. Thus the oppoſite ſides *rm* and *sn* of any body, as a piece of lead for inſtance, that is immerſed in the water ADBC, Plat. XVIII. fig. 10, are at the ſame diſtance from the ſurface of the water, and as the areas of theſe oppoſite ſides are equal, the preſſure upon them will be equal. This concluſion will be equally true, though the ſhape of the body immerſed is not ſo regular as what is here repreſented. For ſuppoſe there ſhould be a little piece of the lead at *m*, which hangs lower than the other parts of the bottom *mn*; then this little piece will

have two oppoſite ſides as well as the reſt of the maſs has: and thoſe oppoſite ſides are at the ſame depth, and their areas are likewiſe equal; for which reaſons the preſſure upon them will be equal. And in the ſame manner this demonſtration may be applyed, however deformed and irregular the ſhape of the body, which is immerſed, may happen to be.

27. *From the preſſure of a fluid againſt the ſides of a body, that is immerſed in it, no motion will be produced.*

A preſſure ſideways can never make the body move either upwards nor downwards: becauſe, by the ſecond law of motion, all motion is in the direction of the moving force, which produces it. But neither can theſe ſide-preſſures make the body move ſideways in any direction whatever: becauſe, by the laſt propoſition, the oppoſite preſſures on all ſides of the body are equal and conſequently deſtroy each other, ſo that no motion can follow from them.

28. *A body, that is immerſed in a fluid, is preſſed more upwards than it is downwards; and the difference of theſe two preſſures is equal to as much of the fluid as would fill the ſpace that the body fills.*

When any hard body, as a piece of lead, is immerſed in water, the lower part of it *mn*, Plat. XVIII. fig. 10, muſt continually be preſſed upwards, juſt as much as the water itſelf is preſſed upwards in the ſame place where *mn* the bottom of the lead is. Now the force, with which the water *mn* is preſſed upwards, is exactly equal to the force, with which it would be preſſed downwards if the piece of lead was out of the way: for every part of a fluid is preſſed juſt as much upwards as it is downwards, by propoſition 16. And the force, with which *mn* would be preſſed downwards, if the lead was out of the way, would be equal to the weight of the incumbent column or of as much water as would fill the whole ſpace EH*mn*; for this would all be filled with water, if the lead did not take up a part of it. Therefore the force, with which *mn* is preſſed upwards, and conſequently the force, with which the piece of lead is preſſed upwards, is equal to the weight of as much water as would fill the whole ſpace EH*mn*, or the whole ſpace HP*no*; if this ſpace HP*no* is taken equal to the other EH*mn*. Let us next conſider the force with which this piece of lead is preſſed downwards. This force is juſt equal to the weight of as much water as there is above it; that is, it is equal to the weight of the column EH*rs*. Therefore the difference between the two preſſures will be the difference of weights between the two columns HP*no* and EH*rs*: for the weight of the former is equal to the preſſure upwards, and the weight

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of the latter is equal to the preſſure downwards; that is, the preſſure upwards will be juſt as much greater than the preſſure downwards, as the weight of water HP*no* is greater than the weight of water EH*rs*. And the difference between theſe weights is juſt the difference between the quantities of water contained in theſe two columns. But this difference is juſt as much water as would fill the ſpace *rsnn*, which the body fills: for ſo much water added to EH*rs* would make it equal to HP*no*. Therefore the piece of lead is preſſed more upwards than it is downwards, by a force equal to the weight of as much water as would fill the ſpace, which the lead itſelf takes up.

The caſe will be the ſame, whatever is the figure of the body immerſed. For, ſuppoſe it to be a cone TSLV, Plat. XVIII. fig. 11, then, ſince by propoſition 16, every equal part of a fluid at the ſame depth is preſſed equally in all directions; if VI is equal to LV, it follows that theſe two parts of the thin lamina or ſheet of fluid FE will be preſſed upwards by equal forces. But, by the ſame propoſition, VI is preſſed as much upwards as it is downwards. Therefore LV is preſſed upwards as much as VI is preſſed downwards. Now the force, which preſſes VI downwards, is the weight of the fluid HPVI that is above it; and conſequently LV, where the bottom of the body is placed, is ſupported by a force equal to the weight of the column of fluid HPVI. And this column HPVI is equal to MHLV, becauſe their baſes VI and LV, are equal to each other, and their height VH is the ſame. Therefore the body is preſſed upwards with a force, that is equal to the weight of as much of the fluid as would fill the whole ſpace MHLV. The ſame body TSLV is preſſed downwards in the mean time by the weight of all the fluid that is above any part of it; that is, by the weight LTSVHM; and not merely by the column ONTS, which reaches from the ſurface to the top of the body. From hence it follows, that the difference between the entire column MHLV, or ſuch a column as this would be if the body was out of the way, and the column LTSVHM, is the difference between the preſſure upwards and the preſſure downwards. But this difference is plainly equal to as much of the fluid as would fill the ſpace which the body takes up. Therefore, upon the whole, the body is acted upon by two forces in contrary directions, but the force, with which the fluid acts upon it to make it aſcend, exceeds the force, by which it preſſes it downwards; and this exceſs is equal to the weight of as much of the fluid, whatever it is, as would fill the ſpace taken up by the body.

29. *A body, that is immerſed in a fluid will ſink, if it is ſpecifically heavier than the fluid.*

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It may not ſeem likely at firſt ſight, that any body, whatever its ſpecific gravity may be, ſhould ſink in a fluid, if the laſt propoſition is true. For why ſhould a piece of lead ſink in water, if the water preſſes it more upwards than downwards? Is not the lead acted upon by two forces in contrary directions; and ought it not therefore to move upwards, that is, in the direction of the ſtronger force? To this we may eaſily reply, that what is here objected would certainly be true, if the lead was acted upon by no other force beſides theſe two. The water preſſes it more upwards than downwards, and conſequently the action of the water would make the lead riſe, if there was nothing elſe to hinder it. But we muſt remember that the lead has weight; and if its weight is greater than the exceſs of preſſure upwards, this exceſs will not only be inſufficient to make the lead aſcend, but it will likewiſe be inſufficient to ſupport it ſo as to keep it from ſinking. The lead is acted upon by three forces, by the two preſſures in oppoſite directions and by its own weight. By the effect of the two firſt, if they acted alone upon it, the lead would be made to aſcend by the exceſs of preſſure upwards. But if the third force or weight of the lead, which always acts downwards, is greater than this exceſs, the lead will deſcend.

Now, whatever is the ſize or ſhape of a body immerſed in a fluid, it endeavours to deſcend by its own weight, and is ſupported by the exceſs of the preſſure upwards above the preſſure downwards; that is, by the laſt propoſition, it is ſupported by a force equal to the weight of an equal bulk of fluid, or of as much fluid as would fill the ſpace taken up by the body. If therefore the body is ſpecifically heavier than the fluid, that is, if, where the maſſes are equal, or bulk for bulk, the body is heavier than the fluid, then its weight is greater than the exceſs of preſſure upwards, which is to ſupport it, and conſequently this preſſure will not ſupport it ſo as to keep it from ſinking.

If a ſtone is thrown into the water it is preſſed by the water more upwards than it is downwards; but notwithſtanding this it will ſink in the water. Becauſe the ſtone endeavours to deſcend by its own weight, the exceſs of preſſure upwards is the weight of a maſs of water, that is juſt as big as the ſtone is. But the ſtone is ſpecifically heavier than the water, that is, where the bulks are equal, the weight of the ſtone is greater than the weight of the water. Therefore the force, with which it endeavours to deſcend, is greater than the exceſs of preſſure upwards, which is all there is to ſupport it; and conſequently this preſſure being too weak to ſupport it, will let it ſink to the bottom.

30. *A body, that is ſpecifically heavier than a fluid, may be ſupported in it by the preſſure upwards, if the preſſure downwards is by any means taken away.*

It has been ſhewn, in demonſtrating the laſt propoſition, that a body, which is immerſed in a fluid, is acted upon by three forces, two of which are the preſſures of the fluid upwards and downwards, and the third is the weight of the body itſelf. But if the fluid, which is above the body and preſſes it downwards, was by any contrivance to be removed; then there would be only two of theſe forces left, the preſſure upwards and the weight of the body. And if the body in theſe circumſtances was ſunk ſo deep in the fluid, as to make the preſſure upwards equal to its weight, then thoſe two equal and contrary forces would deſtroy each other and the body would be ſupported. Let EH*mn*, Plat. XVIII. fig. 10, be a glaſs tube open at both ends. Stop the end *mn* with your finger and pour as much mercury into it as will fill it to the height *ns*, ſuppoſe this equal to 1 inch. Then ſink the end *mn* in water to the depth of 14 inches: and if the tube is long enough for the higher end of it or EH to be above the ſurface, the upper part of the tube EH*rs* will be empty, ſo that there will be no water above the mercury. Therefore by this contrivance the preſſure of the water downwards upon the mercury is prevented; and the mercury, when your finger is taken away from *mn*, will endeavour to deſcend in the water by its own weight only. But, becauſe the bottom of the column of mercury is 14 inches below the ſurface of the water, the preſſure upwards againſt *mn* is equal to the weight of as much water as would fill the whole ſpace EH*mn*, or to the weight of 14 inches of water. Now the ſpecific gravity of mercury being to that of water as 14 to 1, it follows that the weight of 1 inch of mercury is juſt equal to the weight of 14 inches of water, by propoſition 5. Therefore the weight of the column of mercury, or the force, by which it endeavours to deſcend, is equal to the preſſure upwards, or to the force that ſupports it; and conſequently, though your finger is taken away, yet the mercury will not fall out of the tube. If the tube had been long enough and you had ſunk the end *mn* 15 inches below the ſurface, before you had taken away your finger, then the mercury would have been preſſed upwards by 15 inches of water. But 15 inches of water is heavier than 1 of mercury; and therefore the mercury would have riſen in the tube, ſince the preſſure upwards would have been greater than its weight. The mercury however would not riſe in this caſe to the top of the tube, but would ſtop, when the bottom of it was 14 inches from the top of the tube: for then the weight of it would be equal to the preſſure upwards, as before.

From hence it will be eaſy to underſtand a general rule for determining the depth, to which a body ſpecifically heavier than a fluid muſt be ſunk in it, in order to be ſupported, when the preſſure downwards is taken away. The diſtance of the bottom of the body from the ſurface of the fluid muſt be as much greater than the height of the body itſelf, as the ſpecific gravity of the body is greater than that of the fluid. Thus the diſtance of the bottom of the mercury *mn* from AD the ſurface of the water was ſuppoſed 14 inches and the height of the mercury itſelf *ns* 1 inch. But 14 inches are as much greater than 1 inch as the ſpecific gravity of mercury is greater than that of water. The reaſon of the rule is this. The body will be ſupported when the preſſure upwards is equal to its weight: and the preſſure upwards is always equal to the weight of a column of the fluid, whoſe height H*n* is the diſtance of the bottom of the body from the ſurface of the fluid. But theſe two weights will be equal, the weight of the body equal to the weight of the column of fluid, when their bulks are as their ſpecific gravities inverſely, by propoſition 5; that is, when the height of the column or diſtance of the bottom of the body from the ſurface is to the height of the body itſelf, as the ſpecific gravity of the body is to the ſpecific gravity of the fluid. If *rsmn* was a maſs of gold 1 inch high, then *mn* muſt be 19 inches below the ſurface of the water, for the gold to be ſupported by the preſſure upwards, when the water EH*rs* is removed, and the preſſure downwards taken away. Becauſe the ſpecific gravity of gold is to that of water as 1 to 19 inverted, that is, as 19 to 1. If *rsmn* was a maſs of ſilver 1 inch high, then, becauſe the weight of ſilver is to that of water, bulk for bulk, as 11 to 1, the depth H*n* to which the ſilver is ſunk muſt be 11 inches, and if EH*rs* the water, that is above it, is by any contrivance taken away, the preſſure upwards will ſupport the ſilver.

31. *A body, that is immerſed in a fluid, will riſe to the ſurface and ſwim upon it, if it is ſpecifically lighter than the fluid.*

This will be evident from what has been ſaid already under propoſition 29. A piece of cork, when it is immerſed in water, is preſſed by the water both upwards and downwards. But the preſſure upwards exceeds the preſſure downwards; and this exceſs is equal to the weight of as much water as has the ſame bulk with the piece of cork. Therefore as far as the action of the water is concerned, the cork ought to riſe to the ſurface. There is no other force beſides the weight of the cork that could poſſibly prevent its riſing; and this will not be ſufficient to prevent it, For, whilſt the cork endeavours to ſink in the water by its own weight,

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it is preſſed upwards by a force equal to the weight of an equal bulk of water. But cork is ſpecifically lighter than water; that is, where the bulks are equal the weight of cork is leſs than that of water. And conſequently the force with which it endeavours to ſink, is leſs than the force which buoys it up. Therefore the cork muſt move in the direction of the ſtronger force, or muſt riſe in the water, till it comes to the ſurface. From hence we may ſee the reaſon why ſome ſorts of wood ſuch as fir, oak, and elm, that are ſpecifically lighter than water, will ſwim in it, whilſt other ſorts, ſuch as ebony, guiacum, and lignum vitæ, that are ſpecifically heavier, will ſink in it. All metals will ſink in water, becauſe they are ſpecifically heavier than water. But all, except gold, will ſwim in mercury: for all, except gold, are ſpecifically lighter than mercury.

32. *Though a body, which is ſpecifically lighter than a fluid, will float upon it; yet there will always be a part of the body below the ſurface: and this part is equal in bulk to as much of the fluid, as would weigh what the body weighs.*

When a piece of cork floats upon water, a part of the cork is below the ſurface, and as much water, as is equal in bigneſs to the part immerſed, would weigh juſt as much, as the whole piece of cork weighs. Since the cork floats upon the water, there is no water above it, and conſequently no preſſure downwards. Its own weight is the only force that ſhould make it deſcend, and the preſſure of the water upwards is the force that counteracts this weight and is to ſupport it. Now, the preſſure upwards, againſt a given body is as the depth of the bottom of the body below the ſurface of the fluid, by propoſition 24. From whence it follows, that if the bottom *ei* of the cork *ptei*, Plat. XVIII. fig. 12, floated upon the ſurface, ſo that the whole cork ſhould be above the ſurface, there could be no preſſure upwards at all. Since this preſſure muſt be nothing, where the depth of the bottom of the cork below the ſurface is nothing. Therefore in this caſe there would be nothing to ſupport the weight of the cork, for which reaſon it could not be buoyed up or continue at reſt in ſuch a ſituation; but *ei* muſt ſink deep enough to make the preſſure upwards againſt it equal to the weight, which is to be ſupported. This, I imagine, will be ſufficient to ſhew the truth of the firſt part of the propoſition. For, ſince there is no preſſure upwards, that can buoy up the body, when the bottom of it is placed upon the ſurface of the fluid, a part of it muſt neceſſarily ſink below the ſurface.

We are next to determine the ſize of this part and to ſhew, as the ſecond part of the propoſition requires, that *smei* will be equal in bulk to as

much water, as would weigh what *ptei* the whole cork weighs. The force, with which the water at *ei* is preſſed upwards, is evidently equal to the force, with which it would be preſſed downwards if the cork *ptei* was out of the way. Becauſe by propoſition 16 every part of a fluid is preſſed equally in all directions. But the force with which *ei* would be preſſed downwards, if the cork was away, and conſequently the force with which *ei* the bottom of the cork is preſſed upwards, is equivalent to the weight of as much water as would fill the ſpace taken up by *snei*, or by the part of the cork that is below the ſurface. Therefore if the part which is below the ſurface has the ſame bulk or fills the ſame ſpace with as much water as would weigh what the whole cork weighs, then the preſſure upwards will be equal to the weight of the cork, and conſequently will be able to keep it from ſinking any lower.

The ſame thing may be otherwiſe proved. SQ is that lamina, plane, or thin ſheet of the water, upon which the body *ptei* is placed. And the body will not continue at reſt unleſs the part of this lamina *ei* contiguous to the body is preſſed juſt as much as any other equal part ſuch as *iv*. For neither the upper ſurface AB nor any other lamina of a fluid will reſt unleſs it is preſſed equally every where. But *ei* is preſſed by the weight of the whole body *ptei*, and *iv* is preſſed by as much of the fluid as fills the ſpace *nriv*, which ſpace is equal to *snei* the part of the body immerſed. Therefore, when the body reſts upon the fluid ſo as neither to ſink deeper nor riſe higher in it, the weight *nrvi* which preſſes *iv*, or the weight of as much water as is equal in bigneſs to *snei* the part of the body, that is below the ſurface, muſt be equal to the weight, which preſſes *ei*, or to the weight of the whole body.

Iron is ſpecifically lighter than mercury: but the quantity of mercury made uſe of may be too ſmall for the iron to float in it. Becauſe a part of the iron will be below the ſurface of the mercury; and there may be ſo little mercury, as to let this part immerſed ſtand upon the bottom of the veſſel, inſtead of buoying up the iron ſo as to make it ſwim. A boat, that is ſpecifically lighter than water, will not float upon it, unleſs the water is deep enough to keep the part of it, which will always be below the ſurface, from touching the bottom of the river. Upon this account it is, that a boat, when it is loaded, requires a deeper water to float it, than when it is empty. For the heavier the boat is, ſo much greater muſt be the part of it, that is immerſed below the ſurface. Becauſe this part is always equal in bulk to as much water as weighs what the boat weighs: and the weight of the boat and lading together, is greater than the weight of the boat alone. It is poſſible to load a boat ſo deep with wood

as

as to ſink it, notwithſtanding both the boat and the wood are ſpecifically lighter than the water. For the boat is ſunk, when the edges of it are below the ſurface: and though the ſpecific gravity of the boat is not made greater than that of the water by loading it with wood, yet the abſolute weight of the body to be floated is encreaſed. Therefore the part immerſed muſt be greater, when the boat is loaded, than when it is empty. And the lading may be encreaſed till this part is greater than the boat; by which means the edges of it will be below the ſurface, that is, the boat will be ſunk.

33. *A body, that is ſpecifically lighter than the fluid, into which it is immerſed, will not riſe in the fluid, if the preſſure upwards is prevented.*

If a piece of board is held down cloſe againſt the bottom of a veſſel, whilſt the veſſel is filled with mercury; then, as no mercury could get between the board and the bottom, the board cannot be preſſed upwards by the fluid, becauſe there is no fluid under it: and though the force, which held it down againſt the bottom is removed, yet it will not riſe, notwithſtanding it is ſpecifically lighter than the mercury. But if the edges of the board are raiſed a little from the bottom, ſo as to let the mercury get under it, then the preſſure upwards will buoy it up and make it float upon the ſurface. It is more difficult to try this with a piece of board held down againſt the bottom of a veſſel, whilſt it is filled with water: becauſe the water cannot eaſily be kept from inſinuating itſelf between the board and the bottom, ſo that the preſſure upwards can ſcarce be prevented.

The reaſon of what is here affirmed appears from propoſition 31. For a body, that is ſpecifically lighter than a fluid, is buoyed up in it, becauſe the preſſure of the fluid upwards exceeds the weight of the body. Therefore, if this preſſure is taken away or prevented, there is nothing to raiſe the weight of the body, and conſequently the body will continue at the bottom of the veſſel.

This propoſition may ſerve to prove that there is no ſuch property belonging to matter as abſolute levity, no property, by which bodies are made to riſe from the center of the earth in the ſame manner, as by the force of gravity they are made to fall towards it. For when bodies aſcend in fluids, which are heavier than themſelves, as cork in water, or wood in mercury, they aſcend by their comparative levity and not by their abſolute levity, they riſe from the center, becauſe they are lighter than the fluid, and not becauſe they have a tendency contrary to that of gravity.

They

They are buoyed up by the preſſure of the fluid, and not by any property of their own. This plainly follows from what has juſt been ſhewn concerning a piece of board in a veſſel full of mercury. For the board floated in the mercury by any abſolute levity in the board itſelf, it ought to float notwithſtanding the preſſure of the mercury upwards was taken away. Since removing the mercury from under it could not deſtroy any property, which belonged to the board itſelf.

Indeed the ſame thing may be ſhewn from another more familiar example. A piece of copper will aſcend in mercury: but it cannot be the abſolute levity of the copper which makes it aſcend; becauſe, if it was, the ſame property would make it aſcend in water. But ſince this or any other body immerſed in a fluid will aſcend, if it is ſpecifically lighter than the fluid, and will deſcend if it is ſpecifically heavier; it follows that the aſcent of a body in a fluid is not owing to the abſolute but to the comparative levity of it; not to its being light in itſelf, but to its being lighter than the fluid, into which it is immerſed.

34. *A body, that has the ſame ſpecific gravity with the fluid, into which it is immerſed, will reſt in any part of the fluid, wherever it happens to be placed.*

For the body endeavours to deſcend by its own weight, and is prevented from deſcending by a force equal to the weight of an equal bulk of the fluid. But, ſince the body and the fluid are ſuppoſed to have the ſame ſpecific gravity, equal maſſes of the body and the fluid have the ſame weight, and conſequently the force with which the body endeavours to deſcend and the force which prevents it from deſcending are equal to each other; and, becauſe they act in contrary directions, the body will reſt between them; ſo as neither to ſink by its own weight nor to aſcend by the preſſure of the fluid upwards.

We may now account for the well-known motion of little glaſs images in a jarr of water. The jarr has a proper quantity of water put into it, and is made of glaſs that the motion of the images may be ſeen through it: the images are hollow and being filled with air are ſpecifically lighter than the water, and conſequently will float at the top. There muſt be a bladder tyed over the mouth of the jarr; and by preſſing this bladder down with the hand, the images in the water below it will be made to ſink to the bottom of the jarr. Each of theſe images has a ſmall hole in ſome part of it, and by preſſing down the bladder the water below it is preſſed againſt this hole, ſo that the air in the image will be ſqueezed into a leſs compaſs and ſome of the water will get into the hole.

By

By this means the ſpecific gravity of the image is encreaſed; for it will certainly be heavier when there is both air and water in it, than it was when there was nothing in it but air. And as the image is contrived to be very little lighter than the water a very ſmall encreaſe of weight will be ſufficient to ſink it. When the force, which preſſed the bladder, is removed, then the water is no longer preſſed againſt the hole of the image; therefore the air within the image will expand itſelf and drive out the water, ſo as to reſtore the former ſpecific gravity of the image; that is, ſo as to make it again lighter than the water; for which reaſon it will riſe as the preſſure is taken off from the bladder, and will float at the top. If the bladder is preſſed again, whilſt the image is riſing, it may either ſtop in the place where it is, or ſink back again to the bottom. For if the preſſure is juſt ſufficient to ſqueeze ſo much water into the hole, as will make the image of the ſame ſpecific gravity with the water, then it will reſt in any part of the water, where it happens to be. But if the preſſure upon the bladder is greater than this, then the image becomes heavier than the water and cannot be ſupported by it.

Three or more images may be ſo contrived that only one of them ſhall ſink at a time. Thus if all three of them are lighter than water, they will all float. But they may all of them be lighter than water, and yet one of them be ſpecifically heavier than another. In this caſe by preſſing the bladder that image, whoſe ſpecific gravity is the greateſt, will have water enough ſqueezed into it, to make it ſpecifically heavier than the water, in which it floats, ſooner than either of the other two will. Therefore this image will ſink the firſt. By preſſing the bladder a little harder, water enough will be forced into another of the images to make that ſink. But the bladder muſt be preſſed ſtill harder to ſqueeze water enough into the lighteſt of the three images to make that follow the reſt. If, when they are all at the bottom, the preſſure is removed from the bladder at once, they will all begin to riſe at the ſame time. But if it is removed by degrees, the lighteſt, or that which ſunk the laſt, will riſe the firſt; and the heavieſt, or that which ſunk the firſt, will riſe the laſt of the three. For that, which required the greateſt force to ſink it, will become too light to be kept at the bottom, ſooner than the other two will. And all or moſt of the preſſure muſt be taken off from the bladder, before that which is the heavieſt will riſe to the top.

35. *All bodies, when they are immerſed in a fluid, loſe the weight of an equal bulk of that fluid.*

A piece of lead or of any other ſubſtance, when it is immerſed in water, is not ſo heavy, as when it is out of water. For, by propoſition 28, the

the water preſſes it more upwards than downwards, and the exceſs of preſſure upwards will ſupport a part of its weight. This exceſs was ſhewn, in the ſame propoſition, to be equivalent to the weight of as much water, as has the ſame bulk with the lead. And conſequently, ſince the body immerſed muſt loſe as much of its weight as the fluid can ſupport, the lead will loſe the weight of an equal bulk of water. A cubic foot of lead *rsmn* hanging to the ſtring LI will weigh leſs in the water, than it does out of it. Becauſe the water, by its preſſure upwards againſt the lead, will ſupport the weight of a cubic foot of water, or 1000 averdupois ounces, for ſo much a cubic foot of water weighs, and conſequently ſo much of its weight the lead muſt loſe.

A body endeavours to deſcend by its whole weight, but, according to what has been ſhewn in propoſition 28, when it is immerſed in a fluid, it is ſupported by a force equivalent to the weight of an equal bulk of that fluid. And, ſince theſe two forces act in contrary directions, the weight which the body retains in the fluid will be the difference between them. Therefore it loſes the weight of an equal bulk of the fluid.

This propoſition ſupplies us with another method of explaning what has been already demonſtrated concerning the ſinking or floating of bodies in water or any other fluid.

A cubic foot of copper weighs 9000 averdupois ounces, if it is weighed in the air, but in the water it loſes the weight of a cubic foot of water or 1000 ounces. Therefore it will ſink, becauſe notwithſtanding the preſſure upwards, there remains the weight of 9000—1000=8000 ounces, which has nothing to ſupport it. This will always be the caſe, when the body is ſpecifically heavier than the fluid, into which it is immerſed, The weight loſt by it, is the weight of an equal bulk of the fluid. But by the ſuppoſition the body is ſpecifically heavier than the fluid; for which reaſon the weight of it is greater than that of an equal bulk of the fluid: and conſequently, ſince the difference of theſe two weights has nothing to ſupport it, the body will ſink with a force proportional to that difference.

A cubic foot of cork weighs 240 averdupois ounces, if it is weighed in the air; but in water it loſes the weight of a cubic foot of water, or 1000 ounces. Therefore it loſes more weight than it has, or the force, which preſſes it upwards, is greater than its weight. Now to loſe more weight than it has is to acquire a negative gravity; that is, to be preſſed upwards with a force greater than its own weight is to be made to riſe. Therefore the cork will riſe with a force, which is proportional to the difference between the weight, that it is to loſe, and the weight, that it has.

Or

Or with a force proportional to 1000—240=760. But then it is plane that the cork will rise no higher in the water, after the pressure upwards is equal to its weight. A part of it will continue below the surface, which part will be equal in bulk to 240 ounces of water. For a part of the cork of this bigness loses in water the weight of 240 ounces: and, as this is just the weight of the whole cubic foot of cork, it loses just all its gravity and no more, when it is in this situation; and consequently it can sink no deeper, since it has lost all its weight, nor will it rise any higher, since it has lost no more than all. Therefore in this situation, and in this alone, the cork will continue at rest.

A bucket, which holds a cubic foot, weighs, when it is full of water, 1000 averdupois ounces; if we neglect the weight of the bucket itself and consider only the weight of the water contained in it. But this bucket, when it is below the surface of a well, weighs nothing, though it be full of water. For the water contained in it loses the weight of an equal bulk of water, or 1000 ounces, which is just all the weight, that it has. And consequently it will continue at rest in any place, where it happens to be. It cannot sink deeper than the place where it is, because it has lost all its weight; nor can it rise higher, because it has lost no more than all. This is not peculiar to water immersed in water: it is equally true of all bodies immersed in fluids of the same specific gravity with themselves. For the body will always lose the weight of an equal bulk of fluid. But this is precisely all its weight, where the specific gravities of the body and the fluid are the same. Therefore the body will continue at rest in any part of the fluid, where it shall happen to be.

From what has been said in this proposition and the foregoing ones, in regard to the pressure of fluids upwards and the loss of weight when bodies are immersed in them, we may understand whence that loss of weight proceeds. The weight we see is no otherwise lost, than as it is sustained by the action of a contrary force. And therefore we may now conclude upon principles, which have been demonstrated, as we did in proposition 9, upon principles, which we then took for granted, that, when a bucket is full of water, the weight of it cannot be perceived, whilst it is below the surface of the well, not because that weight is destroyed, but because it is supported, not because fluids do not gravitate, when they are in fluids of the same sort, but because there is a pressure in the contrary direction, which is exactly equal to their gravity.

Since the weight, which a body loses, when it is immersed in a fluid, is always the weight of as much of that fluid as is equal in bulk to itself; it follows, that the weight lost by the body cannot at all depend either on

the depth of the fluid itſelf, or on the depth, to which the body is immerſed in it. Becauſe as much of the fluid as is equal in bulk to the body will have exactly the ſame weight, whether the fluid is ſhallow or deep, and whether the body is immerſed juſt below the ſurface, or to any depth how great ſoever. An anchor loſes no more of its weight, when it is at the bottom, than when it is juſt below the ſurface; for in either caſe it loſes the weight of as much water as is equal in bulk to itſelf. It is not more eaſy to ſwim in deep water than in ſhallow, provided the water is not ſo ſhallow as to prevent one from ſtriking freely. For whatever is the depth of the water, a man loſes the weight of as much water as is equal in bulk to his own body: for which reaſon ſhallow water will buoy him up with as great a force as deep water would. Indeed it is eaſier to ſwim in the ſea, than in a river: not becauſe the ſea is deeper than the river, for it is eaſier to ſwim in a ſhallow ſea than in a deep river: but the reaſon is, becauſe ſalt water is ſpecifically heavier than freſh: and, ſince a man in the ſea loſes the weight of as much ſalt water as is equal in bulk to his body, and in the river loſes only the weight of an equal bulk of freſh water; the weight loſt in the ſalt water will be the greateſt, and conſequently ſalt water will buoy him up with the greateſt force.

36. *The weight, which is loſt by a body immerſed in a fluid, is communicated to the fluid.*

If the veſſel ADBC, Plat. XVIII, fig. 10, was ſet in a ſcale, and was balanced by an equal weight in the oppoſite ſcale; then upon dipping the body *rsmn* into the water contained in the veſſel, we ſhould find that the weight of the veſſel was encreaſed. For as the body loſes the weight of an equal bulk of water, ſo the veſſel, notwithſtanding the body is held up by the ſtring IL, would gain the weight, which the body loſes; that is, it will gain juſt as much weight by having the body dipped into it, as it would have done by pouring into it a quantity of water equal in bulk to the body.

The action of the water upwards ſuſtains the weight, which the body loſes. And ſince, by the third law of motion, the reaction of the body downwards will be equal to this action; it follows, that juſt as much weight as the water takes off by its preſſure upon the body, juſt ſo much the body adds to the water by its reaction. This may be otherwiſe expreſſed and perhaps more intelligibly. A part of the bodys weight is loſt, as was obſerved under the laſt propoſition, not becauſe it is deſtroyed, but becauſe it is ſupported. Now the weight, which the body loſes, is not ſupported by the ſtring LI, but by the water. And conſequently

this

this weight, which the water bears, muſt act upon whatever ſupports the water, that is, upon the veſſel that contains it, and upon the ſcale, in which the veſſel ſtands.

37. *The weights, which different bodies loſe, when they are immerſed in the ſame fluid, are proportional to their bulks.*

The weight, which gold or ſilver or lead or ſtones, or any other body loſes in water does not at all depend upon the ſort, or the figure, but upon the ſize of the body. A ſtone of ever ſo irregular a ſhape, and a cube of lead will loſe exactly the ſame weight in water, provided they are of the ſame bigneſs. For, by propoſition 35, the ſtone loſes the weight of an equal bulk of water, and ſo does the lead. Therefore ſince they are of the ſame bigneſs, they each of them loſe the weight of the ſame quantity of water; that is, they each of them loſe an equal weight. But if two bodies of the ſame ſort differ in ſize, they will loſe different weights in proportion to their ſize. For each of them loſes the weight of a bulk of water equal to itſelf: and the weights of different bulks or quantities of water are proportional to the quantities themſelves. So that each of the bodies will loſe a weight proportional to a bulk of water equal to itſelf, that is, a weight proportional to its own bulk. A cubic foot of lead loſes, when it is weighed in water, the weight of a cubic foot of water or 1000 ounces. And any other body of any ſort or of any ſhape, that is twice as big, will loſe the weight of two cubic feet of water, or 2000 ounces, that is, twice the weight which the other loſes. So that the two bodies loſe weights in proportion to their bigneſs.

From hence it will be eaſy to deduce a rule for determining the ſolid contents of any body, when we have found out the weight, which it loſes in water. For ſince a body, whoſe ſolid content is a cubic foot, loſes 1000 ounces of its weight in water, and all bodies loſe weight in proportion to their bulk, it follows, that the bulk of any body whatever will always be juſt as much greater or leſs than a cubic foot, as the weight, which it loſes in water, is greater or leſs than 1000 averdupois ounces.

If a piece of gold in one ſcale weighs juſt as much as a piece of copper in the other ſcale, when they are weighed in the air; then by putting them both, as they lie in the ſcales, into water, the gold will out-weigh the copper. This is not owing to any encreaſe of weight in the gold upon its being dipped in water, for that and the copper and all other bodies whatever loſe weight inſtead of gaining it by being in water. But ſince the gold and the copper have equal weights, therefore their bulks are inverſely as their ſpecific gravities, by propoſition 5, that is, the gold

will be as much leſs than the copper as the ſpecific gravity of gold is greater than that of copper. Now they are both of them to loſe weights in proportion to their bulks, when they are weighed in water. Therefore the gold, becauſe it is the leſſer of the two bodies, will loſe leſs of its weight than the copper does, and conſequently will out-weigh the copper. If the gold and the copper are made of the ſame weight, whilſt they are under water; then by drawing them out of the water into the air the copper will become the heavier. For when they were below water and their weights were equal, they each of them loſt as much of their weight as the water could ſuſtain, that is, each of them loſt a weight in proportion to its bulk. But the copper, being the bigger body of the two, loſt the greater weight. And, as the weights, which they loſt in water, are recovered upon their being drawn up into the air, the copper recovers a greater weight than the gold does, and therefore will out-weigh the gold.

This indeed is univerſal: for if any two bodies of different ſizes, which have equal weight in one fluid, are immerſed in any other fluid of a greater denſity or ſpecific gravity; then the leſs of the two bodies will become the heavier. For the ſame reaſon that the gold, which has the ſame weight with the copper, when they are both in the air, out-weighs it, when they are both in the water. But if any two bodies of different ſizes, which have equal weights in one fluid, are immerſed in any other fluid of a leſs denſity or ſpecific gravity, then the greater of the two bodies will become the heavier: for the ſame reaſon that the copper, which has the ſame weight with the gold, when they are both in water, out-weighs it, when they are both in air.

Upon this principle we may explane a very ſimple baroſcope, or inſtrument for finding the variations in the ſpecific gravity and in the denſity of the air. In the middle ſtate of the airs denſity, let a piece of gold in one ſcale be balanced againſt a piece of cork in the oppoſite ſcale, ſo that the ſcales may hang even. Then, if the denſity of the air encreaſes, this is the ſame as immerſing the two bodies in a fluid of a greater denſity than that, in which they were balanced; and conſequently the leſs body or the gold will become the heavier. But, if the denſity of the air is diminiſhed, this is the ſame as immerſing the bodies in a fluid of a leſs denſity than that, in which they were balanced; and conſequently the greater body or the cork will become the heavier. I mention gold and cork as two proper bodies for this purpoſe; becauſe they differ much as to their ſpecific gravities, and conſequently will have very different bulks, when their weights are equal. Upon which account the variation of their

weights

weights arising from any alteration in the airs denſity will be the more ſenſible.

38. *The weight, which a body loſes in a fluid, is to its whole weight, as the ſpecific gravity of the fluid is to that of the body.*

By finding how much weight a body loſes, when it is immerſed in a fluid, according to propoſition 35, we find the weight of as much of the fluid as is equal in bulk to the body. And if beſides this, we know the weight of the body, then theſe two are the weights of equal bulks. Now, by propoſition 2, the ſpecific gravities are as the weights of equal bulks. Therefore the ſpecific gravity of the fluid is to that of the body, as the weight loſt in the fluid is to the whole weight.

A certain piece of gold weighs 38 grains in the air. Now though this weight, properly ſpeaking, is not the true weight of the gold, becauſe by being weighed in air it loſes the weight of an equal bulk of air; yet, ſince this loſs is inconſiderable, we may neglect it. This piece of gold, when it is weighed in water, is found to weigh only 36 grains. Therefore it has loſt 2 grains. From whence, by the help of propoſition 35, we may reaſon thus. The gold loſes the weight of as much water as is equal in bulk to itſelf. Therefore as much water as is equal in bulk to the gold weighs 2 grains. But the gold itſelf weighs 38 grains. And conſequently, bulk for bulk, the weight of water is to that of gold, or the ſpecific gravity of the fluid is to that of the body, as 2 to 38, that is, as the weight loſt in the fluid is to the whole weight.

A certain piece of ſilver weighs 33 grains in the air; but upon being weighed in water is found to weigh only 30 grains. Therefore in water it loſes 3 grains out of 33. By this means I know, from propoſition 35, that a maſs of water equal in bulk to the ſilver weighs 3 grains, and I know before that the ſilver weighs 33 grains. So that, where the bulks are given, I find that the weights and conſequently the ſpecific gravities of water and ſilver are to each other as 3 to 33, or as the weight loſt by the ſilver in water is to its whole weight.

Having thus found that the ſpecific gravity of water is to that of gold as 2 to 38, and to that of ſilver as 3 to 33; if I would compare the ſpecific gravity of gold and ſilver together, it may eaſily be done, by reducing the expreſſion of the ſpecific gravity of water to 1 in both the proportions ſo as to make it a common ſtandard. Thus, I ſay, by the rule of three, as 2 is to 38, ſo is 1 to 19. Therefore the ſpecific gravity of water is to that of gold either as 2 to 38, or, which is the ſame proportion, as 1 to 19. Again as 3 is to 33, ſo is 1 to a fourth proportional, which

which will expreſs the ſpecific gravity of ſilver, when water is called 1, as well as 33 expreſſes it, when water is called 3. Now this fourth proportional is 11. Therefore the ſpecific gravity of water is to that of ſilver either as 3 to 33, or, which is the ſame proportion, as 1 to 11. Since therefore the ſpecific gravity of gold is to that of water as 19 to 1, and the ſpecific gravity of water to that of ſilver as 1 to 11. It follows that theſe three ſpecific gravities are to each other reſpectively as 19, 1, 11. So that the ſpecific gravity of gold is at laſt found to be to that of ſilver as 19 to 11. Thus likewiſe it will be in all caſes whatever, when water is made the ſtandard with which to compare other bodies, and its ſpecific gravity is reduced in this manner to 1. The ſame numbers which expreſs the ſpecific gravity of different bodies in reſpect of this common ſtandard, will expreſs the proportion that their ſpecific gravities bear to each other. Juſt as 19 and 11, which expreſs the proportion that the ſpecific gravities of gold and ſilver bear to that of water, when it is reduced to 1, expreſs likewiſe the proportion that their ſpecific gravities bear to each other.

Thus the whole art of comparing the ſpecific gravity of bodies, with that of water, conſiſts in finding out what the bodies weigh in air, and how much of that weight is loſt in water. And, when we have found the proportion, which the ſpecific gravity of water bears to the ſpecific gravities of ever ſo many different bodies, then to compare theſe bodies with each other, we have nothing more to do but by the rule of three to reduce the number, by which water, the common ſtandard, is expreſſed, to 1.

There is ſome difficulty in finding how much weight a body loſes in water, if it is lighter than the water, and will not ſink in it. But this may be done by the following contrivance. I have a piece of cork, which I find weighs 120 grains in air. And I have a piece of copper which weighs 450 grains. Therefore both of them together weigh 120+450=570 grains. When theſe two are tyed together with a horſe-hair, the maſs will be heavier than the water and will ſink in it. And by weighing this maſs in water, I find that it weighs no more than 20 grains. So that the whole maſs has loſt in water the difference between 570 and 20 or 570—20=550 grains. After this is done, by weighing the copper alone in water, I find that it weighs 400 grains, and conſequently loſes no more than 50 grains of its weight. From whence we may reaſon in this manner. The maſs of cork and copper together loſes 550 grains in water. But of theſe 550 grains, only 50 are loſt by the copper, therefore the other 500 are loſt by the cork. And when the weight, which is loſt by the cork, is known, we may proceed as before. The ſpecific gravity of

water

water is to that of cork, as the weight loſt in water is to the whole weight or as 500 to 120; that is, as 1 to 0,24, or 100 to 24, or 1000 to 240. In this experiment the copper and cork are tyed together with a horſe-hair; becauſe horſe-hair has very nearly the ſame ſpecific gravity as water, and therefore weighs nothing in water.

The ſame proceſs muſt be uſed when the body, whoſe ſpecific gravity is to be eſtimated, would diſſolve in water. Only in this caſe inſtead of tying a heavy body to it in order to ſink it, we make uſe of ſomething which will not diſſolve in water to caſe it with. Thus if I weigh a maſs of ſalt and a maſs of putty in the air. And by weighing the putty alone in water find how much weight it loſes. When I have caſed the ſalt with the putty and weigh them both in water, ſo as to find what weight is loſt by the whole maſs made up of both together. Then becauſe I know beforehand what weight the putty alone loſes, I am ſure that the reſt of what is loſt muſt be loſt by the ſalt. And as the weight loſt by the ſalt is to its whole weight, ſo is the ſpecific gravity of water to the ſpecific gravity of ſalt.

39. *The weights, which the ſame body loſes when it is weighed in different fluids, are proportional to the ſpecific gravities of thoſe fluids.*

For a body, when it is weighed in a fluid, loſes the weight of as much of the fluid as has the ſame bulk with itſelf. Therefore the ſame body, when it is weighed in different fluids, loſes in each of them the weight of its own bulk, that is, the weight of the ſame bulk of that particular fluid. And, ſince the ſpecific gravities are as the weights of equal bulks, by propoſition 2, it follows that the ſpecific gravities of the fluids are to each other in the ſame proportion as the weights, that are loſt in them by a given body.

A certain maſs of glaſs, when it is weighed in common water loſes 100 grains of its weight, but when it is weighed in ſea-water, it loſes 103 grains. Now in common water it loſes the weight of its own bulk of common water, and in ſea-water it loſes the weight of its own bulk, that is, of the ſame bulk of ſea-water. Therefore as much common water as is equal in bulk to the glaſs weighs 100 grains, and as much ſea-water as is equal in bulk to it weighs 103 grains. And, ſince the ſpecific gravities of theſe two ſorts of water are, as the weights of equal bulks, it follows that the ſpecific gravity of common water is to that of ſea-water as 100 to 103, or as 1 to 0, 03.

A certain piece of lead loſes 50 grains of its weight by being immerſed in water. The ſame piece of lead will loſe 43 grains by being immerſed in

in rectifyed ſpirits of wine. Therefore as much water as is equal in bulk to the lead weighs 50 grains: and an equal bulk of ſpirits, or as much of the ſpirits as is equal in bulk to the ſame lead, weighs 43 grains, by propoſition 35. And ſince, where the bulk is given, the weight of water is to that of ſpirits as 50 to 43, their ſpecific gravities are likewiſe in the ſame proportion. But here again in order to compare the ſpecific gravity of ſpirits with that of other bodies, whether fluid or hard, we muſt by the rule of three reduce the ſpecific gravity of water or the common ſtandard to 1. And as 50 is to 43, ſo is either 100 to 86, or 1 to 0,86.

The propoſition does not require that the body, which is weighed in the fluids, ſhould be ſpecifically heavier than they are. For their ſpecific gravities are proportional to the weight which is loſt in them, by any body of a given ſize, whether the ſpecific gravity of the body is greater or leſs than theirs. Only in practice it is eaſier and the proceſs more ſhort, if we make uſe of a body, which is heaver than the fluids, whoſe gravities are to be compared: becauſe ſuch a body will ſink in them of itſelf, without having any other weight tyed to it in order to ſink it. The proceſs to be made uſe of, if we would find the weight that a body which is ſpecifically lighter than a fluid will loſe in that fluid, has been explaned at large under the laſt propoſition.

40. *When a body, which is ſpecifically lighter than a fluid, floats upon the ſurface of it, the bulk of the part that ſinks below the ſurface is to the bulk of the whole body, as the ſpecific gravity of the body is to the ſpecific gravity of the fluid.*

Under propoſition 38, I explaned a method of comparing the gravity of a fluid with that of a body, which is ſpecifically lighter than the fluid. That method requires that the body ſhould have ſuch a weight tyed to it, as will be ſufficient to ſink it. But this propoſition points out a way of comparing their gravities, though the body floats upon the ſurface. If a piece of cork floats upon the water, the part of the cork that ſinks below the ſurface is juſt as much leſs than the whole maſs, as the ſpecific gravity of the cork is leſs than that of the water. For the part, that ſinks below the ſurface, is equal in bulk to as much water as weighs what the whole cork weighs, by propoſition 32. Therefore the part ſunk is to the whole, as the bulks of equal weights: the part is the bulk of a given weight of water, and the whole is the bulk of an equal weight of cork. But, by propoſition 5, the ſpecific gravities of bodies are as the bulks of equal weights inverſely. Therefore the part ſunk below the ſurface is to the whole, as the ſpecific gravity of water to that of cork when theſe gravities are inverted, that is, as the ſpecific gravity of cork to that of water.

This

This way of comparing the gravity of bodies with that of water, or of any other fluid, is not eaſily reduced to practice with any exactneſs. For it is very difficult to determine what proportion the bulk of the part, which ſinks below the ſurface, bears to the bulk of the whole body.

41. *If there are ſeveral fluids to be compared, and a given body, which is ſpecifically lighter than any of them, is made to float upon their ſurface; the parts of the body, that ſink below the ſurface in theſe different fluids, will be inverſely as their ſpecific gravities.*

A piece of cork will ſink deeper in ſpirits of wine than in water: and the part of it, which ſinks below the ſurface of the ſpirits, will be to the part, which ſinks below the ſurface of the water, as the ſpecific gravity of the ſpirits is to that of the water, when they are inverted, that is, as the ſpecific gravity of the water is to that of the ſpirits. The part, which ſinks below the ſurface in ſpirits, is the bulk of as much ſpirits as is equal in weight to the whole cork. And the part, which ſinks below the ſurface in water, is the bulk of as much water as is equal in weight to the ſame cork, by propoſition 32. Therefore theſe parts are to each other as the bulks of equal weights of ſpirits and water. But theſe bulks, and conſequently the parts of the cork that ſink below the ſurface, are, by propoſition 15, inverſely as the ſpecific gravities of ſpirits and water.

The method of comparing liquors with each other by means of the hydrometer, areometer, or water-poiſe, depends upon this principle. The inſtrument is well known amongſt thoſe who have occaſion to make groſs tryals of the ſtrength of wine, brandy, or other liquors. It is ſometimes made of ivory, but more frequently of glaſs. It is ſpecifically lighter than water and conſiſts of a round hollow ball, and a long ſlender neck or ſtem. At the bottom of the ball oppoſite to the ſtem is a drop of mercury, by the weight of which the inſtrument, when it ſwims in any liquid, will always have its ſtem upright. The ſtem is graduated, or divided into parts, by which the different depths that it ſinks to in different fluids may be noted. For the part of the ſtem, that ſinks below the ſurface, is ſo much the greater, in proportion as the fluid is lighter into which the inſtrument is put.

CHAP. IV.

Of the general properties of an elaſtic fluid.

42. *A fluid will be elaſtic, or compreſſible, if the particles, that compoſe it, repell each other with forces, which are inverſely as the diſtances between their centers.*

INDEED to make an elaſtic fluid it is only requiſite that the particles ſhould repell each other with a finite force, and that this repulſion, or force, with which they avoid and fly off from each other, ſhould encreaſe, when by any external force they are brought nearer together. For ſince the force, which keeps the particles aſunder, is finite, they may be ſqueezed cloſer together; and ſince, as they are made to approach each other, the repulſion between them encreaſes; it is evident that they will not continue ſo cloſe together unleſs they are held in that ſituation; but, when the force, which ſqueezed them together, is removed, their mutual repulſion will ſeparate them again to the ſame diſtances from each other, at which they were placed before they were compreſſed. The poſſibility therefore of compreſſing the fluid depends upon the mutual repulſion of the particles being finite, ſo that we may apply a force ſufficient to overcome it, ſo far as to bring the particles nearer together. And the elaſticity, or ſpring, with which the fluid reſtores or expands itſelf after the compreſſing force is removed, depends upon the encreaſe of this repulſion as the particles mutually approach each other.

Now this repulſion may encreaſe by very different laws. It may be inverſely as the diſtances, that is if the diſtances between the particles decreaſe in the proportion of 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. the repulſion may encreaſe in the proportion of 1, 2, 3, 4, &c. Or it may be inverſely as the ſquares of the diſtances, that is, if the diſtances decreaſe in the proportion of 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. the repulſion may encreaſe in the proportion of 1, 4, 9, 16, &c. Or it may be inverſely as the cubes of the diſtances, that is, if the diſtances between the particles decreaſe in the proportion of 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. the repulſion may encreaſe in the proportion of the cubes of thoſe numbers inverted or as 1, 8, 27, 64. &c. And a fluid will be elaſtic if the repulſive force encreaſes by any of theſe laws in the approach of the particles. We have ſingled out one of theſe laws from the reſt, and have ſuppoſed the particles to repell each other with forces, that are inverſely as the diſtances between their centers; not becauſe this peculiar law of repulſion is neceſſary to conſtitute an elaſtic fluid; but becauſe we ſhall find hereafter, that the particles of the air, which is the only elaſtic fluid we are concerned with, do repel each other by this law.

43.

143. *The particles of an elaſtic fluid when the fluid is compreſſed equally every where, will not continue at reſt, unleſs they are all of them at equal diſtances from each other.*

If the particles are nearer to each other in any one place of the fluid, than they are in the reſt, they will be repelled moſt on that ſide where the diſtances between them are the leaſt; and conſequently will move towards the other ſide, on which there is a leſs force acting upon them. This will always be the caſe, till they are acted upon by equal forces on all ſides. But, by the laſt propoſition, the repulſions cannot be equal on all ſides, till the particles are all diſpoſed at equal diſtances from each other.

Indeed, if one part of the fluid is compreſſed or ſqueezed together by a greater force than the other parts are, then the particles in that place, where the greateſt compreſſing force acts, will be kept cloſer together than they are any where elſe. Becauſe, though the repulſion is encreaſed by bringing the particles nearer, yet the additional compreſſing force which brought them together, will be ſufficient to keep them from flying aſunder again. This is the reaſon for ſuppoſing in the propoſition that the fluid is every where compreſſed equally.

It is almoſt needleſs to obſerve here, that an elaſtic fluid, when it is equally compreſſed in all its parts, will be every where of the ſame denſity. For, ſince the particles will diſpoſe themſelves every where at equal diſtances from each other, it is evident that the ſame number of particles can neither be ſqueezed cloſer together, ſo as to fill a leſs ſpace nor be more diffuſed ſo as to fill a greater ſpace in one part of the fluid than in another. But if there is the ſame number of particles or ſame quantity of matter within the ſame compaſs in every part of the fluid, then the denſity of it muſt be every where the ſame.

144. *The elaſticity or ſpring, with which a compreſſible fluid endeavours to expand itſelf, is equivalent to the force that compreſſes it.*

If the ſpring with which an elaſtic fluid endeavours to expand itſelf, when it is compreſſed, was leſs than the compreſſing force, then it would yield ſtill farther to that force ſo as to be ſqueezed cloſer together: and if its ſpring or elaſticity was greater than the compreſſing force, then it would not have yielded ſo far, nor have ſuffered itſelf to be reduced into ſo narrow a compaſs as it is. A compreſſing force greater than the fluids elaſticity will condenſe it or bring its particles nearer to each other, till their mutual repulſion becomes equal to that force. And an elaſticity greater than the compreſſing force will, notwithſtanding the action of

that force, expand the fluid or drive the particles farther aſunder, till by the encreaſe of diſtance between them their mutual repulſion becomes equal to the compreſſion, which acts upon the fluid. Therefore, when any force has compreſſed an elaſtic fluid, and keeps it within the compaſs to which it is reduced, ſo as neither to condenſe it more nor ſuffer it to expand itſelf, this force can neither be greater nor leſs than the ſpring or elaſticity of the fluid, and conſequently muſt be equal to it.

45. *An elaſtic fluid, when its ſpring is in equilibrio with the compreſſing force that acts upon it, preſſes equally in all directions.*

This has been proved univerſally, in propoſitions 15 and 16, of all fluids whatever, when they cannot be condenſed. But as unelaſtic fluids are incapable of being condenſed from the nature of them, ſo elaſtic ones cannot be condenſed in fact, when they are in equilibrio with the compreſſing force that acts upon them. Therefore elaſtic fluids as well as unelaſtic ones preſs equally in all directions.

46. *If a fluid conſiſts of particles, which repel each other with forces that are inverſely as the diſtances between their centers; the denſity of ſuch a fluid will be proportional to the force that compreſſes it.*

In order to prove the truth of this propoſition, I muſt deſire the reader to remember, what has been ſhewn already, under propoſition 43, that an elaſtic fluid is every where of the ſame denſity, or that the particles, which compoſe it, are every where at equal diſtances from each other. Now in conſequence of this we may ſhew farther that the denſity of ſuch a fluid is inverſely as the cube of the diſtance between the centers of its particles. Suppoſe two equal maſſes, two cubes for inſtance, of an elaſtic fluid, as A and B, Plat. XVIII, fig. 13. Then if the line *de* in the cube A contains 4 particles, it is evident that the whole ſquare ſurface *defg* contains as many particles as are equal to the ſquare of 4 or to $4\times4=16$. But in the whole cube there will be juſt as many ſurfaces equal to *defg* as there are particles in the line *eo*: and ſince there are as many particles in the line *eo* as in the line *de*, that is, 4; therefore there will in the whole maſs be 4 ſuch ſurfaces as *defg*. But as *defg* contains 16 particles, 4 ſuch ſurfaces or the whole cube will contain $4\times16=64$ particles. And ſince 64 is the cube of 4 it follows that the number of particles in the whole maſs A is equal to the cube of the number of particles in the line *de*. In like manner, if the line *hi* in the cube B contains 7 particles, then the ſurface *hilm* muſt contain $7\times7=49$, and the whole cube or 7 ſuch ſurfaces muſt contain $49\times7=343$. And, ſince 343 is the cube

of

of 7, it follows, that the number of particles in the whole mass B is equal to the cube of the number of particles in the line *bi*. But the denſity of theſe two equal cubes A and B will be proportional to the quantity of matter, that is, to the number of particles contained in them. And the number of particles in the maſs is equal to the cube of the number of particles in one line. Therefore the denſity of the body A is to the denſity of the body B as 64 to 343, or as the cube of the number of particles in the line *de* to the cube of the number of particles in the line *bi*. Now it is farther evident, that in any line of a given length ſuch as *de* and *bi*, the number of particles will be greater in proportion as they lie cloſer together, and leſs as they are farther aſunder; that is, the number of particles in ſuch a given line is greater in proportion as the diſtance between their centers is leſs, and leſs as that diſtance is greater. This is what we mean when we ſay that the number of particles in any given line of the maſs is inverſely as the diſtance between their centers. But if the number of particles in a given line is inverſely as the diſtance between their centers, then the cube of the number of particles in ſuch a line muſt neceſſarily be inverſely as the cube of the diſtance between their centers. And it has been ſhewn already that the denſity of A is to that of B as the cube of the number of particles in *de* to the cube of the number of particles in *bi*. Therefore the denſity of A is likewiſe to the denſity of B inverſely as the cube of the diſtance between the centers of the particles.

Having premiſed this we may now go on to prove, as the propoſition requires, that the denſity of a fluid, whoſe particles repel each other with forces that are inverſely as the diſtances between their centers, will always be proportional to the force that compreſſes it. We have ſhewn already that the denſity is inverſely as the cube of the diſtance between the centers of the particles: and if we can ſhew that the force, which compreſſes ſuch a fluid, muſt likewiſe be inverſely as the cube of the diſtance between the centers of the particles; then it will plainly follow, that the denſity muſt be proportional to the compreſſing force, becauſe they are both of them inverſely as the cube of the diſtance between the centers of the particles, which compoſe the fluid. Now the compreſſing force muſt be equal to the elaſticity, or ſpring of the fluid, by the laſt propoſition; that is, it muſt be equal to the force, with which all the particles of the maſs to be compreſſed repel each other; and not merely to the force with which any two particles repel each other. For compreſſing the fluid does not bring only two particles cloſer together, it ſqueezes all the particles, that are contained in the maſs, into a narrow compaſs; and conſe-

ſequently the force, which can do this, muſt be proportional to the joynt repulſion of them all. But in compreſſing any maſs of an elaſtic fluid ſuch as A is Plat. XVIII. fig. 13, every ſurface, that is contained in the maſs, is ſqueezed cloſer to the ſurface that lies next it: thus for inſtance, the ſurface *defg* is ſqueezed cloſer to the ſurface next beyond it, in which *s* is one particle: and any force, which compreſſes the fluid, muſt be proportional to the repulſion between theſe two or any other two ſurfaces in the maſs. Firſt, then the repulſion of the ſurface *defg* againſt the next beyond it, if all other circumſtances are equal, will be proportional to the number of repelling particles in that ſurface. And the number of particles in the ſurface *defg* is as the ſquare of the number of particles in the line *de*. But the number of particles in the line *de* is inverſely as the diſtance between their centers. Therefore the ſquare of the number of particles in the line *de*, that is, the number of particles in the ſurface *defg*, and conſequently the repulſion of this ſurface againſt the next beyond it, is inverſely as the ſquare of the diſtance between the particles. And ſecondly, where the number of particles in each ſurface is given, the repulſion of one ſurface againſt another, as they are brought cloſer together, will be inverſely as the diſtance between them. Becauſe each particle in one ſurface repells each particle in the other according to this law. And the diſtance between any two ſurfaces is the ſame as the diſtances between two particles, of which one is in one of the ſurfaces and the other is in the other: thus the diſtance between the ſurface *defg* and that, which lies next beyond it, is the ſame as the diſtance between the particle *e* which is in one of them and the particle *s* which is in the other. Since then the repulſion is inverſely as the diſtance between two ſurfaces, and this diſtance is proportional to the diſtance between two particles; it follows that the repulſion between two ſurfaces is inverſely as the diſtance between two particles. Thus we have ſeen that upon the firſt account the repulſion in the maſs of fluid is inverſely as the ſquare of the diſtance between the centers of the particles; and that upon the ſecond account it is inverſely as that diſtance itſelf. Therefore, upon both accounts together, it is inverſely as the ſquare of the diſtance and inverſely as the diſtance, that is, it is inverſely as the ſquare of the diſtance multiplyed into the diſtance, or inverſely as the cube of the diſtance between the centers of the particles.

Since therefore the force, that compreſſes the fluid, muſt be proportional to the force of elaſticity or ſpring of the fluid, and ſince this ſpring or repulſion is inverſely as the cube of the diſtance between the particles, provided the repulſion between any two particles which lie next each other, is inverſe-

verſely as the diſtance between them; it follows, that where this is the law of the repulſive force, the compreſſing force muſt be inverſely as the cube of the diſtance between the particles. But the denſity of an elaſtic fluid is inverſely as the cube of the diſtance between the particles, as has juſt been proved. Therefore in a fluid, where this is the law of the repulſive force, the denſity is proportional to the compreſſing force.

CHAP. V.

Of the Air.

45. *By the atmoſphere is meant, all the air, which encompaſſes the earth.*

IN the word atmoſphere we commonly include ſomething beſides the air. For there are many bodies of very different ſorts conſtantly floating in the air, and all theſe, in the common uſe of the word, are conſidered as parts of the atmoſphere.

46. *The air has weight; and this weight is what ſupports the mercury in the Torricellian experiment.*

The air certainly has weight, becauſe a veſſel is heavier, when it is full of air, than it is after the air is exhauſted. The effect of this weight appears in ſeveral inſtances. Firſt it appears in the inſtance of two hollow braſs hemiſpheres, which, being made to fit each other exactly ſo as to form a ſphere, will adhere ſo ſtrongly to each other, when the air is exhauſted out of this ſphere, that a very great force will be requiſite to part them. When there is no air within, the preſſure of the air from without ariſing from the weight of it, is what keeps theſe hemiſpheres ſo firmly united. Secondly; if all the vents of a pair of common bellows are cloſe ſtopped, we ſhall find great difficulty in opening them; which difficulty ariſes from the weight of the air, that preſſes upon the boards of the bellows, and may be removed by unſtopping the vents ſo as to let the air get into the bellows as we open them: for then the preſſure of this internal air is a balance to the weight of what preſſes upon the boards, as we ſhall ſee more planely hereafter. But thirdly, the effect of this weight appears very remarkably in the Torricellian experiment. Making the Torricellian experiment is only making a common barometer or weather-glaſs. It is called by this technical name from Torricelli, who was the inventor of it. The glaſs tube made uſe of in the experiment muſt be about 36 inches long, and muſt be hermetically ſealed at one end. It is thus ſealed by heating the end of it in a lamp, till it grows ſoft and is

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ready to melt and then by twiſting it cloſe together with a pair of pinchers, ſo as to make the ſoft glaſs entirely ſtop up the orifice. The chemiſts ſhould tell us the reaſon why they call this ſealing hermetically or putting on Hermes's ſeal. Perhaps it means no more than ſealing chemically; and may be called ſealing hermetically upon a ſuppoſition that Hermes Triſmegiſtus was the inventor of chemiſtry. When the tube is thus prepared, the ſealed end of it is held downwards, and mercury is poured into the open orifice of it, till the tube is full. You then ſtop this orifice with your finger and invert the tube ſo as to have the ſealed end uppermoſt, and having a baſon of mercury FG ready, Plat. XIX. fig. 8, you plunge the end that is ſtopped with your finger below the ſurface of the mercury in the baſon. When this is done, you take away your finger, and part of the quickſilver will fall out of the tube into the baſon, ſo as to leave the top C quite empty for about 6 inches. The reſt of the mercury, which, becauſe the tube is 36 inches long, will be a column of about 30 inches, continues ſuſpended in the tube.

Before I explane the reaſon why this mercury is ſupported, I muſt deſire the reader to remember, that there is not the leaſt air in the 6 inches, which are left by the mercury at the top of the tube. For till the finger that cloſed the orifice in the baſon was taken away, theſe 6 inches were filled with mercury: and when by removing the finger that orifice was opened, and the mercury, which was 36 inches high before, ſunk in the tube, ſo as to ſtand at the height of only 30 inches, and left thoſe 6 inches empty, it was impoſſible any air ſhould ſucceed in its place. Becauſe no air could get into the tube; unleſs it was poſſible for it either to penetrate through the ſealed end of the tube, or to force its way through the mercury in the baſon and the 30 inches of mercury beſides, that remain ſuſpended. But, as neither of theſe can be done, it follows, that in that part of the tube, which the mercury leaves, there is a vacuum. This and any other vacuum, which is made in the ſame manner, is called a Torricellian vacuum: and this name ſerves to diſtinguiſh it from a vacuum that is made with an air-pump, which, from Mr. Boyle the improver of the air-pump, is called a Boylean vacuum. But the Torricellian vacuum is diſtinguiſhed from the Boylean not only by the different manner of making it, but by the perfection of it. For it is plane that in the Torricellian vacuum there is not the leaſt air, whereas there always will be ſome air left in the Boylean vacuum after pumping ever ſo long. The reaſon why all the air cannot be perfectly exhauſted out of a veſſel by means of an air-pump will appear, when we come to explane the ſtructure of an air-pump and the manner in which it works.

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The mercury in the baſon will not continue at reſt, unleſs all the parts of it are preſſed equally. For as mercury is a fluid, its parts yield to every impreſſion: and conſequently, if there is a greater force preſſing upon one part of the ſurface than upon another, the part, which is preſſed the leaſt, will aſcend, till the equality of preſſure is reſtored. Now the reſt of the ſurface of the mercury in the baſon is expoſed to the air, and has the weight of the air reſting upon it. Whereas that part, which is under the tube C or D or E, has no air above it, and conſequently would not be preſſed equally with the reſt, unleſs there was ſo much mercury in the tube as is equivalent to the weight of air, which preſſes upon any other equal part of the other ſurface. Thus, for inſtance, if all the mercury was to fall out of the tube C, there would be a vacuum over that part of the ſurface FG, which the tube C covers. Therefore, as much of the mercury in the baſon, as is under this tube, having no air above it would not be preſſed at all. Whereas any other part of the ſurface FG, which is equal to what C covers or to the baſe of the tube C, is preſſed by the weight of a column of air, whoſe baſe is equal to the part preſſed upon or to the baſe of the tube C, and whoſe height is the height of the atmoſphere. Now in theſe circumſtances two ſuch parts of the ſame ſurface FG could not be in equilibrio or could not continue at reſt. For one of them is preſſed by the air, and the other is not preſſed at all. Therefore the mercury will riſe into the tube C, till the weight of it becomes equivalent to the weight of air, which preſſes any other part of the ſurface FG equal to the baſe of the tube C or to the part which this tube covers. But the ſame cauſe, which would raiſe the mercury back into the tube, if it had all fallen out, will prevent it from falling out at firſt. So that it is the weight of the air preſſing upon the ſurface of the mercury in the baſon, which keeps 30 inches of mercury ſuſpended in the tube, when the finger is taken away from the open end of it. And, ſince 30 inches of mercury reſting upon that part of the ſurface, which is under the tube, produces a preſſure equivalent to the weight of the air upon any other equal part of that ſurface; it follows, that where the baſes of the columns or parts of the ſurface, that are preſſed by thoſe columns, are given, a column of mercury 30 inches high weighs as much as a column of air, whoſe height is equal to that of the atmoſphere.

That this preſſure or weight of the atmoſphere is the true cauſe why the mercury is ſupported, appears from what happens when this preſſure is removed. Suppoſe a barometer with the mercury ſuſpended at the uſual height of 30 inches was placed in the receiver *oooo* of an air-pump, Plat. XX. fig. 1. Now beſides the barometer, which is ſuppoſed to be

in the receiver, there is commonly joyned to the air-pump as a part of it a tube *nl* open at both ends. This tube is called the mercurial gage. It has one end at *u* in the receiver, and the other in a baſon of mercury *mm*. Whilſt the air is exhauſting from the receiver by means of the pump, the mercury ſinks in the barometer and riſes in the gage; till the air is all exhauſted: and then there will be little or no mercury in the tube of the barometer, which is ſuppoſed to be in the receiver, for it will all have ſunk out of the tube into the baſon. In the mean time the mercury will have riſen from *mm* the baſon of the gage into the tube *ll* where it will ſtand at the height of about 30 inches. Firſt then we muſt obſerve, that all the mercury keeps ſinking out of the barometer in the receiver, whilſt the air is exhauſting, and when there is no air left, no mercury is ſupported. Since therefore 30 inches of mercury are ſuſpended, whilſt the air preſſes freely upon the baſon, and ſince the height of it decreaſes, as that preſſure grows leſs, and none of it is ſupported, when that preſſure is quite taken away; it ſeems very obvious to conclude that the weight of the air is the cauſe that ſupports it. This concluſion is ſtill farther confirmed upon re-admitting the air into the receiver; for the mercury which had before fallen out of the tube into the baſon is immediately raiſed into the tube again, ſo as to ſtand at the height of 30 inches, as it did before the receiver was exhauſted. Secondly, let us conſider what happens to the mercury in the baſon *mm* and the gage *ll*. Before any air is exhauſted from the receiver there is no mercury in the gage: becauſe the air which preſſes upon the other parts of the mercury, preſſes equally through the open tube *ll* upon that part which is immediately under the tube. Therefore the ſurface of the mercury *mm*, being preſſed equally every where, will continue level, and no one part of it will riſe higher than the reſt. But as the receiver is exhauſted, the preſſure of the air from within it upon that part of the ſurface *mm*, which is under the tube *ll*, will be conſtantly decreaſing, whilſt the reſt of the ſurface is preſſed as much as uſual. Upon which account that part of the mercury under the tube will keep riſing. And when the receiver is well exhauſted, there will be a Boylean vacuum in the receiver and the top of the gage, for which reaſon the mercury will then ſtand in it nearly as high as in a common barometer. I ſay nearly as high, becauſe the reader will remember, that the Boylean vacuum is an imperfect one: and ſince upon this account there muſt be always a ſmall quantity of air at the top of the gage, this will keep the mercury from riſing ſo high as it uſually ſtands in the barometer; becauſe in the Torricellian vacuum, at the top of the tube of a barometer, there is no air at all. That we have here aſſigned the true cauſe

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why the mercury riſes and is ſupported in the gage, appears upon re-admitting the air into the receiver: for then the air preſſes again as much upon the ſurface of the mercury in the tube *ll* as upon any equal ſurface in the baſon *mm*: and if the inequality of preſſure before was the cauſe which ſupported the mercury, upon theſe preſſures being made equal it ought to fall out of the gage into the baſon, and ſo upon tryal it is found to do.

I have been the more particular in explaning this matter, that my reader may be the better able to underſtand an experiment which Mr. Azout made with a double barometer *gb*, Plat. XIX. fig. 1. This experiment I ſhall now deſcribe, after obſerving that of theſe two barometers the upper one *ge* anſwers to the barometer in the receiver, as made uſe of in the other experiment; and the lower one *cb* anſwers to the mercurial gage. The only difference is this; in the experiment already explaned the upper barometer is put into a Boylean vacuum; and Azouts experiment is a contrivance for putting the upper barometer *ge* into a Torricellian vacuum. In the experiment already mentioned the gage or lower barometer, when the receiver is exhauſted, has a Boylean vacuum at the top of it; and the lower barometer in Azouts experiment has a Torricellian vacuum inſtead of it. The experiment is as follows, *bcib* is a bent tube open at both ends, which is ſcrewed into the glaſs globe *cdef* by means of a braſs collar at *c*. *gfe* is a ſtrait tube open likewiſe at both ends, which is ſcrewed into the ſame globe in the ſame manner at *f*. A wet bladder is tyed over the orifice *b*: and when the whole inſtrument is filled with mercury quite up to the orifice *g*, then another wet bladder is tyed over this orifice, which anſwers the purpoſe of ſealing it hermetically. If the tube *bc* is more than 30 inches long, and the tube *ge* is likewiſe more than 30 inches long, then the reader, I imagine, will eaſily underſtand that the whole inſtrument conſiſts of two barometers. *ge* is the tube and the glaſs globe is the baſon of the upper one; and *bcb* is the tube and *ba* is the baſon of the lower one. They are not indeed barometers, whilſt the inſtrument is quite full of mercury, and the bladder tyed over the orifice at *b*. For as each of the tubes is more than 30 inches long, and both of them are full of mercury, it is plane that the mercury in each of them is at a greater height than it ever ſtands at in a barometer: and it is kept at this height by the bladder at *b*, which confines it in the tubes and prevents it from ſinking. But if the lower baſon *ba* has ſome mercury put into it, and the bladder at *b* being immerſed below the ſurface of this mercury is cut full of ſlits with a ſmall penknife, then the mercury will begin to fall out of the inſtrument at the orifice *b*. And firſt the

whole tube *gfe* will be left empty. Then the mercury will run out of the glaſs globe through *b* the orifice of the lower tube, till the ſurface of it has ſunk below this orifice; and then no more can run out, but the remainder will be left at the bottom of the globe as in the figure. Laſtly, the mercury will ſink in the tube *beb* till it ſtands at the height of 30 inches. Thus there will be no mercury left in the upper barometer; but there will be 30 inches in the lower. Now *ge* is the tube of the upper barometer, and the lower part of the glaſs globe, with the remainder of mercury in it, which could not run out at the orifice *b*, is the baſon of this barometer. But ſince the upper part of the globe was left empty by the mercurys running out from thence, it follows that in this upper part above the ſurface *id* there is a Torricellian vacuum. Therefore the upper barometer *ge* has a vacuum over its baſon; juſt as the upper barometer in the former experiment had by being put into an exhauſted receiver. The lower barometer *bb* like the gage in the former experiment has an exhauſted receiver above it; only the gage here, for ſuch we may call it, has a Torricellian vacuum in *bde* its receiver, whereas in the receiver of the former experiment there was a Boylean vacuum over the gage. The event of this experiment is juſt the ſame as that of the former. For upon unſcrewing the tube *gfe* at *f* ſo as to let the air into the vacuum in the upper part of the globe, the mercury riſes to the height of 30 inches in the upper tube *eg*; and ſinks entirely out of the lower tube *bb*. Juſt as in the former experiment, upon letting the air into the receiver, the mercury roſe to the height of 30 inches in the barometer, which was placed in the receiver; and at the ſame time all the mercury ſunk out of the gage, which had been raiſed into it, whilſt the receiver was exhauſting. Both theſe experiments are exactly the ſame and ſucceed in exactly the ſame manner, ſo that we may draw the ſame concluſion from either of them. Upon this account Azouts experiment ſeems to be quite uſeleſs ſince the improvement of the air-pump: for it proves no more than what we can prove without it; and the other experiment is much more eaſy both to make and to underſtand than his is.

I have all along called the height of the mercury in the barometer 30 inches; but if the weight of the atmoſphere is the cauſe, which ſupports it, then in different ſtates of the atmoſphere this height muſt vary: it will be greater when the weight of the atmoſphere is greater, and leſs when that weight is leſs. Nor indeed is the middle height 30 inches but about 29½. I called it 30 only for the ſake of having a round number.

From this propoſition it appears that a column of mercury 30 inches high is equal in weight to the preſſure of the atmoſphere upon any ſur-

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face, which is equal to the base of that column. And, in propoſition 5, we took notice, that a column of mercury 30 inches high has juſt the ſame weight with a column of water 35 feet high, if their baſes are equal. Therefore if a vacuum is made in ſyringes, ſiphons, or pumps, by exhauſting the air out of them, the preſſure of the atmoſphere will raiſe water into them, and will ſupport it when raiſed, to the height of 35 feet.

47. *The air is an elaſtic fluid.*

The nature of an elaſtic fluid, as it has been already explained in the foregoing chapter, is, that the parts of it may be ſqueezed cloſer together, and that, when the force, which thus brings them nearer to each other, is removed, the fluid will again expand itſelf and fill the ſpace, which it took up before. In order to prove that the air is a fluid of this ſort we need not have recourſe to the condenſer, an engine, to be deſcribed hereafter, by which a greater quantity of air may be forced into any veſſel, than that veſſel uſually contains; nor to the air-pump, which, as it exhauſts the air from a receiver, leaves the remainder in the veſſel leſs denſe than common air is. For any one, by preſſing a blown bladder between his hands, will find that this force will reduce the air contained in the bladder into a leſs ſpace, and as ſoon as he ceaſes to preſs it, that air will immediately expand itſelf and fill the ſame ſpace as uſual.

48. *The elaſticity or ſpring of the air is equivalent to the force that compreſſes it.*

This has been proved already, in propoſition 44, of elaſtic fluids in general. The reader will be able without any aſſiſtance to apply what is there ſaid to the air in particular.

The ſpring of the air will produce the ſame effects that the preſſure of it produces: becauſe the ſpring is equivalent to the compreſſing force, or to the weight of the incumbent atmoſphere, that is, to the preſſure of the atmoſphere. Barometers are not always made in the manner that has been deſcribed under the foregoing propoſition. For the open orifice of the tube, inſtead of being put into a baſon of mercury, is ſometimes turned up into the ſhape of a crook, and the open end of it terminates in a large bubble, which contains the ſtagnant mercury and ſo ſupplys the place of a baſon. Now if this bubble was to be cloſe ſtopped up or to be ſealed hermetically, ſo that the outward air could not preſs upon the mercury, which is contained in it, yet the mercury in the tube would ſink no lower, on account of the preſſure of the atmoſphere being excluded. Becauſe

the

the ſpring of the air, that is included in the bubble above the ſurface of the mercury, being equal to the preſſure of the atmoſphere, is likewiſe equal to the weight of the mercury in the tube and will ſupport it.

Let a hole be bored length-ways through a cork, and a glaſs tube open at both ends be thruſt into the hole: then ſqueeze the cork hard into a glaſs bottle, that has its bottom covered with water deep enough for the lower end of the tube to reach below the ſurface of it: and laſtly cover the top of the cork with cement ſo as to prevent any air from paſſing by the ſides of it. The outward air preſſes by its weight through the tube upon that part of the water, which is under it; and the air included in the bottle preſſes by its elaſticity upon the reſt of the ſurface. But the whole ſurface continues level; that part of it which is within the tube neither riſes higher nor ſinks lower than the reſt. Therefore this ſurface muſt be preſſed equally every where: from whence it follows that the ſpring of the air, that is included in the bottle, is equal to the weight of the external air, that preſſes through the tube. But if this bottle was to be ſet into a receiver; whilſt the air is exhauſting, the water would riſe into the tube. For the ſpring of the included air is equal to the weight of the incumbent atmoſphere in its natural ſtate; but as the air is exhauſted, this weight conſtantly decreaſes: and conſequently the ſpring of the included air will preſs more upon the other parts of the ſurface, than the weight of the rarifyed air in the receiver preſſes upon that part which is immediately under the tube.

A barometer, when there is a vacuum at the top of it, ſtands at the height of 30 inches. But if there is any air in the upper part of the tube, then the mercury will not ſtand ſo high. The weight of 30 inches of mercury is equal to the preſſure of the atmoſphere: and when there is no air in the upper part of the tube, the weight of the mercury is the only counterpoiſe to that preſſure: and therefore 30 inches of mercury will be ſupported. But if there is air in the top of the tube, then the counterpoiſe from the tube againſt the preſſure of the atmoſphere is the weight of mercury and ſpring of the included air together: for which reaſon ſo great a weight of mercury cannot poſſibly be ſupported, as would be, if there was nothing elſe but that weight to counteract the external preſſure. If it ſhould be aſked why any mercury at all is ſupported in this caſe, ſince the elaſticity of the included air is alone equivalent to the preſſure of the atmoſphere; and conſequently theſe two forces being equal ought to balance each other, and if they did, no mercury at all ought to be ſupported? In anſwer to this enquiry, the reader muſt remember, that, as the mercury ſinks out of the tube, the ſpace at the top

of

of the tube is enlarged, and the included air, expanding itſelf ſo as to fill that ſpace, becomes leſs elaſtic. Therefore the ſpring of this air, when it is expanded, will not of itſelf be a counterpoiſe to the external preſſure; but as much mercury muſt be ſupported in the tube as is equal in weight to the difference between the ſpring of the included air, when it is expanded, and the preſſure of the external atmoſphere. Now, it appears from this propoſition that the elaſticity of the air is always equivalent to the compreſſing force. But we have ſeen that the expanded air at the top of the tube has leſs elaſticity than common air; and conſequently the compreſſing force muſt be leſs. Let us therefore enquire how it comes to be leſs. The uſual compreſſing force, that acts upon the air in the lower parts of the atmoſphere, is the weight of the incumbent column of air, of a column, whoſe baſe is the part preſſed, and whoſe height is the height of the atmoſphere. This weight is equal to 30 inches of mercury, as was proved in propoſition 46. The preſſure from without againſt the baſe of the tube is the weight of the atmoſphere, or a weight equal to 30 inches of mercury. But the air at the top of the tube is not compreſſed by all this weight; for there is ſome mercury in the tube, ſuppoſe 20 inches; and the weight of this is a counterpoiſe to 20 parts in 30 of the external preſſure. Conſequently the difference between the preſſure of the air from without and of the mercury from within, that is, in the caſe ſuppoſed, the difference of weight between 30 and 20, or $30-20=10$ inches of mercury, is all the force, with which the external air acting againſt the baſe of the tube compreſſes the air included in the top of it; for all the reſt of the external preſſure is balanced by the weight of mercury in the tube.

49. *The ſpace, which any given quantity of air fills, is inverſely as the force that compreſſes it.*

The air at the bottom of the atmoſphere, that is, the air, which is near the ſurface of the earth, ſuſtains the weight of all the reſt that is above it, quite to the top of the atmoſphere. This weight is the force, that compreſſes it, and is equal, when the part compreſſed is given, to the weight of a column of mercury 30 inches high: for, in the laſt propoſition, it was proved that ſuch a column of mercury, when the baſe is given, weighs as much as a column of air, that reaches from the bottom of the atmoſphere to the top of it. Thus we ſee that the air, which we breathe, is in its ordinary ſtate compreſſed by a weight equivalent to 30 inches of mercury. Now, if the compreſſing force is made greater than this, the air is ſqueezed cloſer together, and a given quantity of it will

fill

fill a leſs ſpace; but if the compreſſing force is made leſs it expands itſelf, and the ſame quantity will fill a greater ſpace. We are to prove, that this ſpace is inverſely as the compreſſing force, that is, that the ſpace is leſs in the ſame proportion as the compreſſing force is greater, or greater in the ſame proportion as the compreſſing force is leſs. This will appear to be the caſe, in one inſtance at leaſt, if we can ſhew, when a certain quantity of air is preſſed by a weight equal to $3\times30=90$ inches of mercury, that is, with a weight equivalent to 3 atmoſpheres or with 3 times the ordinary compreſſing force, that then it will be ſqueezed into a ſpace 3 times leſs than uſual, or into $\frac{1}{3}$ of the ſpace that it commonly fills. And if we can ſhew likewiſe, when a certain quantity of air is preſſed by a weight equal only to 10 inches of mercury, which is but $\frac{1}{3}$ of the atmoſpheres weight or of the ordinary compreſſing force, that then it will expand itſelf ſo as to fill 3 times the ſpace that it commonly fills.

Let there be a bent tube *nkig*, Plat. XIX. fig. 2, which is open at *n*, and hermetically ſealed at *g*. Pour a little mercury into it, ſo as juſt to fill *ki*. Then *gi* is filled with common air, that is, with air compreſſed by the weight of the atmoſphere, which weight is equivalent to 30 inches of mercury. If more mercury is poured into the open orifice *n*, the weight of this mercury is an additional compreſſing force upon the air contained within *ig*: and this additional force ſqueezes it cloſer together and makes the mercury riſe into the leg *ig*. It cannot riſe ſo high in the leg *ig*, as it ſtands in the other leg *kn*: becauſe the air in *ig* prevents it; and this air cannot be driven out at *g* where the tube is ſealed. But the weight of the column *bi* in one leg is a counterpoiſe to the weight of an equal column *kl* in the other, by propoſition 12. Therefore all the additional weight, that preſſes the air, is the weight of *lm* a column of mercury, which is the difference of its height in the two legs *gi* and *nl*. Now if this column *lm* is 60 inches high, it is equivalent to the weight of two atmoſpheres. So that the air in *ig*, which bears the weight of the atmoſphere and the weight of this mercury beſides, is preſſed by 3 times a greater force than uſual; for both theſe weights are equivalent to 3 atmoſpheres, and the air is commonly preſſed by the weight of only 1. And in this caſe the air, which at firſt filled the whole ſpace *ig*, is reduced into the ſpace *bg* or $\frac{1}{3}$ of *ig*. Therefore by tripling the compreſſing force the ſpace, which the air fills, is reduced to $\frac{1}{3}$: that is, the ſpace filled is inverſely as the compreſſing force. Here we muſt obſerve that the ordinary compreſſing force is 1 atmoſphere or 30 inches of mercury, and that the compreſſing force in the experiment is *lm* added to 1 atmoſphere or to 30 inches of mercury. And it is

found

found that not only in the particular inſtance, which I have given, but univerſally, that as much as 1 atmoſphere or the ordinary compreſſing force is leſs than 1 atmoſphere added to *lm*, or than the compreſſing force in the experiment, ſo much greater is the ſpace *ig*, which the given quantity of air commonly fills, than the ſpace *bg*, which it fills in the experiment; that is, the ſpace filled by the air decreaſes in the ſame proportion that the compreſſing force encreaſes, or is as the compreſſing force inverſely.

But this is not enough to prove the truth of the propoſition, unleſs we can ſhew farther, that the ſpace filled by the air encreaſes likewiſe, when the compreſſing force is diminiſhed, ſo as to be in this caſe as well as in the former inverſely as that force. The experiment, by which this is ſhewn, is a little more perplexed than the other, and ſomething more attention will be requiſite in order to underſtand it. Let the ſealed end of a tube 50 inches long be held downwards, and mercury be poured into it till it is filled within 10 inches of the top: then, if the open orifice is ſtopped with the finger, there will be 10 inches of air between the finger and the ſurface of the mercury, in that part of the tube, which was not filled with mercury. This air, when the tube is inverted, will riſe to the top, that is, to the ſealed end, for this by inverting the tube becomes the upper end. Thus if *eb*, Plat. XIX. fig. 3, is the tube, 10 inches *ef* at the top of it will be filled with air, as long as the orifice *b* is cloſed with the finger. But if, upon immerſing *b* into the mercury contained in the baſon *a*, the finger is taken away, the mercury will ſink as in the Torricellian tube; only in the preſent caſe it muſt ſink lower than it does in the barometer. Becauſe there is no air at the top of the barometer, but in this experiment there is enough to fill the ſpace *ef*. And, as this included air endeavours to expand itſelf, it will preſs upon the ſurface of the mercury, and will prevent it from ſtanding at ſo great a height as it does, when there is a vacuum at the top of the tube. Let *db*, equal 30 inches or the ſtandard height in the barometer. Then the mercury in this experiment will ſtand only at the height *cb* equal to 20 inches. And the included air expanding itſelf will fill the whole ſpace *ec* from the top of the tube to the ſurface of the mercury, which ſpace is 30 inches. For if *eb* the whole length of the tube=50 inches, and *cb* the height of the mercury=20 inches, then *ec*=50—20=30. We have therefore in this experiment the ſame parcel of air in two different ſtates of compreſſion: in one of them it fills the ſpace *ef* or 10 inches; in the other it fills *ec* or 30 inches. Let us next enquire what the forces are, which compreſs it in theſe two different ſtates, and what proportion they bear to each other.

When the air fills *ef* or 10 inches, it is in its natural ſtate; it is nothing elſe but common air, or air of its uſual denſity, and therefore is compreſſed by the weight of the atmoſphere, that is, by a weight equivalent to the weight of 30 inches of mercury, or of as much mercury as would fill the ſpace *bd* which is the ſtandard height of the barometer. But when the ſame parcel of air fills the ſpace *ec* the compreſſing force is leſs than the weight of the atmoſphere. The whole weight of the atmoſphere does indeed preſs upwards againſt *b* the baſe of the tube; but yet the air in the top of it is not compreſſed by this whole weight. For there is a counter-preſſure downwards from the mercury *cb* within the tube, which takes off a part of this weight. Therefore the force, with which the atmoſphere acts upwards to compreſs the air in the top of the tube, is the difference between its own weight from without, and the weight of the mercury from within, or the difference between 30 and 20, that is 10 inches of mercury, or the difference between *db*=30 inches, and *cb*=20 inches, which difference is *dc*=10 inches. Upon the whole the ordinary compreſſing force is equivalent to the weight of a column of mercury *bd* 30 inches high; and the compreſſing force in this experiment is equivalent to the weight of a column of mercury *cd* 10 inches high. Now when the compreſſing force is equal to 30 inches of mercury the air fills 10 inches of the tube; and when the compreſſing force is equal to 10 inches of mercury the air fills 30 inches. Therefore, when the compreſſing forces are to each other as 30 to 10, the ſpaces filled by the air are to each other as 10 to 30, or as theſe forces inverted. The caſe is the ſame in all other inſtances. For as much as *bd* the ſtandard height of the mercury, or ordinary compreſſing force, is greater than *cd* the difference between the ſtandard height and the height in this experiment, or than the compreſſing force in the experiment, ſo much leſs is the ſpace *ef*, which a given parcel of air ordinarily fills, than the ſpace *ec* which it fills in the experiment. Therefore, univerſally, the ſpace which the air fills encreaſes as the compreſſing force diminiſhes, or the ſpace which any given quantity of air fills is always inverſely as the force that compreſſes it.

50. *The denſity of the air is proportional to the force that compreſſes it.*

The ſpace, which a given quantity of air fills, has been proved in the laſt propoſition to be inverſely as the compreſſing force; or, which amounts to the ſame thing, the compreſſing force is inverſely as the ſpace which a given quantity of air fills. And it is evident, from propoſition 5, that the denſity of a given quantity of air is likewiſe inverſely as its bulk,

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that is, inverſely as the ſpace filled by it. From hence it follows that the compreſſing force and the denſity of the air muſt be proportional to each other: becauſe they are both of them inverſely as the ſpace, which a given quantity of it takes up. Thus a double compreſſing force produces a double denſity in the air, a triple compreſſing force produces a triple denſity, and ſo on, the denſity always encreaſing as the compreſſing force encreaſes.

From this propoſition it appears, that if the air conſiſts of particles which repel each other, and if its elaſticity is owing to their mutual repulſion, then this repulſion is inverſely as the diſtances between the centers of the particles. For it has been proved, in propoſition 46, that if an elaſtic fluid conſiſts of particles that repel each other, their repulſion muſt be inverſely as the diſtance between them in order to make the fluids denſity proportional to the force that compreſſes it. And ſince the denſity of the air has now been ſhewn to be in this proportion; it follows, that this muſt be the law, by which the particles that compoſe it, mutually repel each other; upon a ſuppoſition, that the airs ſpring or elaſticity is owing to the repulſive force, by which the particles, that compoſe it, avoid each other.

This ſuppoſition is now generally admitted; as a repulſive force between the particles ſeems to be more natural, and accounts better for the great expanſion, of which air is found to be capable, than the old opinion, that its elaſticity is owing to the figure and ſhape of its particles. I ſhall explane this opinion and then leave it with the reader to make his own conjectures, whether this or the other is the more probable. If the particles of air conſiſt of ſeveral branches, which will yield as other elaſtic bodies would do, and when the force, which bent them is removed, would reſtore themſelves; or if they conſiſt of an elaſtic ſubſtance, like the ſpring of a watch, and each of theſe ſprings is rolled up into a ſmall ſphere, this figure and quality of the particles will make them capable of being compreſſed or ſqueezed into a leſs compaſs, and of expanding themſelves upon removing the force that compreſſed them, ſo as to fill the ſame ſpace that they filled before.

The air in a diving-bell is compreſſed by the weight of the atmoſphere, before the bell is let down into the water. But when it has ſunk 35 feet below the ſurface, the air contained in it is compreſſed by the weight of the atmoſphere as before, and by the weight of 35 feet of water beſides, which is equivalent to another atmoſphere. Therefore the compreſſing force at this depth is doubled, and conſequently the air in the bell will then be twice as denſe as the common air, that we breathe. As much

air likewiſe, as juſt fills the bell, when it is at the ſurface of the water, will, at the depth of 35 feet, only fill half of it, by the laſt propoſition, for as the compreſſing force is doubled, the ſame quantity of air will be reduced to half its uſual dimenſions. For this reaſon the water would riſe into the bell, through the baſe or bottom of it, which is always open, and would fill the other half of it; if there was not a contrivance for bringing down additional air enough to force out this water and to keep the whole capacity of the bell full of air. However the air, which fills it will, at the depth of 35 feet, have twice the denſity that common air has, and at the depth of 70 feet, where it will be compreſſed by the weight of another atmoſphere it will have triple the denſity of common air.

I ſhall here give a ſhort account of the contrivance for bringing down additional air to the diving-bell: becauſe it will ſerve to ſhew, that if a veſſel full of air is ſunk into water, and the water communicates with the air in the veſſel, then the preſſure upon that air will be ſo much the greater as the veſſel is ſunk farther below the ſurface of the water. The contrivance is this. A barrel is made uſe of, which has one bung-hole in the lower part of it, and another in the upper part. A leathern pipe is faſtened to the hole in the upper part, and this pipe is ſo long that, when it hangs down on the outſide of the barrel, its orifice reaches below the bung-hole in the lower part. If this barrel, by the help of weights faſtened to it, is made to ſink with its bottom downwards, the water by preſſing againſt the lower bung-hole will condenſe the air contained in the barrel: for, notwithſtanding this preſſure, none of the air can eſcape through the upper hole, becauſe it is kept in by a greater preſſure againſt the orifice of the leathern pipe which hangs below the bottom of the barrel, and conſequently being deeper in the water ſuſtains a greater preſſure than what acts againſt the lower bung-hole. If the barrel is let down in this manner, till it gets below the bell, and then the end of the leathern pipe is lifted up into the bell; the lower bung-hole will then be more preſſed than the orifice of the pipe, and therefore the air contained in the barrel will be driven up through the pipe, and will be received into the bell. And becauſe the barrel is deeper in the water than the bell is, the water will preſs more againſt the baſe of the barrel to force the air out of it than it does againſt the baſe of the bell: for which reaſon the air will ruſh out of the barrel with force enough to drive out any water which had riſen into the bell, whilſt it was deſcending.

By the ſame contrivance, freſh air is brought down to the bell as often as there is occaſion for it. The air, which has been heated by frequently breathing it, is let out through a ſtop-cock in the top of the bell, and riſes

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in bubbles to the ſurface of the water, whilſt freſh air is received from the leathern pipe of a barrel contrived in the manner already deſcribed.

The air is moſt denſe of all near the ſurface of the earth. For here it has the whole weight of the atmoſphere upon it; and therefore is compreſſed by a force equal to the weight of 30 inches of mercury. But any part of the atmoſphere higher up, or at ſome diſtance from the ſurface, is compreſſed by a leſs force. Becauſe it bears only the weight of as much air as is above it. Therefore as the compreſſing force is greater near the ſurface of the earth, than at any diſtance from it; it follows that the denſity of the air cannot be the ſame every where, but is greateſt at the ſurface, and decreaſes as we riſe from thence towards the top of the atmoſphere.

51. *The air preſſes equally in all directions.*

For the air is an elaſtic fluid, and all elaſtic fluids preſs equally in all directions, when they are in equilibrio with the force that compreſſes them, by propoſition 45.

When we ſtretch out our hand, with the back of it upwards; we do not feel the weight of the air which preſſes upon it: becauſe the lower air preſſes the palm upwards, as much as the incumbent column preſſes the back downwards, and theſe two equal and contrary forces mutually deſtroy each other, ſo that neither of them is felt. But if there is an orifice *rr* at the top of the receiver, Plat. XX. fig. 1, and a perſon cloſes it with the palm of his hand. Then as the receiver below is exhauſted, the air, which before preſſed upwards againſt the palm of his hand, is removed, whilſt the external air ſtill continues to preſs with as great a weight as uſual upon the back of it. By this means he will find his hand puſhed downwards with force enough to give him pain, and to make him feel the weight of the incumbent atmoſphere.

But though the air preſſes upwards as much as downwards, and though this may be the reaſon why, when we ſtretch out our hand, we ſhould not feel the weight, which preſſes upon the upper part of it: yet it may ſtill be aſked, why theſe two contrary forces do not bring the upper and lower parts, as the back and the palm, of the hand cloſer together, and ſqueeze a perſons hand between them juſt as it would be ſqueezed between the two cheeks of a vice. In anſwer to this, we muſt obſerve, that the hand is not preſſed by two forces only, of which one acts upwards and the other downwards, for the air preſſes equally in all directions: and therefore the ſides of the hand are preſſed as much as any equal ſurface

either

either in the back or the palm of it. And theſe innumerable equal forces in contrary directions will deſtroy each other, ſo that none of them will be felt; but the hand will continue in all reſpects at reſt between them. The figure of it cannot be in the leaſt altered by a preſſure, which acts equally upon every part of it. The palm and the back for inſtance cannot be brought cloſer together, becauſe to do this the ſides muſt be removed farther aſunder, and the ſides are kept in their places by a force, which is equal to what acts upon any other equal part of the hand.

If any one was to aſk farther, does not ſuch a preſſure, though it is equal on all ſides, ſqueeze all the parts of the hand cloſer together, and by this means make it leſs? We might aſk him in return — leſs than what? Does he mean leſs, when it is ſtretched out, than when it is held cloſe to the body? If this is his meaning, it ought not to be leſs in one caſe than it is in the other, ſince it is equally preſſed upon by the air in both caſes. But does he mean leſs, when it is in the air, than if it was in a vacuum? To this we ſhould reply, that it is leſs. For it would ſwell in a vacuum; as the fleſh does under a cupping-glaſs, or the body of an animal in an exhauſted receiver.

Notwithſtanding the weight of the air, a cone of lead or of any other ſubſtance ought not to weigh more when its bottom or baſe is upwards than when its vertex or point is upwards. For it is preſſed downwards by an equal column of air in both caſes: as is evident from a bare inſpection of Plat. XVIII. fig. 11. The column of air, which preſſes the cone TSLV downwards, when its vertex is upwards, has LV for its baſe and is as high as the atmoſphere. And if the bottom of the cone or LV was upwards it would be preſſed by a column of the ſame baſe and height. The weight of the cone, if it was encreaſed by this preſſure, would be encreaſed equally in either ſituation. But ſince the preſſure upwards is equal to the preſſure downwards, theſe two equal and contrary forces will deſtroy each other, and will leave the weight of the cone the ſame that it would be if it was not preſſed at all. When I ſay the weight of this or any other body will be the ſame, notwithſtanding the preſſure of the air, as if it was not preſſed at all; I neglect what was proved in propoſition 28. For in fact the weight of a body inſtead of being encreaſed by the airs preſſure is diminiſhed by it. For the air preſſes it more upwards than downwards, juſt as any other fluid would: and the body, in conſequence of this unequal preſſure, loſes the weight of as much air as is equal in bulk to itſelf.

That the airs weight will be felt upon removing the preſſure upwards, which ſupports that weight, has been obſerved already in the caſe of a per-

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Plate XVIII.

Fig. 1.

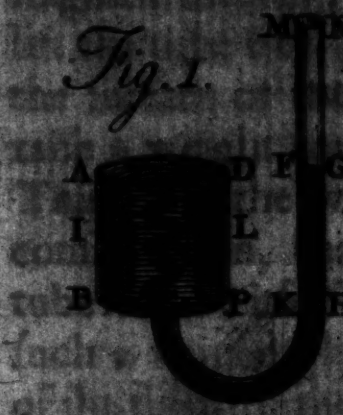


Fig. 2.

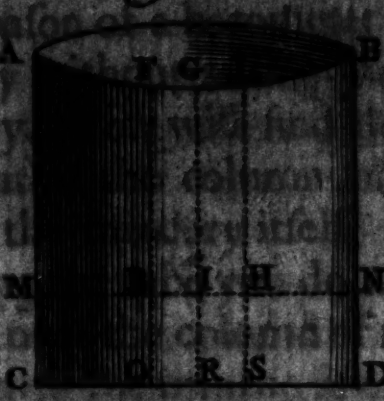
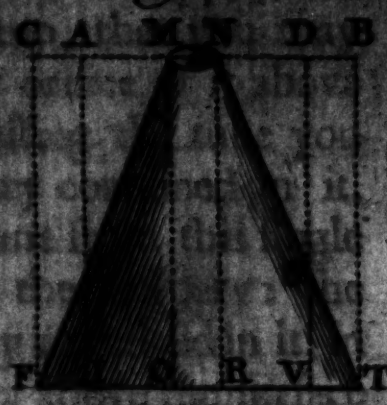


Fig. 3.



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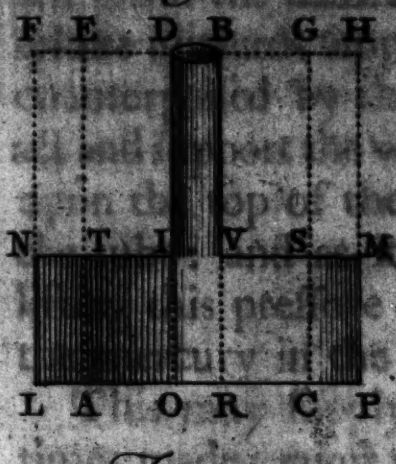
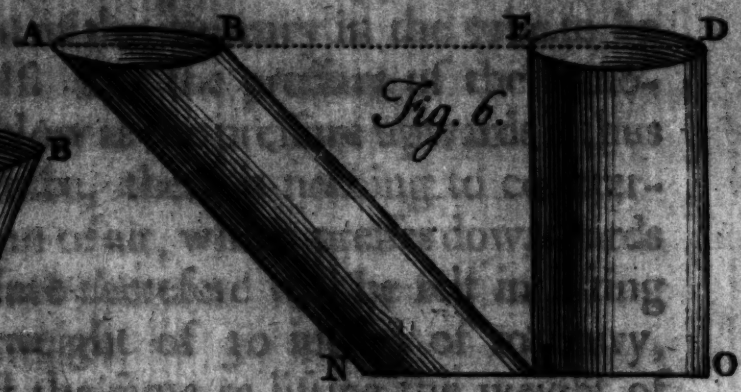


Fig. 5.

Fig. 6.



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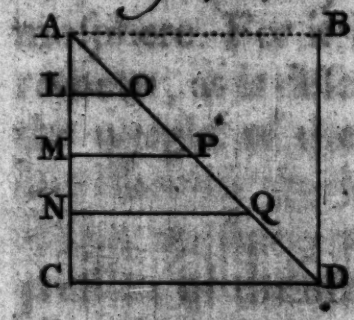

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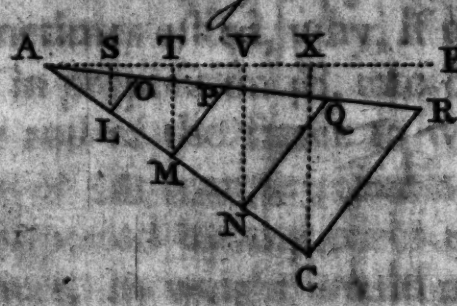


Fig. 9.

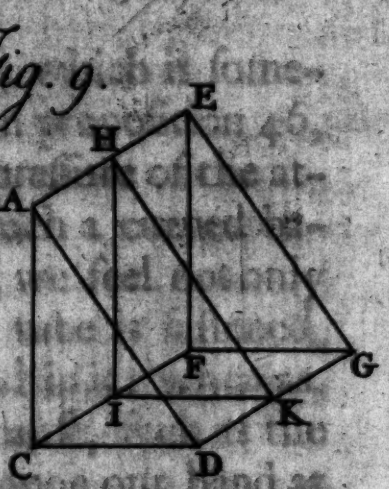


Fig. 10.

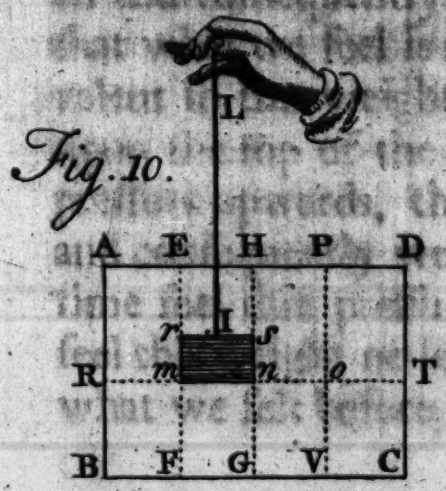
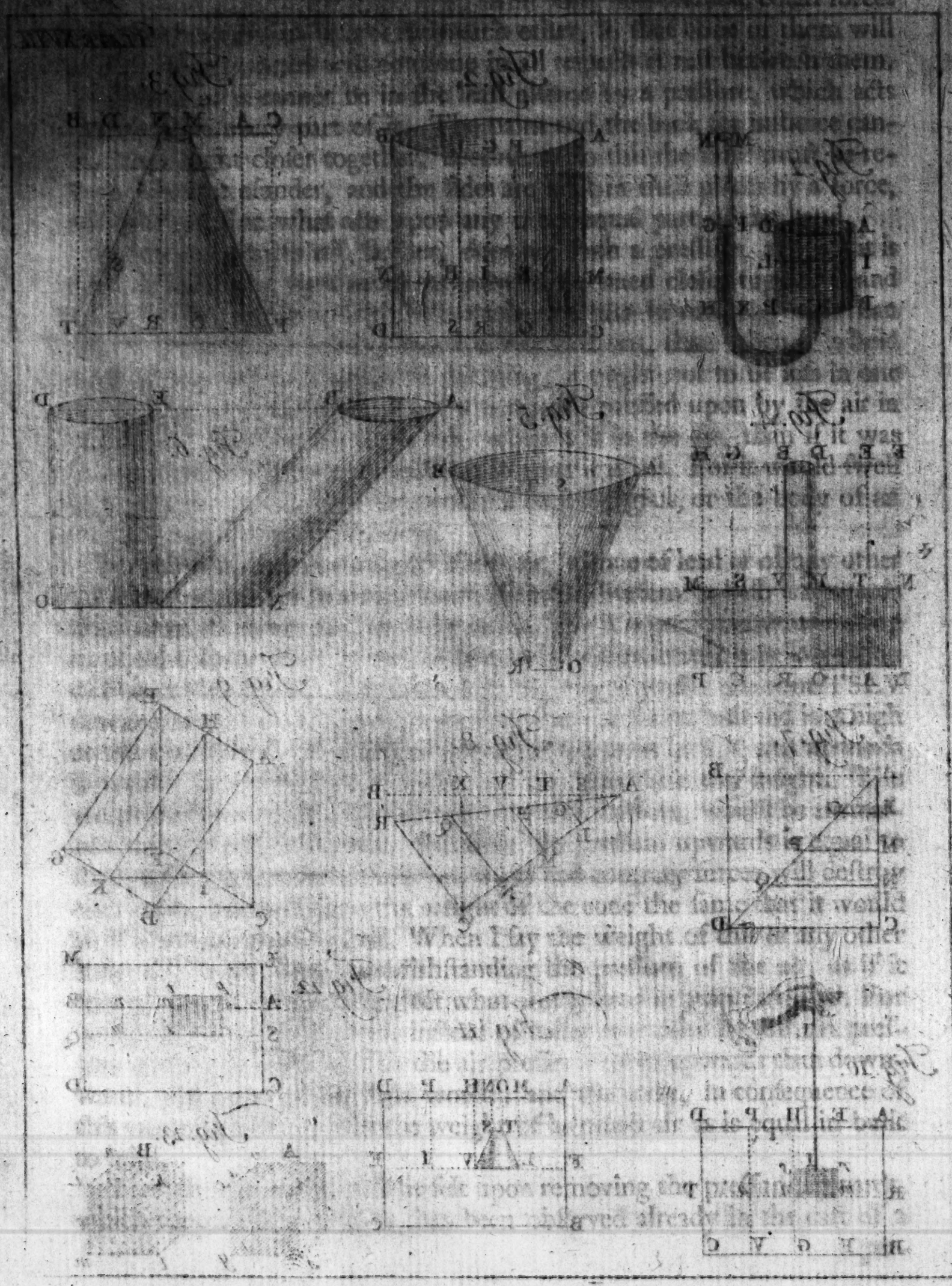


Fig. 11.

Fig. 12.

Fig. 13.



perſons laying his hand upon an orifice in the top of a receiver, whilſt the receiver is exhauſted. This weight may likewiſe be felt if the preſſure upwards, which in common caſes ſupports the weight of the incumbent atmoſphere, is counterpoiſed by ſome other weight. Thus ſuppoſe the ſtagnant mercury in the baſon of a barometer is ſo deep, that you may lift the tube up a good way without raiſing the lower orifice of it above the ſurface of the mercury. You will find that in lifting the tube you raiſe a weight equal to that of the column of mercury contained in it. This is not the weight of the mercury itſelf, as is plane from this ſingle conſideration, that in lifting the tube you do not raiſe the mercury: the tube riſes by itſelf and ſlips over the column of mercury contained in it, in ſuch a manner that the top of the column is no farther above the ſurface of the ſtagnant mercury, after you have lifted up the tube, than it was before. Neither indeed ought the weight of the mercury in the tube to be felt from the nature of the thing itſelf. For the preſſure of the atmoſphere upwards ſupports that weight. Now as the preſſure upwards is thus counterpoiſed by the mercury in the tube, there is nothing to counteract and ſupport the weight of the column of air, which preſſes downwards upon the top of the tube. This preſſure therefore will be felt in lifting the tube: and as it is equal to the weight of 30 inches of mercury, lifting this preſſure is, as to ſenſe, juſt the ſame as lifting the weight of the mercury in the tube.

This may ſupply us with an anſwer to an objection, which is ſometimes made againſt what I have endeavoured to prove, in propoſition 46, that the mercury in the barometer is ſupported by the preſſure of the atmoſphere. For it is ſometimes aſked, why, if we take ſuch a curved barometer, as is deſcribed in propoſition 48, into our hand, we feel not only the weight of the tube and of the frame to which the tube is faſtened, but likewiſe the weight of the mercury that is contained in it: whereas, if that mercury was ſupported by any other external cauſe, ſuch as the preſſure of the atmoſphere, its weight would not preſs upon our hand at all and conſequently ought not to be felt by us? But to this we reply, that what we feel is not the weight of the mercury, but a preſſure equivalent to that weight: we feel the incumbent column of air, that preſſes upon the top of the tube. For as the mercury is a counterpoiſe to the preſſure upwards, there is nothing to ſupport the preſſure downwards, and conſequently we cannot ſupport the tube, but we muſt at the ſame time feel that preſſure. If we empty the mercury out of the tube we feel this weight no longer. Not becauſe the weight of the mercury was what we felt before, but becauſe when the tube is empty, the preſſure

of

of the air upwards being counterpoiſed by no other weight ſupports the preſſure downwards. That this is the true anſwer, and that what we feel, when the tube is full, is not the weight of the mercury but the preſſure of the air upon the top of the tube, is plane from what has been already obſerved in the foregoing paragraph. For in lifting up the tube of a common barometer, you feel the ſame weight, notwithſtanding in this caſe you lift the tube only without raiſing the mercury.

52. *When the compreſſing force is given, the elaſticity of the air is neither encreaſed by heat nor diminiſhed by cold.*

The force, which compreſſes the air, is the weight of the incumbent atmoſphere. This weight is not always the ſame. The greater it is the greater weight of mercury it will ſupport in the barometer; and the leſs it is, ſo much the leſs weight of mercury will be ſupported by it. Accordingly we find that the mercury in the barometer is higher at ſome times than at others. But, ſince the elaſticity of the air is as the compreſſing force, by propoſition 48, ſince the compreſſing force is the weight of the atmoſphere, and ſince the weight of the atmoſphere is proportional to the height of the mercury in the barometer, it follows that the elaſticity of the air will be greateſt, when the mercury ſtands the higheſt, and leaſt, when the mercury ſtands the loweſt.

Heat and cold make no alteration in the elaſticity of the air; but either in the day-time or in the night, either in ſummer or in winter, if the mercury is at the ſame height in the barometer, the weight of the atmoſphere or the compreſſing force is the ſame, and conſequently, by propoſition 48, the elaſticity of the air muſt likewiſe be the ſame, notwithſtanding the difference of heat and cold. Indeed, if any certain quantity of air is included in a cloſe veſſel and does not communicate with the open atmoſphere, this air by being heated will become more elaſtic. Thus if a thin glaſs vial with air in it is cloſe corked, and the air within the vial is heated by ſetting it before a fire, the included air will be made more elaſtic and by endeavouring to expand itſelf will burſt the vial. From hence we may conclude, that, when the particles of air are kept at the ſame diſtance from one another, heat will make them repel one another more forcibly. But if the heated air is at liberty to expand itſelf and can mix with the open atmoſphere; then as much more forcibly as the particles repel one another for being heated, ſo much farther they will depart from one another. For the ſame compreſſing force, that could keep them within a certain diſtance of one another, when they were cool and repelled each other leſs, will not be ſufficient to keep them ſo cloſe to-

together, when their repulſive force is encreaſed. Upon the whole therefore the heat will produce two effects in the air, it will encreaſe the repulſive force of the particles, and in conſequence of this it will rarefy the air. An encreaſe of repulſive force would make the air more elaſtic, if its particles were to continue at the ſame diſtances from one another. But, ſince the heated air will expand itſelf, the repulſive force, by this encreaſe of the diſtance of the particles from one another, will be juſt as much diminiſhed, as the heat had encreaſed it, Therefore by the heat and by the rarefaction in conſequence of the heat the repulſion or elaſticity will continue the ſame.

In like manner cold diminiſhes the mutual repulſion of the particles. But when we ſay it diminiſhes the repulſion, we mean only that it would do ſo, upon ſuppoſition that the repelling particles continued at the ſame diſtance from one another. For if the air, when it is warm, is compreſſed with a certain force, and this ſame compreſſing force continues, when the air is cold; it is plane that, as the cold particles avoid one another with a leſs force than the warm ones, the ſame compreſſing force will bring the cold particles nearer to one another than it could bring the warm ones. Therefore juſt as much as the repulſion or elaſticity is diminiſhed by the cold, juſt ſo much it will be encreaſed by the particles being brought nearer together. And conſequently by both cauſes taken together the elaſticity of the air will continue the ſame, provided the height of the mercury in the barometer, the weight of the incumbent atmoſphere, or the compreſſing force, is the ſame when the air is cold, as when it is warm.

CHAP. VI.

Of Sound.

IT is the province of the metaphyſicians to conſider the idea of ſound in the mind, and of the anatomiſts to deſcribe the impreſſions, that it makes upon the ear, and the manner, in which it is conveyed from thence to the brain. Our enquiries are confined to two other particulars; firſt, what ſound is in the body, that produces it; and ſecondly what it is in the air, through which it is propagated or conveyed from the ſounding body to the ear.

The firſt of theſe particulars will be the more readily underſtood if we conſider it in the moſt ſimple inſtance. We will therefore explane the motions or vibrations of an elaſtic ſtring, ſuch as the cat-gut of a vi-

olin or the wire of a harpſichord, and afterwards will ſhew in what manner ſuch ſtrings produce ſound. And this will lead us to the ſecond particular to ſhew in what manner it is propagated through the air after it is produced by the ſounding body.

53. *An elaſtic ſtring, whilſt it is vibrating, is accelerated and retarded by the ſame law as a pendulum is, that vibrates in a cycloid.*

If ADB, Plat. XIX. fig. 4, is a piece of wire or cat-gut, which is ſtretched upon two pins or ſcrews one at A and the other at B; this wire or cat-gut, when any force bends it into the poſition AEB, will by its elaſticity reſtore itſelf to its former poſition ADB. But it will not ſtop, when it is at ADB; but the velocity, which it acquired in returning, will, by the firſt law of motion, carry it forwards in the ſame direction in which it was moving, and will bend it into the contrary poſition ACB. The ſame elaſticity, which ſtraitened it, or brought it back from the bent poſition AEB into the ſtrait one ADB, will likewiſe ſtraiten it again or bring it back from the bent poſition ACB into the ſtrait one ADB. And the velocity, which it acquires in returning, will, as before, make it over-run its proper place ADB, and bend it once more into AEB. Thus the ſtring, when it is bent one way, acquires velocity enough in returning to bend it as far the other, and continues to vibrate ſo as to make the middle point of it run backwards and forwards upon the line EDC, till the reſiſtance of the air and the friction upon the pins deſtroys its motion: and then it will reſt in the poſition ADB; upon which account ADB is called its proper place.

The proper place of a pendulum is the loweſt point of the curve, in which it vibrates, or that point, which is directly under the center of motion; by propoſition 42 of mechanics. Thus if a pendulum vibrates in the cycloid ACD, Plat. V. fig. 2, C or the loweſt point in the cycloid is the proper place of the pendulum or is the only place where it can ſtop. The pendulum is accelerated, whilſt it deſcends from A to C and is retarded, whilſt it aſcends from C to D; by propoſition 133 of mechanics. And the law, by which a pendulum ſo vibrating is accelerated, whilſt it deſcends, and is again retarded, whilſt it aſcends, is this; the accelerating force of gravity in the deſcent of the pendulum, and the retarding force of gravity in the aſcent of it, is proportional to its diſtance from C its proper place, by propoſition 136 of mechanics.

Now to prove the truth of the propoſition it will be neceſſary to ſhew, firſt, that, if an elaſtic ſtring is bent, the elaſticity of it will accelerate it, whilſt it ſtraitens itſelf, and will retard it again, whilſt it bends it-

ſelf

ſelf the contrary way; and ſecondly, that the accelerating force of its elaſticity in one caſe, and the retarding force in the other, is always proportional to the diſtance of the ſtring from its proper place, or to the diſtance of the middle point of the ſtring from D, Plat. XIX. fig. 4.

That the ſtring, if it is bent, will be accelerated, whilſt it ſtraitens itſelf, is evident. For the force of elaſticity, which makes it return, or which ſtraitens it, continues to act upon it during the whole time of its return. Thus if the ſtring is bent into the poſition AEB, Plat. XIX. fig. 5, the elaſticity of it would ſtraiten it again, though this elaſticity ſhould act only once upon it: for, by the firſt law of motion, all bodies perſevere to move on uniformly by the action of one ſingle moving force; and conſequently one ſingle impulſe of elaſticity would make the ſtring return to its proper place with an uniform velocity. But the force of elaſticity, by which the ſtring is ſtraitened, does not act once only and then ceaſe, but continues to impel the ſtring during its whole return. For as this elaſticity, if the ſtring had been bent at firſt into the poſition ADB, would have ſtraitened it, ſo when the ſtring, in returning from AEB, has got as far as ADB, the force of elaſticity acts upon it, and conſequently the velocity, which was communicated by the firſt impulſe of elaſticity, will be encreaſed by this new impulſe. In like manner, when the ſtring, in returning from the poſition AEB, is got as far as ACB, the force of elaſticity impels it again. For if the ſtring had been bent at firſt into ACB, its elaſticity would have ſtraitened it: and ſince the force of elaſticity impells the ſtring, when it is at ACB, this impulſe muſt neceſſarily be the ſame, whether it was originally bent no farther than ACB, or acquires that poſition in returning from AEB. And conſequently the velocity, which the ſtring had before it came to ACB, will be encreaſed; that is, the ſtring will be accelerated by the impulſe of elaſticity, which acts upon it, when it is in this poſition. We have here taken notice of no other impulſe of elaſticity, but what acts upon the ſtring in the three poſitions AEB, ADB, and ACB. But it muſt be remembered, that the force of elaſticity is a continued impulſe, which acts upon it in every inſtant of its motion, and in every place that it paſſes through from AEB, till it arrives at AFB, that is, till it becomes ſtrait. And theſe continued impulſes conſtantly encreaſe the velocity of the ſtring or accelerate it, whilſt it is ſtraitening itſelf.

But when the ſtring has arrived at AFB and is quite ſtrait, it will not reſt in that poſition; becauſe the velocity, which it has acquired in returning, will, as was before obſerved, carry it on in the ſame direction in which it was moving before, and will make it over-run its proper

place and bend it as much on the other ſide as it was bent towards E before. Now as the force of elaſticity will always act upon the ſtring unleſs it is ſtrait, and as this force acted in the ſame direction that the ſtring moved in from AEB to AFB, and conſequently accelerated it, whilſt it was returning to its proper place; ſo this ſame force, when the ſtring overruns its proper place and bends itſelf the contrary way into AGB, will act in the oppoſite direction to that in which the ſtring is moving, and therefore will retard it. And becauſe the velocity, which the ſtring acquired by the force of elaſticity, whilſt it was returning, will be deſtroyed in the ſame manner and by the ſame degrees, whilſt it is bending itſelf the contrary way; it follows that as FE was the diſtance to which the middle point of the ſtring was drawn from its proper place, the velocity acquired by it in returning will not be loſt, till it has run as far to the other ſide from F to G.

But the ſtring, in returning, is not accelerated uniformly; that is, the velocity, though it keeps conſtantly encreaſing, as was proved before, does not encreaſe by equal degrees. For the elaſticity, or force that accelerates the ſtring, and conſequently the acceleration or velocity produced, is always proportional to the diſtance of its middle point from F, which is its proper place. For ſuppoſe the ſtring AFB was to be bent from its rectilineal poſition by a weight hung to the middle point F: it is found by experiment that the diſtance, to which F is drawn from its proper place, is always proportional to the weight which bends the ſtring. Thus if a weight of one pound would bend the ſtring into the poſition ACB, and ſo draw the point F to the diſtance CF from its proper place; a weight of two pounds will bend it as far as ADB, and will draw the middle point F to the diſtance FD, or to twice the diſtance FC from its proper place; and a weight of three pounds will draw this point three times as far as a weight of one pound would; that is, a weight of three pounds would bend the ſtring to AEB ſo as to make FE three times FC. This may be expreſſed in fewer words by ſaying that the ſagitta produced is always proportional to the bending force or to the force that produces it: for the diſtance, to which the middle point of the ſtring is drawn from its proper place, is called the ſagitta. And it is found by experiment that, if any weight, which is applyed to the middle of the ſtring at F in order to bend it, produces the ſagitta FC, a double force or weight will produce a double ſagitta FD, and a triple force will produce a triple one FE. Now it is evident, that any weight, which can bend a ſtring and keep it bent, muſt be equal to the elaſticity of the ſtring, that is, to the force, with which the ſtring endeavours to ſtraiten itſelf. For whatever is the degree

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that the ſtring is bent to, a weight leſs than the elaſticity could not have bent it ſo far, and a weight greater than the elaſticity would bend it farther. Therefore the weight, which juſt keeps the ſtring bent to that degree, muſt be in equilibrio with its elaſticity. But the weight or bending force is proportional to the ſagitta produced by it: and the ſtrings elaſticity is proportional to this bending force: therefore the ſtrings elaſticity is always proportional to the ſagitta. And ſince the elaſticity of the ſtring is the force that accelerates it in its return, and the ſagitta is the diſtance of the middle point of the ſtring from its proper place; it follows that the accelerating force is proportional to this diſtance. Thus, when the ſtring is bent to AEB and the ſagitta is FE, the elaſticity of it is juſt as much greater than when it has got to ADB as FE is greater than FD. And in like manner in the whole return of the ſtring, its elaſticity, that is, the accelerating force decreaſes in the ſame proportion with the ſagitta or diſtance of the middle point from its proper place.

What has already been ſaid concerning the force of elaſticity, by which the ſtring is accelerated in its return, will be eaſily applyed to the ſame force, as it retards the ſtring, whilſt it is bending itſelf the contrary way. For the force of elaſticity in this caſe as well as in the other muſt be proportional to the ſagitta; and conſequently the retarding force, which is this elaſticity, is proportional to the diſtance of the middle point of the ſtring from its proper place.

This reaſoning is applicable not only to ſtrings, that are bent by weights hung to them, but to ſuch as are bent by any other force: for inſtance, it is applicable to the ſtring of a violin, when it is bent by drawing the bow over it, and to the ſtring of a harpſichord, when it is bent by the ſtroke of the quill againſt it. For the elaſticity is proportional to the ſagitta: and ſince the ſagitta is found to be proportional to the bending force in one caſe, it muſt be ſo in all others, whatever that force may be or in whatever manner it acts. So that theſe ſtrings, as well as thoſe that are bent by weights, are accelerated and retarded by their own elaſticity in proportion to their diſtance from their proper place, that is, they are accelerated and retarded by the ſame law with a pendulum, which vibrates in a cycloid.

54. *All the vibrations of the ſame ſtring are performed in equal times.*

When a ſtring is firſt ſtruck it deſcribes a greater ſpace in its vibrations, than it does after the friction upon the pins and the reſiſtance of the air have deſtroyed a part of its motion. The ſtring AFB, Plat. XIX. fig. 5, if it is bent to AEB, will return upon the line EF, and will bend itſelf

.the

the contrary way enough to make the middle point F deſcribe a line FG equal to EF on the oppoſite ſide. So that the whole vibration is performed upon a line GE equal to twice EF. But as the ſtrings motion is loſt, the vibrations are performed upon ſhorter lines. After it has been vibrating ſome time, the decreaſe of its motion ariſing from the two cauſes before mentioned may be ſuch, that the ſtring can only bend itſelf as far as D one way making the ſagitta DF; and then the ſagitta on the contrary ſide will be only equal to DF, and the whole vibration will be performed upon a line equal to twice DF. In like manner after the motion has decreaſed ſtill farther, the ſtring will bend itſelf one way only to C, ſo as to make the ſagitta CF, and conſequently it will over-run its proper place only to an equal diſtance on the other ſide, and the vibration will be performed upon twice CF. Thus every vibration is performed upon a ſhorter line than the preceding one; till at laſt the ſtring ſtops in the rectilineal poſition AFB. But though theſe vibrations are performed upon longer lines at firſt, than they are at laſt, yet all of them, from the firſt beginning of the ſtrings motion till it ſtops, are performed in equal times. The ſtring is no longer in vibrating upon twice EF, than upon twice DF, or twice CF.

By the laſt propoſition the accelerating force is always proportional to the diſtance of the ſtring from its proper place; that is, the farther the ſtring is from a rectilineal poſition, or the greater the ſpace is that it has to deſcribe in returning, ſo much greater is the force of elaſticity, by which it is made to return: and conſequently, ſince the quantity of matter in the ſtring is given, the velocity, with which it returns, will be proportional to this force, or to the ſpace that the ſtring has to deſcribe in returning. But where the velocity of a body is proportional to the ſpace that it has to deſcribe, that ſpace, whether it is great or ſmall, will be deſcribed in a given time. Therefore the ſtring, ſince it returns with a velocity proportional to the ſpace it has to deſcribe, will always return in an equal time. It takes up no longer time to return from E to F, than it does to return from C to F. For though EF is a longer line than CF, yet the time taken up to deſcribe EF will be no longer than what is taken up to deſcribe CF. Becauſe juſt as much longer as EF is than CF, juſt ſo much faſter the ſtring moves, when in returning it deſcribes the line EF, than it does, when it has only CF to deſcribe. But the time of returning is half the time of a vibration. And ſince half the time of vibration upon the line EF is equal to half the time of vibration upon the line CF, it follows that the whole time of vibration upon twice EF muſt be equal to the whole time of vibration upon twice CF.

In

In this propoſition there was no occaſion to take notice of a diſtinction, which we muſt attend to in ſome of thoſe that follow. As we diſtinguiſhed in mechanics, propoſitions 31, 32, between the moving force and the accelerating force of gravity; ſo here we muſt diſtinguiſh between the moving force and the accelerating force of elaſticity. The moving force of elaſticity is greater or leſs in proportion to the quantity of motion produced in the ſtring. And the accelerating force of elaſticity is greater or leſs in proportion to the velocity, with which the ſtring moves. There is no difference between theſe forces, where the quantity of matter in the ſtring is given: for where the quantity of matter is given the quantity of motion is proportional to the velocity; and therefore the moving force muſt be proportional to the accelerating one. Upon this account, when we are conſidering one and the ſame ſtring, as in this propoſition, we may neglect the diſtinction. Becauſe in one and the ſame ſtring the quantity of matter is always given. But in two ſtrings, which contain unequal quantities of matter; the velocity will be different if the quantity of motion is the ſame. For, by propoſition 14 of mechanics, where the moment is given, the velocity is always inverſely as the quantity of matter. Therefore though two ſuch ſtrings were equally elaſtic, and ſo were to be moved with equal forces, and conſequently were to have equal moments; yet, as they have unequal velocities, the accelerating force of elaſticity is not the ſame in both.

55. *If ſtrings, that have equal diameters, are ſtretched with equal forces, the times, which they take up to perform a vibration, will be proportional to their lengths.*

If ſtrings are ſtretched upon ſcrews, or in the ſame manner as the ſtrings of a harpſichord, it is difficult to determine, when the forces, which ſtretch them, are equal. But if a ſtring AB, Plat. XIX. fig. 5, is faſtened to a pin at A, and is drawn over a pully at B ſo as to hang a weight at H the other end of it: there will be no difficulty in determining, when any other ſtring is ſtretched by an equal force. For ſince the weights thus hung at the ends of the ſtrings are the forces, which ſtretch them; theſe forces are equal, if the weights are ſo.

Now if two ſtrings had equal diameters and were ſtretched by equal forces, but one of them was twice as long as the other, that is, if their lengths were to each other as 2 to 1, they would not perform their vibrations in equal times; for the longer of them would take up twice the time to perform a vibration that the ſhorter would. And in like manner in ſtrings of any other lengths the times of vibration will bear the ſame proportion to each other with the lengths of the ſtrings.

It has been ſhewn already, in propoſition 53, that elaſtic ſtrings vibrate by the ſame law with pendulums, which vibrate in a cycloid. And, by propoſition 137 of mechanics, the ſquares of the times, in which ſuch pendulums perform their vibrations, are inverſely as the forces, that accelerate them. Therefore the ſquares of the times, in which elaſtic ſtrings perform their vibrations, will likewiſe be inverſely as the forces, by which they are accelerated. The reader ſhould keep this concluſion in his mind; becauſe it will be of uſe in demonſtrating, not only this propoſition, but likewiſe the two that follow.

Now the ſquares of the lengths of the ſtrings are inverſely as the forces that accelerate them. For firſt, if ſtrings of equal diameters but different lengths were equally elaſtic, they would move with unequal velocities. For their moments would be equal; becauſe their elaſticities or the moving forces, that act upon them and make them vibrate, are equal by the ſuppoſition. And conſequently their velocities muſt be inverſely as their quantities of matter, by propoſition 14 of mechanics. But in ſtrings of equal diameters, the quantities of matter are as the lengths. Therefore the velocities, with which they vibrate, will be inverſely as their lengths. Now the accelerating forces are proportional not to the quantities of motion, for theſe by the ſuppoſition are equal, but to the velocities, with which the ſtrings vibrate. From whence it follows that the accelerating forces are inverſely as the lengths of the ſtrings. But ſecondly, we muſt obſerve, that if ſtrings of different lengths are ſtretched with equal forces, their elaſticities will be unequal. For ſuppoſe one ſtring to be as long again as the other; it is evident, that their elaſticities are proportional to the forces, that can bend them equally, ſo as to produce an equal ſagitta in each of them. But whatever force is requiſite to produce a given ſagitta in the longer ſtring, a double force is found by experiment to be requiſite to produce an equal one in the ſhorter. Or otherwiſe. By the ſuppoſition, there is twice as much of the longer ſtring to be ſtretched or held ſtrait, as there is of the ſhorter; and conſequently, when equal forces are applyed to ſtretch both of them, the longer will be but half as much ſtraitened as the ſhorter is, that is, the ſhorter will be twice as ſtiff or twice as elaſtic as the longer is. From whence it follows in this caſe, and by the ſame method of reaſoning it may be proved in all other caſes, that the elaſticity of theſe ſtrings is inverſely as their lengths. Therefore ſince the moving force is inverſely as the lengths of the ſtrings, if the quantity of matter contained in them was equal, their velocities and conſequently their accelerating forces would likewiſe be inverſely as their lengths.

We

We ſee therefore that, when ſtrings have equal diameters and are ſtretched by equal forces, but are of different lengths, their accelerating forces are inverſely as their lengths upon two accounts. Firſt, becauſe they contain unequal quantities of matter, for which reaſon their velocities would be inverſely as their lengths, if their elaſticities were equal. And ſecondly, becauſe their elaſticities and conſequently their velocities would be inverſely as their lengths, if the quantities of matter contained in them were equal. From hence then it follows, that the accelerating forces, being as their lengths inverſely upon two accounts, are as their lengths and their lengths inverſely, that is, inverſely as their lengths multiplyed into themſelves, or inverſely as the ſquares of their lengths.

Since then we have ſhewn that the ſquares of the times, in which ſtrings, that have equal diameters, and are ſtretched by equal forces, perform their vibrations, are inverſely as the forces that accelerate them; and likewiſe that the ſquares of the lengths of the ſtrings are inverſely as the ſame forces; it follows that the ſquares of the times of their vibration are proportional to the ſquares of their lengths, becauſe they are both of them inverſely as the accelerating forces.

But if the ſquares of the times of vibration are proportional to the ſquares of the accelerating forces, the conſequence is, that the times of vibration are proportional to the lengths.

56. *When ſtrings of equal lengths are ſtretched by equal forces, the times of their vibrations are proportional to their diameters.*

If, for inſtance, two ſtrings of equal lengths were ſtretched by equal forces, but one of the ſtrings was twice as thick in the diameter as the other; the thicker ſtring would take up as much more time to perform a vibration, as the other takes up. And ſo, in all other inſtances, whatever proportion the diameters of the ſtrings bear to each other, the times, that they take up to perform a vibration, will bear the ſame proportion to each other.

Since the ſtrings have equal lengths and are ſtretched or held ſtrait by equal forces, they will be equally ſtiff, ſo that whatever force is requiſite to bend the thicker, the ſame will likewiſe be requiſite to bend the other equally. From hence it follows, that both of them are equally elaſtic; becauſe the elaſticity of a ſtring, or the force with which it endeavours to ſtraiten itſelf, can neither be greater nor leſs than the force, which is ſufficient to bend it and to keep it bent to a given diſtance from its proper place. But if the ſtrings are equally elaſtic, the quantities of motion produced by their reſpective elaſticities, when they unbend themſelves, muſt neceſſarily be equal; becauſe their elaſticities are the moving

forces, which make them return to their proper place or unbend them, and the quantity of motion is always proportional to the moving force impreſſed. And ſince the quantities of motion are equal, but the quantities of matter different, the velocities generated in a given time, and conſequently the accelerating forces, will be inverſely as the quantities of matter in the ſtrings. But the quantities of matter in cylindrical ſtrings of equal lengths is proportional to the ſquares of their diameters. Therefore the accelerating forces are inverſely as the ſquares of the diameters. But the accelerating forces are likewiſe inverſely as the ſquares of the times of vibration, by propoſition 53, and propoſition 137 of mechanics. And conſequently ſince the ſquares of the times of vibration and the ſquares of the diameters of the ſtrings are both of them inverſely as the accelerating forces, the ſquares of the times of vibration are in the ſame proportion with, or directly as, the ſquares of the diameters. But if the ſquares are directly as each other, the ſquare roots are ſo likewiſe. Therefore the times of vibration are directly as, or proportional to, the diameters.

57. *If ſtrings of equal lengths and equal diameters are ſtretched by unequal forces; the times of vibration are inverſely as the ſquare roots of the forces that ſtretch them.*

If one ſtring is ſtretched by any force equal to a weight of 9 pounds, and any other ſtring is ſtretched by a force equal to 4 pounds; the ſquare roots of theſe forces are to each other as 3 to 2, and theſe roots inverted are to each other as 2 to 3. And the times of vibration will be as theſe roots inverted, that is, as 2 to 3. This inſtance will ſerve to ſhew the meaning of the propoſition in all other inſtances: for univerſally, if we find the ſquare roots of the ſtretching forces, the times, in which ſtrings of equal lengths and equal diameters perform their vibrations, will be as theſe ſquare roots inverted.

Strings, that have equal lengths and equal diameters, contain equal quantities of matter. Therefore, when they are made to vibrate by their elaſticity, their velocities will be proportional to their quantities of motion, by propoſition 13 of mechanics. But the accelerating forces are as their velocities, and the moving forces are as their quantities of motion. Conſequently the accelerating forces will be as the moving forces, that is, as the elaſticities of the ſtrings. But, ſince elaſtic ſtrings and pendulums, that move in a cycloid, vibrate by one and the ſame law, by propoſition 53, it follows, from propoſition 137 of mechanics, that the ſquares of the times, in which equal ſtrings perform their vibrations, will be inverſely as their elaſticities. Now the elaſticity of a ſtring is that force, which keeps

keeps it ſtrait, or which, when it has been bent, ſtraitens it again: and the force, which keeps a ſtring ſtrait, when it is ſtretched between two pins or ſcrews, is the force that ſtretches it. Therefore in equal ſtrings the elaſticities are proportional to the forces that ſtretch them. But the ſquares of the times of vibration, as we have ſeen already, are inverſely as the elaſticities: conſequently, the ſquares of the times of vibration are inverſely as the forces that ſtretch the ſtrings. From whence it follows, that the ſquare roots of the ſquares of the times of vibration, that is, the times of vibration, will be inverſely as the ſquare roots of the forces, by which the ſtrings are ſtretched.

58. *Sound, in the body that produces it, is a trembling motion: this motion is communicated to the air; and the air conveys it to the ear.*

When any elaſtic body is ſtruck, ſo as to produce a ſound, the body or ſome part of it is made to vibrate. This is evident to ſenſe in the ſtrings of a violin or a harpſichord; for either the eye may ſee or the hand may feel the trembling of the ſtrings, when by ſtriking them they are made to ſound.

If a bell is ſtruck by its clapper on the inſide, the bell is made to vibrate. The baſe of the bell is a circle; but by ſtriking any part of this circle on the inſide, the part which is ſtruck will fly out a little way, ſo that the diameter, which paſſes through this part of the circular baſe, will become longer than another diameter, which croſſes this at right angles. Therefore by the ſtroke the baſe will be changed into an ellipſis, whoſe longer axis will paſs through the part, againſt which the clapper ſtruck. But the elaſticity of the bell will reſtore the figure of the baſe, and will make the part, which was forced out of its place, return back. This part in returning will acquire velocity, in the ſame manner as an elaſtic ſtring would in the ſame circumſtances, by propoſition 53. And ſince it acquires velocity in returning to its place, it will not ſtop at that place, but will over-run it. Thus the circular figure of the baſe will be changed into an ellipſis again, only now the ſhorter axis will paſs through the part that was firſt ſtruck. If the bell was to be ſtruck at firſt by a hammer on the outſide, the part ſtruck would move inwards; and ſuch a motion would likewiſe change the baſe into an ellipſis: only in this caſe the ſhorter axis of the ellipſis would paſs through that part where the blow was given. The elaſticity of the bell will reſtore its figure; and as the part, which was ſtruck, will acquire velocity in returning to its proper ſituation, the acquired velocity will not ſuffer it to reſt there, but will carry it farther out from the oppoſite ſide; and the baſe will by this means be again changed into an el-

lipſis having the longer axis at that part where the blow was firſt given. Thus we have ſeen, that wherever the bell is ſtruck, the parts of it will perform one vibration; the part, which is ſtruck will yield to the blow; the elaſticity of the bell will bring it back to its former ſituation; in returning it will acquire velocity; and as far as the blow had driven it one way, ſo far the acquired velocity will carry it the other. But ſince, after one vibration is thus performed, the figure of the baſe will be elliptical; the parts of the bell will vibrate a ſecond time and ſo on, in the ſame manner, that an elaſtic ſtring vibrates. The laws of theſe vibrations have already been ſhewn in the caſe of elaſtic ſtrings. For where the cauſe of vibration and the manner of vibrating is the ſame, the vibrations will in all caſes be performed by the ſame laws: in explaning thoſe laws the caſe of elaſtic ſtrings was made choice of rather than any other, only becauſe it is of all others the moſt ſimple.

The ſame ſtroke, which makes a bell vibrate, makes it ſound too: and as the vibrations decay the ſound grows weaker. Our ſenſes may convince us that the parts of a bell are in a trembling or vibratory motion, whilſt the bell ſounds: for if we lay our hand upon it, we may feel it jar: or if ſmall ſtraws or pieces of paper are thrown upon it, we may ſee that the jarring or trembling of the bell will put them in motion.

But the air muſt convey this vibratory motion to the ear: for otherwiſe, though the ſounding body is made to vibrate, no ſound will be heard. Thus if a bell is rung in the receiver of an air-pump, the ſound will grow weaker as the air is exhauſted: and, when all the air is drawn out of the receiver, no ſound at all will be heard. When the air is admitted again into the receiver, the ſound will at the firſt entrance of the air begin to be heard, and will grow louder as more air returns. If the bell was to be rung in like manner in a veſſel where the air is condenſed, the ſound of it would be much louder than it is in common air. And accordingly, when divers are let down to any great depth of water, becauſe the air in the diving-bell, by propoſition 50, is much condenſed, they ſeem to one another to ſpeak much louder than uſual.

When an elaſtic ſtring ADB, Plat. XIX. fig. 4, is bent into the ſituation ACB, and, being let go, ſtraitens itſelf firſt, and then bends itſelf as far the contrary way; or when any other elaſtic body vibrates in the ſame manner; the air, which lies before the elaſtic body, will be propelled or driven forwards, and by that means will be condenſed. When this condenſed air expands itſelf, it will expand itſelf not only towards the ſtring, that is, it will not only return with the ſtring; but as the elaſti-

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city of this condenſed air acts in all directions, it will likewiſe expand itſelf forwards, and ſo will condenſe the air, which lies beyond it. This laſt condenſed air in expanding itſelf will produce the ſame effect or will condenſe the air, which lies ſtill farther forwards. And thus the motion produced in the air by the vibration of the elaſtic body being conſtantly carryed forwards will be conveyed to the ear.

As theſe condenſations of the air are what ſtrike upon the ear, or upon any obſtacle, which is in their way, they are called pulſes. Sound, in the air through which it is conveyed, conſiſts of ſuch pulſes. And it will be neceſſary in the following propoſitions to conſider more particularly in what manner theſe pulſes are produced and propagated from the ſounding body.

But here we ſhould obſerve, that theſe pulſes are ſometimes produced without ſuch vibrations of the ſounding body. And in theſe caſes if we look farther than the air itſelf for the cauſe of the ſound, it is only to find out by what accident theſe condenſations or pulſes may be produced without any vibrations in what is looked upon as the ſounding body. We have two inſtances of this ſort, one in wind-inſtruments ſuch as a flute or an organ-pipe, the other in the diſcharge of a gun. In an organ-pipe, or in a flute, the air, which is driven through the pipe, ſtrikes againſt the edge of the lip in its paſſage; and by being accumulated there it is condenſed: and from this condenſation the pulſes begin to be propagated. When a gun is diſcharged, the powder upon taking fire expands the air about it all at once, and conſequently condenſes the air, which encompaſſes the ſpace where this expanſion happens. For whatever air is driven out of that ſpace, where the expanſion is made, will be forcibly driven into the ſpace all round it. This condenſation is the firſt pulſe: and, as this expands itſelf by its elaſticity, pulſes of the ſame ſort will be produced and propagated forwards.

To make the theory of ſound as intelligible as we can, it will be neceſſary to explane it in the moſt ſimple inſtance: we will therefore conſider the manner, in which theſe pulſes are produced by an elaſtic ſtring.

59. *When a particle of air is puſhed forwards by an elaſtic ſtring; the particle in moving forwards will be accelerated firſt and then retarded, in the ſame manner that the ſtring is: and this ſame acceleration and retardation will ſucceſſively be produced in other particles, which lie beyond it.*

If an elaſtic ſtring is bent into the ſituation ACB, Plat. XIX. fig. 4, upon being let go it will be accelerated as it unbends itſelf and returns into

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As theſe condenſations of the air are what ſtrike upon the ear, or upon any obſtacle, which is in their way, they are called pulſes. Sound, in the air through which it is conveyed, conſiſts of ſuch pulſes. And it will be neceſſary in the following propoſitions to conſider more particularly in what manner theſe pulſes are produced and propagated from the ſounding body.

But here we ſhould obſerve, that theſe pulſes are ſometimes produced without ſuch vibrations of the ſounding body. And in theſe caſes if we look farther than the air itſelf for the cauſe of the ſound, it is only to find out by what accident theſe condenſations or pulſes may be produced without any vibrations in what is looked upon as the ſounding body. We have two inſtances of this ſort, one in wind-inſtruments ſuch as a flute or an organ-pipe, the other in the diſcharge of a gun. In an organ-pipe, or in a flute, the air, which is driven through the pipe, ſtrikes againſt the edge of the lip in its paſſage; and by being accumulated there it is condenſed: and from this condenſation the pulſes begin to be propagated. When a gun is diſcharged, the powder upon taking fire expands the air about it all at once, and conſequently condenſes the air, which encompaſſes the ſpace where this expanſion happens. For whatever air is driven out of that ſpace, where the expanſion is made, will be forcibly driven into the ſpace all round it. This condenſation is the firſt pulſe: and, as this expands itſelf by its elaſticity, pulſes of the ſame ſort will be produced and propagated forwards.

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If an elaſtic ſtring is bent into the ſituation ACB, Plat. XIX. fig. 4, upon being let go it will be accelerated as it unbends itſelf and returns into

into its rectilinear position ADB, and will be retarded as it
the contrary way into the position AEB, by propoſition 53.
particle of air, which is contiguous to any point of the ſtrin
cularly to C the middle point of it, being impelled by the ſtr
moved in the ſame manner that the ſtring is; it will be accele
C the middle point is moving from C to D, and will be retar
it is moving from D to E.

The particle *a*, Plat. XIX. fig. 6, muſt be ſuppoſed cloſe
It could not well be drawn ſo without making the two figure

Suppoſe therefore *a*, Plat. XIX. fig. 6, to be the particle o
is contiguous to the ſtring. Then, whilſt the middle point
is returning and is accelerated, it is evident, that this particle,
ſtantly impelled by the ſtring, muſt be moved juſt as faſt as
and conſequently muſt be accelerated in the ſame manner.
the middle point of the ſtring is over-running its proper plac
tarded, the particle *a* will move no faſter than the ſtring,
quently in this part of its motion forwards it will be retarded
particle *a*, when the ſtring is retarded, can move no faſter th
moves, will appear from the following conſiderations. A
puſhes the particle *a* forwards, this particle approaches to
the next particle beyond it: and the mutual repulſion of th
ticles is inverſely as their diſtance from each other, by propoſi
Therefore the nearer *a* is to *b*, the more *a* will be repelled
Now whilſt the ſtring continues to puſh *a* forward, and is
ſo long *a* being driven before the ſtring will continue to be
notwithſtanding this repulſion. But as the force of the ſtrin
it will abate when the ſtring is retarded, this repulſion of *a*
prevent *a* from moving forwards any faſter than the ſtring mo
fore, when the ſtring is retarded *a* will ſtill move forward
ſtring, but will be retarded as the ſtring is.

But the propoſition affirms farther, that this acceleration
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into its rectilinear position ADB, and will be retarded as it bends itself the contrary way into the position AEB, by propoſition 53. Now any particle of air, which is contiguous to any point of the ſtring, as particularly to C the middle point of it, being impelled by the ſtring, will be moved in the ſame manner that the ſtring is; it will be accelerated whilſt C the middle point is moving from C to D, and will be retarded, whilſt it is moving from D to E.

The particle *a*, Plat. XIX. fig. 6, muſt be ſuppoſed cloſe to C, fig. 4. It could not well be drawn ſo without making the two figures confuſed.

Suppoſe therefore *a*, Plat. XIX. fig. 6, to be the particle of air, which is contiguous to the ſtring. Then, whilſt the middle point of the ſtring is returning and is accelerated, it is evident, that this particle, being conſtantly impelled by the ſtring, muſt be moved juſt as faſt as the ſtring is, and conſequently muſt be accelerated in the ſame manner. But, whilſt the middle point of the ſtring is over-running its proper place and is retarded, the particle *a* will move no faſter than the ſtring, and conſequently in this part of its motion forwards it will be retarded. That this particle *a*, when the ſtring is retarded, can move no faſter than the ſtring moves, will appear from the following conſiderations. As the ſtring puſhes the particle *a* forwards, this particle approaches to *b*, which is the next particle beyond it: and the mutual repulſion of theſe two particles is inverſely as their diſtance from each other, by propoſitions 50, 56. Therefore the nearer *a* is to *b*, the more *a* will be repelled back from *b*. Now whilſt the ſtring continues to puſh *a* forward, and is accelerated, ſo long *a* being driven before the ſtring will continue to be accelerated, notwithſtanding this repulſion. But as the force of the ſtring abates, as it will abate when the ſtring is retarded, this repulſion of *a* from *b* will prevent *a* from moving forwards any faſter than the ſtring moves. Therefore, when the ſtring is retarded *a* will ſtill move forwards before the ſtring, but will be retarded as the ſtring is.

But the propoſition affirms farther, that this acceleration and retardation of the particle *a*, which is neareſt to the ſtring, will ſucceſſively be communicated to *b*, *c*, *d*, &c. which are other particles lying beyond *a*. It is evident that, when the particle *a* is driven forwards, it muſt approach to the particle *b*, or that the diſtance *ab* muſt be diminiſhed: and conſequently, by propoſition 46, the mutual repulſion between *a* and *b* muſt be encreaſed. Now as the motion begins from *a*, the diſtance of *a* from *b* muſt be diminiſhed, before the diſtance of *b* from *c* can be diminiſhed. Therefore this particle *b* will be nearer to *a* than to *c*, and for that reaſon, being repelled more by *a* than it is by *c*, it will begin to move

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from *a* towards *c*. By this motion of the particle *b* the diſtance *bc* is diminiſhed: and, ſince the particle *b* begins to move before the particle *c* is moved, the diſtance *bc* will be diminiſhed, before the diſtance *cd* is diminiſhed, or by the motion of *b*, the diſtance of *b* from *c* will become leſs than the diſtance of *c* from *d*. Therefore, by propoſition 46, *c* will be repelled more by *b* than it is by *d* in proportion as its diſtance from *b* is leſs than its diſtance from *d*. And for this reaſon the particle *c*, being acted upon by two repulſions one ſtronger than the other, will begin to move in the direction of the ſtronger or from *b* towards *d*. In like manner the motion will be communicated from *c* to *d*, from *d* to *e*, from *e* to *f*, &c.

Before we prove that each particle, as it moves forwards, will be accelerated and retarded in the ſame manner that *a* was, the reader will do well to obſerve, that the motion, which begins from *a*, is communicated in ſucceſſion to thoſe, which lie beyond it, or is communicated to them one after another, and not to all of them at once. For at the inſtant when *a* begins to move the diſtances *ab* and *bc* are equal, by propoſition 43, and conſequently *b* is equally repelled both by *a* and *c*, ſo that it will move neither way, till *a* has firſt moved out of its place and the diſtance *ab* is by this means diminiſhed. In like manner *b* muſt be moved out of its place, and muſt come nearer to *c*, ſo that *bc* may be leſs than *cd*, or ſo that the particle *c* may be at a leſs diſtance from *b* than from *d*, before this partcle *c* will begin to move. For the ſame reaſon *c* muſt be moved out of its place and muſt come nearer to *d*, before *d* will move. And this holds good in all the other particles; each particle muſt be moved out of its place before the next beyond it can be made to move. Therefore the motion, which begins from *a*, is communicated in ſucceſſion to the particles, which lie beyond it, or they are moved one after another, and not all at once. But, ſince the particles are moved one after another, it will take up ſome time to communicate the motion along the line *ax*.

Now each particle, that lies beyond *a*, as it moves forwards in its turn, is accelerated and retarded, in the ſame manner that *a* is. Since *a* begins to move ſooner than *b*, *a* will approach to *b* before *b* approaches to *c*; and conſequently the diſtance *ab* will become leſs than the diſtance *bc*, and *b* being more repelled from *a* than from *c*, by propoſition 46, will move from *a* towards *c*. Whilſt *b* is thus moving forwards it will be accelerated firſt, and then retarded. Since *a* in the firſt part of its motion forwards is accelerated, though *b* is likewiſe moving forwards, yet *a* will gain upon it, or will keep approaching to it, ſo that the diſtance *ab* will decreaſe as long as *a* is accelerated: for as the motion begins from *a*, ſo

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every degree of velocity will likewiſe begin from *a*; that is, whilſt *a* and *b* are both moving forwards, the velocity of *a* will keep encreaſing, and though *b* is likewiſe moving forwards, yet *a* will have every additional degree of velocity in this accelerated motion, before the ſame degree of velocity is communicated to *b*: and conſequently in every infinitely ſmall inſtant of time *a* will be moving faſter than *b*. But if *a* gains upon *b*, whilſt both of them are moving forwards, becauſe *a* in each inſtant moves faſter than *b*, it follows, that the diſtance *ab* will decreaſe faſter than the diſtance *bc*, or *a* will approach faſter to *b* than *b* approaches to *c*. Now as long as *ab* is leſs than *bc*, the particle *b* will be more repelled by *a* than by *c*; and conſequently *b* will keep moving forwards, and, being in the mean time conſtantly repelled from *a*, will in every inſtant of its motion have freſh velocity communicated to it, or will be accelerated. Thus in the acceleration of the particle *a*, each degree of velocity begins from *a*, and is communicated from thence to *b*; and conſequently *b* is accelerated in the ſame manner that *a* is. After the particle *a* has been accelerated, it will be retarded as has been ſhewn already. And as in the acceleration every degree of motion began firſt from *a* and then was communicated to *b*; ſo in the retardation every degree of motion will be loſt by the particle *a* before it is loſt by the particle *b*. Conſequently, ſince in every inſtant, after *a* begins to be retarded, *a* will move ſlower than *b* in the ſame direction, the diſtance between *a* and *b* will encreaſe. Whilſt the diſtance of *b* from *a* is thus encreaſing, *b* will ſtill move forwards with the velocity that it acquired during its acceleration. And ſince *b* by moving forwards approaches to *c*, and conſequently is perpetually repelled back by *c*, this repulſion will retard its motion, till all the velocity, that it had acquired before, is loſt. Now as *a*, after it had been accelerated, was retarded by its approach to *b*; ſo *b*, after it had been accelerated, is retarded by its approach to *c*. Therefore *b* is retarded in the ſame manner that *a* was.

What has been proved of the particle *b* may be proved likewiſe of the particle *c*. Since *b* begins to move before *c*, every degree of velocity, whilſt *b* is accelerated, will begin from *b*, before it is communicated to *c*; and thus in every inſtant *b* will be moving faſter towards *c*, than *c* is moving towards *d*. From hence it follows that, *c* will move forwards, becauſe it will be more impelled from *b* than from *d*: and the force, which impells it forwards, by acting continually upon it, will produce velocity in every inſtant or will accelerate the particle *c*. When *b* communicates the laſt degree of velocity to *c*, it begins to be retarded itſelf; ſo that *b* will begin to loſe velocity, when *c* is the moſt accelerated. Conſe-

ſequently, after this, *b* will in every inſtant move ſlower than *c*: and though they are both moving the ſame way, yet as *c* moves faſter than *b*, they will mutually depart from each other, or the diſtance *bc* will encreaſe. But as *c*, with the velocity that was acquired during its acceleration, moves forwards and approaches to *d*, it will be continually more and more repelled back from *d*, by propoſition 42, and therefore will be retarded in its motion forwards. Now ſince *b* is firſt accelerated by the approach of *a* towards it, and then is retarded by its own approach towards *c*, and ſince *c* is firſt accelerated by the approach of *b* towards it, and then is retarded by its own approach towards *d*; it follows, that *c* is accelerated and retarded in the ſame manner with *b*. But *b* is accelerated and retarded in the ſame manner with *a*, as has been ſhewn already. Therefore *c* is accelerated and retarded in the ſame manner with *a*.

This method of reaſoning might be applyed to all the other particles, that lie beyond *c*, ſuch as *d*, *e*, *f*, &c. And conſequently, as the particle *a*, which lies neareſt to the ſtring, is accelerated and retarded in the ſame manner with the ſtring; ſo this acceleration and retardation will be ſucceſſively communicated to the other particles, which lie beyond it.

60. *As the particles of air are puſhed forwards by the vibration of an elaſtic ſtring, the air is condenſed, whilſt the particles are accelerated, and its natural denſity is reſtored, whilſt they are retarded.*

Whilſt the particle *a*, Plat. XIX. fig. 6, is accelerated in its progreſs, it approaches to *b*; whilſt the particle *b* is accelerated it approaches to *c*; whilſt the particle *c* is accelerated it approaches to *d*, and ſo on; as has been proved under propoſition 59. But the air is condenſed, where the particles approach to one another. Therefore the air is condenſed, whilſt the particles are accelerated in their progreſs.

Whilſt the particle *a* is retarded in its progreſs, the diſtance between *a* and *b* encreaſes, whilſt the particle *b* is retarded, the diſtance between *b* and *c* encreaſes; whilſt the particle *c* is retarded the diſtance between *c* and *d* encreaſes; and ſo on; as has been proved under propoſition 59. But as the diſtance between the particles encreaſes, the air recovers its natural denſity. Therefore the air, whilſt the particles are retarded in their progreſs, recovers its denſity.

Now the acceleration in the progreſs of the particles begins from the particle *a*, and then is propagated forwards to the particles *b*, *c*, *d*, &c. which lie beyond it: in like manner the retardation begins from *a*, and then proceeds forwards to *b*, *c*, *d*, &c; by propoſition 59. But the air is condenſed where the particles are accelerated in their progreſs, and reco-

vers its natural denſity, where they are retarded. Therefore the condenſations, which begin from *a*, are propagated forwards, and, when the condenſation in any place is over, the air in that place recovers its natural denſity.

Let us now conſider the ſituation of the particles in reſpect of one another, when the ſtring ACB, fig. 4, has performed one vibration, that is, when the middle point of it has moved from C to E. Since the particles come cloſer to one another, as they are accelerated; the diſtance between any particle *a* and the next particle to it will be the leaſt, or the air between *a* and *b* will be the moſt condenſed, when *a* is accelerated up to its greateſt velocity. This happens, as appears from propoſition 59, when the middle point of the ſtring, that is ſuppoſed to impell *a*, has moved from C to D. In like manner, when *b* is in its ſtate of greateſt acceleration, the diſtance *bc* becomes the leaſt. And when *c* is in its ſtate of greateſt acceleration, the diſtance *bc* becomes the leaſt. But every particle as it moves forwards is accelerated firſt and then retarded, by propoſition 59. And ſince the velocity of any particle, like that of the ſtring, is loſt, whilſt the particle is retarded, in the ſame manner and by the ſame degrees that it was acquired; it follows that any particle *a* will be accelerated during one half of its progreſs, and will be retarded during the other half. Therefore any particle will have its greateſt velocity, and conſequently, in reſpect of the next particle beyond it, will be in its ſtate of greateſt condenſation in the middle of its progreſs. But in the other half of its progreſs the acquired velocity decreaſes, till it becomes nothing. And as the particle is retarded, its diſtance from the next beyond it encreaſes. Therefore, when the particle has performed this other half of its progreſs, the diſtance between that and the next will have encreaſed juſt as much as it had decreaſed before; and conſequently theſe two particles will then be at their natural diſtance from one another. Suppoſe the motion, which begins from *a*, to be communicated as far as *d*, juſt when *a* has performed half its progreſs; that is, ſuppoſe that *d* is juſt beginning to move at the inſtant when *a* is in its ſtate of greateſt acceleration. At this inſtant all the particles from *a* to *d* will be in motion, and the air from *a* to *d* will be condenſed. But ſince *a* is in its ſtate of greateſt acceleration before any of the particles, that lie beyond it, though the air is condenſed from *a* to *d*, yet it will be the moſt condenſed between *a* and *b*. The particles *d* and *e* will at this time be at their natural diſtance from each other; becauſe, by the ſuppoſition, the motion is only beginning in *d*, ſo that *d* has not made any approach towards *e*. This is the ſtate of the air, when *a* has made half of its progreſs. Now, ſince whilſt

a is performing half of its progreſs, the motion, which began from *a*, reaches to *d*; in an equal time, or whilſt *a* is performing the other half of its progreſs, the motion muſt neceſſarily be communicated from *d* to ſome particle, which lies as far beyond *d* as *d* lies beyond *a*. Therefore, taking *dg* equal to *ad*, when *a* has completed its progreſs, the particle *g* will begin to move. But the acceleration of the particles is propagated forwards in the ſame manner that the motion itſelf is, or the greateſt degree of velocity is communicated from one particle to another in the ſame manner with the firſt degree. Therefore ſince the motion is propagated as far as from *d* to *g*, whilſt *a* is performing the latter half of its progreſs; the greateſt acceleration will in the ſame time be propagated to an equal diſtance. But in the middle of its progreſs *a* was in the ſtate of greateſt acceleration: conſequently, ſince *ad* is equal to *dg*, whilſt the motion is propagated from *d* to *g*, the greateſt acceleration will be propagated from *a* to *d*; or, when *a* has completed its progreſs, *d* will be in the ſtate of greateſt acceleration. At this inſtant, all the particles beyond *g* will be at their natural diſtances from each other; becauſe *g* is only beginning to move and has not made any approach to the particles beyond it. The particles *d*, *e*, *f*, *g*, will be nearer to one another than uſual, or the air in that part will be condenſed. But *d* is in the ſtate of greateſt acceleration: therefore the condenſation about *d* will be the greateſt. The particle *a* during the latter half of its progreſs has been retarded, and the diſtance *ab* has kept encreaſing: conſequently at the end of its progreſs, when it has loſt all its velocity, the diſtance *ab* will be enlarged to the natural diſtance between two particles: whilſt about *b* and *c*, though theſe particles are in a retarded ſtate, the diſtances will be leſs than the natural ones; becauſe theſe particles will not have completed their progreſs or will not have loſt all their velocity ſo ſoon as *a* has. This is the ſtate of the air from *a* to *g*, at the inſtant that *a* has completed its progreſs, or when the ſtring ACB, which is ſuppoſed to put *a* in motion, has performed one vibration, by moving from the ſituation ACB into the ſituation AEB.

If the ſtring was to ſtop in the ſituation AEB, the particle *a* having loſt all its velocity and being at the natural diſtance from *b* would ſtop too. As ſoon as *b* had loſt all its velocity, it would ſtop likewiſe: and ſo would all the other particles *c*, *d*, *e*, *f*, g, in their turns. But then in the mean time, the motion would be communicated forwards to the particles *h*, *i*, *k*, *l*, *m*, *n*, which would in their progreſs be accelerated firſt and then retarded, juſt as the other particles had been before them. Upon this ſuppoſition, when the particles from *a* to *d* were ſtopped in the places, to which they had been puſhed forwards by the firſt vibration of the

ſtring, an equal number of particles from *g* to *n* would be in the ſame ſtate, in which the particles from *a* to *g* were deſcribed to be at the inſtant the ſtring came into the ſituation AEB; that is, the particles about *g* and about *n* would be at their natural diſtances, and thoſe about *k* would be in the ſtate of greateſt condenſation. This condenſation will be propagated forwards, as before, along the ſucceeding particles, *l*, *m*, *n*, *o*, *p*, &c. And thus by one vibration of the ſtring, all theſe particles will be put into an undulating motion; the air will begin to be condenſed at *a*, this condenſation will proceed forwards, and as it proceeds, the particles recover their natural diſtances.

In the two following propoſitions we will conſider the effects, which will be produced, whilſt the ſtring vibrates back again, and returns from the ſituation AEB into the ſituation ACB.

61. *When a particle of air has been puſhed forwards by an elaſtic ſtring, the particle, as the ſtring returns, will return with it, and will be accelerated firſt and then retarded in the ſame manner that the ſtring is: and this ſame acceleration and retardation will ſucceſſively be produced in other particles, which lie beyond it.*

Whilſt E, Plat. XIX. fig. 4, the middle point of the ſtring AEB is returning from E to D, it is accelerated; and, whilſt it is returning from D to C, it is retarded, by propoſition 53. The particle of air *a*, fig. 6, which had been impelled forwards by the ſtring, in the progreſs of its middle point from C to E, will return with the ſtring, and in its return will be firſt accelerated and then retarded in the ſame manner that the ſtring itſelf is.

As the ſtring returns, there will be nothing on that ſide of *a*, where the ſtring was, to keep it in its place: and conſequently, ſince *a* is repelled from *b* and has nothing to keep it in its place, it will move from *b*, or will follow the ſtring. Now this repulſion will act continually upon the particle *a*, whilſt the ſtring is returning from E to D: and this continued action upon *a* will accelerate its motion, ſo that the particle will keep pace with the ſtring. The ſame repulſion will ſtill act upon *a*, after the ſtring is paſſed D and is returning to C. But the particle can move no faſter than the ſtring, becauſe the ſtring will prevent it. And conſequently, ſince the ſtring in this part of its return is retarded, the particle will likewiſe be retarded.

As the particle *a* returns, the diſtance *ab* encreaſes; and, when it has encreaſed ſo as to become greater than *bc*, the particle *b* will be more repelled from *c* than from *a*, and conſequently will move towards *a*, or

will

will begin to return as *a* had done before it. This repulsion will continue greater on the side of *c* than on the side of *a*, as long as the distance *ab* is greater than the distance *bc*, by proposition 42. Therefore so long the repulsion on the side of *c* will continue to communicate new velocity to *b*, or to accelerate it in its return. But when the particle *a* is retarded, and the particle *b* still keeps moving the same way and approaches to *a*; though the velocity, which *b* had acquired, will carry it on, yet because it comes nearer to *a* it will in every instant be more repelled from *a*, and therefore will be retarded. The particle *a* is accelerated, whilst the string returns from E to D. And since *a* begins to move before *b*, or since *a* has the first degree of velocity before *b*, *a* will be accelerated or will have every other degree of velocity, before *b* has the same degree of velocity. Consequently, during the acceleration of *a*, this particle *a* will in every instant move faster than *b*; so that the distance *ab* will keep encreasing. But when the middle point of the string is arrived at D, and the particle *a* begins to be retarded; as it acquired each degree of velocity during its acceleration before *b* acquired the same degree, so it will lose each degree of velocity during its retardation before *b* loses it. Consequently, during the retardation of *a*, this particle *a* will in every instant move slower than *b*; so that the distance *ab* will keep decreasing. But when the distance *ab* is greater than *bc*, *b* will be accelerated by the repulsion of *c*: and when the distance *ab* is less than *bc*, the particle *b* will be retarded. Therefore the particle *b* as well as *a* will be accelerated first and then retarded during its return. Now since in this acceleration each degree of velocity begins first from *a* by the repulsion of *b*, and then proceeds to *b* by the like repulsion of *c*, it follows that *b* will be accelerated in the same manner that *a* is. And since in this retardation each degree of velocity is lost first in *a* by the strings preventing the motion of *a*, and then is lost in *b* by the repulsion of *a* preventing the motion of *b*, it follows that *b* will lose velocity, or will be retarded, in the same manner that *a* is. But *a*, as has been proved, is accelerated and retarded in the same manner with the string. Therefore *b* will likewise be accelerated and retarded in the same manner with the string.

What has been said of the particle *b* may be applyed to the particle *c*, or to any other particle, that lies beyond it. They will each of them return successively, and, as they return, they will be first accelerated and then retarded in the same manner, with *b*, or with *a*, or with the string. Thus in particular the particle *c* will be accelerated and retarded in its return, in the same manner that the particle *b* is. We have already seen that *b* will be accelerated: and since in this acceleration *b* will have each de-

degree of velocity before *c* has it, it follows that the diſtance *bc* will keep encreaſing, and will, during the acceleration of *b*, be greater than *cd*. Therefore *c* will be more repelled from *d* than from *b*, by propoſition 42: and the continued action of this greater repulſion will accelerate *c*. But ſince, in the retardation of *b*, each degree of velocity will be loſt by *b* before it is loſt by *c*, it follows that the diſtance *bc* will keep decreaſing, and, during the retardation of *b*, will be leſs than *cd*. Therefore *c* will be more repelled from *b* than from *d*; and though it will be carryed on towards *b* by the velocity, which it had acquired, yet the continued action of this greater repulſion from *b* will retard it. Now the particle *b*, as has been ſhewn, is firſt accelerated in its return by the greater repulſion on the ſide of *c*, and then is retarded by the greater repulſion on the ſide of *a*. But the particle *c* in its return is firſt accelerated by a like greater repulſion on the ſide of *d* and then is retarded by a like greater repulſion on the ſide of *b*. Therefore the particle *c* in its return is accelerated and retarded in the ſame manner that *b* is. And ſince *b* is accelerated and retarded in the ſame manner that *a* is, and *a* is accelerated and retarded in the ſame manner that the ſtring is, it follows that *c* likewiſe is accelerated and retarded in the ſame manner that the ſtring is.

62. *As the particles of air, after they have been puſhed forwards by an elaſtic ſtring, return to their places, the air is rarefyed, whilſt the particles are accelerated, and its natural denſity is reſtored, whilſt they are retarded.*

Whilſt the particle *a*, Plat. XIX. fig. 6, is accelerated in its return, it departs from *b*; whilſt the particle *b* is accelerated, it departs from *c*; whilſt the particle *c* is accelerated, it departs from *d*; and ſo on; as has been proved under propoſition 61. But the air is rarefyed where the particles depart from one another. Therefore the air is rarefyed, whilſt the particles are accelerated in their return.

Whilſt the particle *a* is retarded in its return, the diſtance between *a* and *b* decreaſes; whilſt the particle *b* is retarded, the diſtance between *b* and *c* decreaſes; whilſt the particle *d* is retarded, the diſtance between *c* and *d* decreaſes; and ſo on; as has been proved under propoſition 61. But the rarefyed air recovers its natural denſity, as the diſtance between the particles encreaſes. Therefore the air recovers its natural denſity, whilſt the particles are retarded in their return.

The acceleration in the return of the particles begins from *a* and then proceeds to the particles *b*, *c*, *d*, &c. one after another: in like manner the retardation begins from *a* and then proceeds to *b*, *c*, *d*, &c. by propoſition 61. Now the air is rarefyed, where the particles are accelerated in their

their return, and recovers its natural denſity where they are retarded. Conſequently the rarefactions are propagated forwards, and when the rarefaction in any place is over, the air in that place recovers its natural denſity.

From what has been ſaid, under this propoſition and under propoſition 60, it appears, that every thing is the ſame in the progreſs and return of the particles, except only, that in the progreſs, when the particles are accelerated, the air is condenſed; but in the return, when the particles are accelerated, the air is rarefyed. And from hence it follows that as, in the progreſs of the particles, the air is moſt condenſed, ſo, in their return, it will be moſt rarefyed, where they are in the ſtate of greateſt acceleration.

Now during the time that the ſtring was vibrating forwards, the motion, which began from *a*, would be propagated as far as *g*, by what was ſhewn under propoſition 60; and at the end of this vibration, when the ſtring is in the poſition AEB, fig. 4, the air is moſt condenſed at *d*, and the particles about *a* and about *g* are at their natural diſtances from one another. And ſince the rarefaction in the return of the particles is produced in the ſame manner as the condenſation in their progreſs; it follows, that, during an equal time, whilſt the ſtring is vibrating back again, the motion, which begins from *a*, will be propagated as far as *g*, and at the end of this vibration, when the ſtring is returned into the poſition ACB, the air will be moſt rarefyed at *d*, and the particles about *a* and about *g* will be at their natural diſtances from one another.

63. *The ſpaces, where the air is condenſed by the vibrations of a ſounding body, are called pulſes; and the ſpaces, where it is rarefyed, are called intervals.*

The reaſon why the ſpaces, where the air is condenſed, are called pulſes has been given already, in propoſition 58. And the reaſon why the ſpaces, where the air is rarefyed, are called intervals is obvious: for they are the intervals between two ſucceſſive pulſes. Thus if *ag* is the firſt pulſe, that is produced by the progreſs of the ſtring: when this pulſe is propagated into the ſpace *gn* during the return of the ſtring, *ag* becomes an interval between the firſt pulſe *gn*, and the next, which the ſtring will produce in its next progreſs: for this ſecond pulſe will begin from *a*; and conſequently the diſtance from *a* to *g* is the interval between the firſt pulſe and the next that follows it.

64. *The pulſes and intervals ſucceed one another alternately, as they are propagated forwards.*

Whilſt

Whilſt the ſtring is vibrating forwards, the air in the ſpace between *a* and *g* is condenſed into a pulſe, and in an equal time, this motion will be propagated forwards, ſo as to condenſe the air or produce a pulſe in the ſpace between *g* and *n*, by propoſition 60. But in this equal time, whilſt the condenſation or pulſe is produced between *g* and *n* the ſtring will vibrate back again by prop. 54: and whilſt the ſtring is vibrating back again the air in the ſpace between *a* and *g* is rarefyed and becomes an interval, by propoſition 62. In an equal time the condenſation will be propagated forwards, from the ſpace *gn* into the ſpace *nt*, by propoſition 60: and in the mean while the rarefaction will be propagated forwards, from the ſpace *ag* into the ſpace *gn*. So that, when *nt* becomes a pulſe, *gn* will become an interval. And for the ſame reaſons when *tz* is condenſed into a pulſe, *nt* will be rarefyed into an interval. In the ſame manner the pulſes and intervals, as long as they are propagated forwards, will alternately ſucceed one another.

65. *When the elaſtic ſtring, which excited the pulſes, ceaſes to vibrate, the particles of air next to the ſtring continue at reſt, and no new pulſes are produced.*

When an elaſtic ſtring, ſuch as AB, Plat. XIX. fig. 4, that is bent into the poſition ACB, ſtrikes upon a particle of air at *a*, this particle and thoſe, that lie beyond it, will ſucceſſively be puſhed forwards, as the ſtring vibrates into the poſition AEB: and each particle in its progreſs will be accelerated firſt, till it comes to the middle of the line of its progreſs, and then will be retarded, till it comes to the end of that line, by propoſition 59. Now, ſince the particle *a* for inſtance, loſes its velocity in the latter half of its progreſs, juſt as it acquired velocity in the former half; when the ſtring comes to the poſition AEB, this particle will have no velocity left, and conſequently can go forwards no farther. And when *a* thus comes to the end of its progreſs, or can go forwards no farther, it is at its natural diſtance from *b*, by propoſition 60. Therefore, if the ſtring was to ſtop in this poſition AEB, the particle *a* would not be ſo forcibly repelled by *b* as to make it move back again. But ſince, if the ſtring was to ſtop, the particle *a* could neither move backwards nor forwards, it muſt ſtop in the place where it was at the end of its progreſs. The ſame may be ſaid of every other particle *b*, *c*, *d*, &c. For each particle will have loſt all its velocity, when it comes to the end of its progreſs, and will be at the natural diſtance from the next particle to it: upon which account, as each particle began to move, and then was accelerated and retarded in its turn, ſo likewiſe it will ſtop in its turn; and conſequent-

quently no new pulſes will be produced from *a*, after the ſtring has ceaſed to vibrate or has ſtopped in the poſition AEB.

But when the ſtring inſtead of ſtopping returns to its firſt poſition ACB; the particles *a*, *b*, *c*, &c. return with it, and in their return are ſucceſſively accelerated firſt and then retarded, by propoſition 61; till the ſtring comes to this poſition ACB: and then the particle *a*, having loſt all its velocity, and being, by propoſition 62, at its natural diſtance from *b*, will ſtop. Now all the particles in their turns are accelerated and retarded in the ſame manner that *a* is; and conſequently all the particles, when they have completed their return, and are placed at their natural diſtances from one another, will ſtop in their turns. In this return of the particles an interval is produced. And as no new pulſe would be produced, if the ſtring was to ſtop in the poſition AEB; ſo no new interval would be produced, if it was to ſtop in the poſition ACB.

Indeed the ſtring will not ſtop of its own accord in either of theſe poſitions, but will continue to vibrate backwards and forwards, by propoſition 53, till the reſiſtance of the medium and the friction at A and B ſtop it at laſt in the rectilineal poſition ADB. But what has already been ſaid will be applicable wherever the ſtring ſtops. Though the pulſes and intervals that were produced during its vibrations will be propagated forwards, yet as ſoon as the ſtring ſtops no new ones will be produced. This is the reaſon why the ſound of a ſtring, or a bell, or any other vibrating body, will ceaſe, as ſoon as the body has done vibrating. For as theſe pulſes are the cauſe of the ſound, there will be no ſound produced, when no pulſes are produced.

66. *The ſpace, in which the air is condenſed during the progreſs of any ſingle particle, is called the thickneſs of a pulſe.*

If in the progreſs of the particle *a*, Plat. XIX. fig. 6, the motion is communicated as far as *g*, ſo as to condenſe the air from *a* to *g*, the diſtance *ag* is the thickneſs of the firſt pulſe. In the return of the particle *a* the air in the ſpace *ag* is rarefyed, and *ag* will be an interval: but in the mean time the air from *g* to *n* will be condenſed during the progreſs of the particle *g*; and this ſpace *gn* is the thickneſs of the ſecond pulſe.

The whole ſpace *an*, through which the motion is propagated during both the progreſs and return of the particle *a*, or the diſtance, to which one pulſe is propagated before the next is produced, is uſually called the thickneſs or breadth of a pulſe, ſo as to include in this breadth both the pulſe *gn* and the interval *ag*. But perhaps the following propoſitions will be more readily underſtood, if we keep to the definition here propoſed.

67. *The particles of air, in their progreſs and in their return, are accelerated and retarded by the ſame law with a pendulum that vibrates in a cycloid.*

The ſtring, which communicates the motion to the particles, is accelerated and retarded, both in its progreſs and in its return, by the ſame law with a pendulum that vibrates in a cycloid, by propoſition 53. But the particles are accelerated and retarded in the ſame manner with the ſtring, or every degree of velocity is ſucceſſively communicated from the ſtring to the particles, both in their progreſs and in their return, by propoſitions 59, 61. Therefore the particles are accelerated and retarded by the ſame law with a pendulum that vibrates in a cycloid.

68. *All the particles of air, which are put into motion by the ſame ſtring, perform their vibrations in equal times.*

We ſhall ſee hereafter, that as the motion is communicated forwards from *a*, Plat. XIX. fig. 6, it decays; that is, the firſt particle *a* will be puſhed farther out of its place than the particle *g*, the particle *g* farther than the particle *n*, the particle *n* farther than the particle *r*, and ſo on. Now the diſtance, to which any particle is puſhed forwards from its proper place, is the line, upon which it vibrates in its progreſs and in its return. Conſequently the particle *a* will vibrate upon a longer line than the particle *g*, the particle *g* upon a longer line than the particle *n*, &c. But though theſe particles vibrate, ſome upon longer and ſome upon ſhorter lines, yet all of them will perform their reſpective vibrations in equal times.

Before we go on to prove the truth of this, it may not be improper to deſire the reader to diſtinguiſh between the thickneſs of a pulſe and the line upon which a ſingle particle vibrates. If *ag* is the thickneſs of the firſt pulſe, it is not neceſſary that *ag* ſhould be the line upon which the particle *a* vibrates. For this particle *a* is moved but a very little way out of its place, the line of its progreſs is very ſhort: but in this progreſs the motion is communicated ſucceſſively to *b*, *c*, *d*, *e*, *f*, *g*; and by this means the air is condenſed as far as from *a* to *g*; though each particle in the mean time, like the particle *a*, is puſhed but to a very ſmall diſtance from its proper place.

That all the particles which are impelled by the ſame ſtring will perform their vibrations in equal times follows from propoſitions 59, 61. For the ſtring performs all its vibrations in equal times, whether the middle point of it vibrates upon a long or upon a ſhort line. And, ſince the particles are accelerated and retarded in the ſame manner that the ſtring is, they will likewiſe perform their reſpective vibrations in equal times.

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The ſtring is accelerated and retarded by the ſame law with a pendulum that vibrates in a cycloid; and for that reaſon it performs all its vibrations either upon long or upon ſhort lines in equal times, by propoſition 54. But each particle of air, to which the ſame ſtring communicates motion, is likewiſe accelerated and retarded by the ſame law with a pendulum vibrating in a cycloid, by propoſition 67: and conſequently all the particles, for the ſame reaſon, will perform their reſpective vibrations in equal times.

Now the tone of a ſound depends on the duration of a ſtroke from one of the pulſes on the ear. Sounds are more grave, as the ſtroke, which a pulſe makes, continues for a longer time, and more acute as it continues for a ſhorter time. But ſince, by propoſition 54, all the vibrations of the ſame ſtring from the firſt to the laſt are performed in equal times; and ſince, by this propoſition, the particles of air, as they are moved by each vibration of the ſtring, perform their vibrations in equal times, whatever is their diſtance from the ſtring: it follows firſt, that the ſame ſtring will ſound the ſame note during the whole time of its vibration, or from the firſt vibration to the laſt, and ſecondly, that the note of a ſound produced by a given ſtring will be the ſame at all diſtances of the ear, at which the ſound can be heard. For if all the particles, that are moved, perform their vibrations in equal times, all the pulſes will be produced in equal times, and conſequently the impreſſion or ſtroke of each upon the ear will continue for an equal time.

69. *The thickneſs of a pulſe is proportional to the time, in which it is produced.*

If the particle *a*, Plat. XIX. fig. 6, by continuing to go forwards for a certain time will produce a pulſe, whoſe thickneſs is *ag*; then, if the time of the particles progreſs had been twice as long, the thickneſs of the pulſe would be *an* or twice *ag*; if the time of progreſs was triple, the thickneſs of the pulſe would be *at* or triple *ag*. And in all other caſes, as the time of progreſs, which is the time of producing a pulſe, is greater or leſs, the thickneſs of the pulſe, which is produced during that progreſs, will be greater or leſs in the ſame proportion.

The particles are moved ſucceſſively or one after another, and not all at one and the ſame inſtant, by propoſition 59. Thus when the particle *a* is puſhed forwards, the particles *b*, *c*, *d*, &c. are not puſhed forwards at one and the ſame inſtant; but ſome little time paſſes between the firſt motion of *a* and the firſt motion of *b*, and again ſome little time paſſes between the firſt motion of *b* and the firſt motion of *c*, and ſo we may ſay

of all the other particles, to which this motion is communicated. The very ſmall part of time, that paſſes after one particle has begun to move before the next to it begins to move, we will call an inſtant. Therefore, ſince in each inſtant the motion of any one particle is communicated to the next particle beyond it, the greater number of inſtants there are in the time taken up by *a* to complete its progreſs, the greater will be the number of particles that are put in motion, and the leſs the number of inſtants, the fewer particles will be moved. Now if the particles from *a* to *g* are moved during the progreſs of *a*, the thickneſs of the pulſe would be *ag*. If twice the number of particles were moved in the time of progreſs, the motion during the progreſs would be communicated to twice the diſtance *ag*, or the thickneſs of the pulſe would be *an* equal to twice *ag*. And univerſally, the thickneſs of a pulſe will be proportional to the number of particles that are moved during the time of any ſingle particles progreſs, or during the time in which the pulſe is produced. But the number of particles, that are moved during the time of progreſs, is proportional to that time or to the number of inſtants contained in that time: Therefore the thickneſs of a pulſe is proportional to the time of progreſs or to the time in which that pulſe is produced.

From hence it follows that the thickneſs of the pulſes is greater in ſounds that are grave than in thoſe that are acute. Becauſe the times of vibration in grave ſounds is greater than in acute ones, as was obſerved under propoſition 68.

70. *All the pulſes, that are produced by the ſame ſtring, are of the ſame thickneſs at all diſtances from the ſtring.*

The thickneſs of a pulſe is proportional to the time in which it is produced, by propoſition 69, that is, to the time in which any particle performs a vibration in its progreſs, by propoſition 66. But at all diſtances from the ſame ſtring the particles perform their vibrations in equal times, by propoſition 68. Therefore at all diſtances from the ſame ſtring the pulſes have the ſame thickneſs.

71. *The pulſes, which are produced by a ſounding body, are hollow ſpherical ſurfaces; ſo that the ſound produced by thoſe pulſes is propagated in all directions.*

We have hitherto conſidered the motion ariſing in the air from the vibration of an elaſtic ſtring as propagated along a right line in one direction only. But if a ſtring was to vibrate at the point *a*, Plat. XIX. fig. 7, the vibratory motion, which it communicates to the air would be pro-

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pagated not only along the line *agntz*E or from *a* to E; for ſince the ſtring moves backwards as well as forwards, it is plane that the ſame vibratory motion will be communicated likewiſe the other way along the line *acdfgb*I, or from *a* to I. But this motion will be produced not only in theſe directions in which the ſtring moves: for the air, when it is condenſed at *a*, will expand itſelf in all directions round *a*; and conſequently, the condenſation will be propagated equally all round as well as towards *g* and *c*, ſo that the firſt pulſe will be a ſpherical ſurface *bcpg* having *a* for its center. The vibratory motion thus produced in all the air within the ſpace *bcpg* will be communicated all round to the particles encompaſſing this ſpace on all ſides, in the manner already deſcribed in propoſition 59, and by this means all the air included between *rdsn* and *bcpg* will be condenſed, or this hollow ſpherical ſurface will be the ſecond pulſe. Whilſt this ſecond pulſe is producing the air contained in the ſpace *bcpg* will be rarefyed, in the manner deſcribed in propoſition 61, and will become an interval. And thus theſe pulſes and intervals will be alternately propagated from *a* towards K and L, and in all poſſible directions. Upon the plane of the paper, where the figure is drawn, ſpherical ſurfaces could be no otherwiſe repreſented than by ſuppoſing theſe ſpheres cut through *a* their common center, and then drawing the ſections of the ſpheres, which ſections are ſo many circles. The reader muſt therefore take care to ſupply by his imagination what is neceſſarily wanting in the figure, and to remember that any pulſe, as the ſecond for inſtance, whoſe thickneſs is *gn*, is a hollow ſphere which encompaſſes the center *a* on all ſides.

If an elaſtic ſtring was to vibrate, or a bell was to be ſtruck, or a flute to be blown, at the point *a*; the ſound would be heard in any place, to which the pulſes produced by the ſounding body are propagated. And ſince the pulſes are propagated all round the ſounding body in all poſſible directions, if the ſound could be heard ſuppoſing the ear to be placed at E or at I, it might likewiſe be heard on which ever ſide the ear was placed, at the ſame diſtance from the point *a*, where the ſound begins.

72. *When the pulſes produced by a ſounding body paſs either through a hole in any obſtacle or by the edges of it, they are propagated from thence in all directions.*

If theſe pulſes paſs through a window into a room, the ſound may be heard, wherever the ear is placed in the room, and not only if it is directly before the window. In like manner, when they paſs over the top or by the end of a wall, the ſound will be heard by an ear placed any where behind

hind the wall. For when the condenſed air or pulſe gets beyond the obſtacle, by paſſing either through a hole in it or by the edges of it, the motion of the pulſe will be communicated to the air all about it, by propoſition 71, and conſequently the pulſes produced beyond the obſtacle will be propagated behind it in all poſſible directions.

For this reaſon it is, that ſound paſſes through bent tubes as well as through ſtrait ones. Becauſe the motion in the air, to which ſound is owing, will be communicated to the air in whatever direction the particles are placed, and not only to ſuch particles as are placed in the ſame right line. But light, which is not owing to any preſſure upon or vibration in a fluid medium, can only paſs through ſtrait tubes, by propoſition 34 of optics.

73. *The intenſity of ſound, at different diſtances from the ſounding body, is inverſely as the ſquares of the diſtances.*

Sounds may differ from one another both in reſpect of their tone and in reſpect of their intenſity or ſtrength. In reſpect of their tone they are diſtinguiſhed into grave and acute: in reſpect of their intenſity, they are diſtinguiſhed into loud or ſtrong, and low or weak. The tone of any ſound depends, upon the time that an impreſſion continues, and is not altered by the diſtance of the ear from the ſounding body. But the intenſity or ſtrength of any ſound depends upon the force with which the particles of air, as they are condenſed, ſtrike the ear: and this force we ſhall find is different at different diſtances, ſo that a ſound, which is very loud, if we are near the body that produces it, would be weaker, if we were farther from it, and our diſtance from it may be ſo great that we cannot hear it at all.

The proportion, in which the intenſity of ſound decreaſes, as the diſtance of the ear from the ſounding body encreaſes, is this: if the different diſtances, at which the ear is placed, are to one another as 1, 2, 3, 4, 5, then the ſquares of thoſe diſtances are 1, 4, 9, 16, 25, and the intenſity of ſound will be inverſely as theſe ſquares, or as the reciprocals of theſe ſquares, that is, the ſtrength of the ſound will decreaſe in the ſame proportion with the fractions $\frac{1}{1}\ \frac{1}{4}\ \frac{1}{9}\ \frac{1}{16}\ \frac{1}{25}$, which are the reciprocals of 1, 4, 9, 16, 25. This is what we mean when we ſay, that the intenſity of ſound is inverſely as the ſquares of the diſtances, or that it decreaſes, in the departure of the ear from the ſounding body, in the ſame proportion that the ſquares of the diſtances encreaſe.

If the ear is at *n*, Plat. XIX. fig. 7, the ſound produced at *a* will be ſtronger, than if the ear was at E. The diſtance of *n* from *a* is *an*; the di-

diſtance of E from *a* is *a*E: the intenſity of the ſound at the diſtances *an* and *a*E will be as the ſquares of *an* and *a*E inverted. Thus ſuppoſing *an* to be to *a*E as 2 to 5, the ſquares of *an* and *a*E are to each other as 4 to 25: ſo that the ſtrength of the ſound, when the ear is at *n*, will be to its ſtrength, when the ear is at E, as 4 to 25 inverted, or as 25 to 4, or as $\frac{1}{4}$ to $\frac{1}{25}$.

In order to prove this we muſt obſerve firſt, that the pulſe, which ſtrikes upon the ear either at *n* or at E or at any other diſtance from the ſame ſounding body, is of the ſame thickneſs, by propoſition 70; and ſecondly, that the orifice of the ear, at which the pulſe or condenſed air enters it, is the ſame. Conſequently the quantity of matter or number of particles, by which the ear is ſtruck, will be the ſame at all diſtances. From hence it follows, by propoſition 13 of mechanics, that the force, with which the ear is ſtruck, will be as the velocity of the particles. This velocity is inverſely as the ſquares of the diſtances from *a*. Each pulſe is a ſpherical ſurface of a given thickneſs, and upon this account the quantity of matter, which each pulſe contains, is as the ſquare of its ſemidiameter. For all ſimilar ſurfaces are as the ſquares of their homologous ſides; Euc. b. VI. prop. 20. corol. 1. and the ſurfaces of all ſpheres are ſimilar. Thus the quantity of matter contained in the ſecond ſpherical pulſe from *a*, the thickneſs of which is *gn*, is to the quantity of matter contained in the pulſe whoſe thickneſs is *z*E, as the ſquare of *an* to the ſquare of *a*E. Now the quantity of motion is the ſame in each pulſe, for the motion, which begins from *a*, is ſucceſſively communicated to all the pulſes that are propagated from thence. But where the quantity of motion is given the velocity will be inverſely as the quantity of matter, by propoſition 14 of mechanics. Conſequently, ſince the quantity of matter in each pulſe is as the ſquare of its ſemidiameter, the velocity, with which the particles move in each pulſe, will be inverſely as the ſquare of its ſemidiameter. But the ſemidiameter of that pulſe, which ſtrikes the ear, is the diſtance of the ear from the ſounding body. Thus, if the ear is at *n*, the ſemidiameter of the pulſe, which ſtrikes it, is *an* or is the diſtance of the ear from *a*: and if the ear is at E the ſemidiameter of the pulſe, which ſtrikes it, is *a*E, or is the diſtance of the ear from *a*. Therefore the ſquare of the ſemidiameter of the pulſe, which ſtrikes the ear, is the ſquare of the ears diſtance from *a*; and conſequently the velocity with which the particles ſtrike the ear, or the ſtrength of the ſound, is inverſely as the ſquare of the ears diſtance from the ſounding body.

74. *The intenſity of ſound decreaſes, as the vibrations in the ſounding body grow weaker.*

If

If an elaſtic ſtring was to ſtop all at once, the ſound produced by it would ceaſe immediately, as was ſhewn under propoſition 65. But if the vibrations of the ſtring decay gradually, the ſound will likewiſe keep growing weaker, till it becomes too weak to be heard. The ſtring performs all its vibrations from the firſt to the laſt in equal times, by propoſition 54; and conſequently each pulſe, that the ſtring produces, is produced in an equal time, by propoſition 60; and upon that account each pulſe from the firſt to the laſt will have the ſame thickneſs, by propoſition 69. But when the thickneſs of the pulſe is given, the quantity of air, or number of particles, by which the ear is ſtruck, will likewiſe be given; and, by propoſition 13 of mechanics, the moment, with which it is ſtruck, or the intenſity of the ſound, will be as the velocity with which the particles move. Now the velocity of the ſtring is ſucceſſively communicated to the particles of air, as they are made to vibrate, by propoſition 59. Therefore as the velocity of the ſtring decays, the velocity of the particles and conſequently the intenſity of the ſound will likewiſe decay.

75. *The intenſity of ſound is encreaſed by a ſpeaking-trumpet.*

When a man ſpeaks without ſuch a trumpet, the pulſes, as they are produced, dilate themſelves in all directions, by propoſition 71, or the motion is immediately communicated to the air all round him. But if he ſpeaks in a trumpet, his voice, that is, the motion produced by his voice, is confined to the ſmall portion of air contained in the trumpet. For this reaſon as there are fewer particles to be moved, than there are when the motion dilates itſelf immediately in all directions, the motion that is communicated will be greater; and conſequently, when the voice comes out of the trumpet, its intenſity or ſtrength will be greater, than it would have been, if it had been propagated in all directions at firſt.

76. *The velocity of ſound is directly as the thickneſs of the pulſes, and inverſely as the time in which each pulſe is produced.*

The velocity of ſound is greater in proportion as it takes up a leſs time to paſs from the ſounding body to a given diſtance: and its velocity is leſs, in proportion as it takes up a greater time to paſs to the ſame diſtance.

Firſt, if the thickneſs of pulſes, which are produced in a given time, could be different; then in proportion as the thickneſs of thoſe pulſes is leſs the ſound will take up more time to paſs from *a* to E, Plat. XIX. fig. 7, or its velocity will be leſs. And as the thickneſs of the pulſes is greater,

greater, the ſound will take up leſs time to paſs from *a* to E, or its velocity will be greater. For ſuppoſe the thickneſs of each pulſe to be equal to *ag*, then five ſuch pulſes will reach from *a* to E, and conſequently, ſince the thickneſs of five ſuch pulſes is equal to *a*E, in the time that theſe five pulſes are produced the ſound will be propagated from *a* to E, or through the ſpace *a*E. But if the thickneſs of each pulſe had been only $\frac{1}{2}$ *ag*, then there muſt have been ten ſuch pulſes to reach from *a* to E, or the ſound could not be propagated through the ſame ſpace *a*E, till ten pulſes were produced. Now if any pulſe, whether its thickneſs is *ag* or only $\frac{1}{2}$ *ag*, was produced in an equal time, then the time of producing ten pulſes each of the thickneſs $\frac{1}{2}$ *ag* would be double the time of producing five pulſes each of the thickneſs *ag*. Conſequently, the time taken up to propagate the ſound from *a* to E, when the thickneſs of each pulſe is $\frac{1}{2}$ *ag*, will be double the time that would be taken up to propagate it through an equal ſpace, when the thickneſs of each is *ag*. Thus by halving the thickneſs of each pulſe the time of conveying ſound to a given diſtance is doubled, and conſequently the velocity of it is halved; that is, the velocity decreaſes in the ſame proportion with the thickneſs of the pulſe. The caſe would be the ſame in any other inſtance: for the leſs the thickneſs of each pulſe is, the more of them are requiſite, or the more of them muſt be produced, in order to convey the ſound to a given diſtance. And ſuppoſing each pulſe, whatever its thickneſs is, to be produced in an equal time, the more pulſes there are to be produced, the longer time will be taken up to produce them. Therefore in proportion as the thickneſs of each pulſe is leſs, the time taken up to propagate ſound to a given diſtance will be greater. But as the time required to propagate ſound to a given diſtance is greater, the velocity of it muſt be leſs. Therefore in proportion as the thickneſs of the pulſes is leſs, the velocity of ſound is leſs, or the velocity decreaſes as the thickneſs of the pulſes decreaſes. In like manner we might ſhew, that as the thickneſs of the pulſes is greater, the time taken up to propagate ſound to a given diſtance will be leſs, and conſequently the velocity of it will be greater, upon ſuppoſition that a pulſe, whatever its thickneſs may be, will always be produced in a given time. But ſince, upon this ſuppoſition, the velocity of ſound is leſs as the thickneſs of the pulſes is leſs, and greater as the thickneſs of the pulſes is greater, it follows that the velocity of ſound is directly as the thickneſs of the pulſes.

Secondly; if we ſuppoſe it poſſible for pulſes of an equal thickneſs to be produced in unequal times, then, where the thickneſs of the pulſes is

thus given, the velocity of ſound will be inverſely as the time, in which a ſingle pulſe is produced: or, upon this ſuppoſition, as the time of producing a ſingle pulſe is greater, the velocity of ſound will be leſs; and as the time is leſs, the velocity will be greater. For when the thickneſs of the pulſes is given, a certain number of them will reach to a given diſtance: thus, if the thickneſs of each pulſe is equal to *ag*, five of them will reach from *a* to E, or the ſound will deſcribe the given ſpace *a*E in the time that five pulſes are produced. From hence it follows, that in proportion as the time is leſs, in which five pulſes are produced, ſound will take up leſs time to deſcribe the given ſpace *a*E, or its velocity will be greater; and in proportion as that time is greater, its velocity is leſs. Now the leſs the time is, which is required to produce every ſingle pulſe, the leſs will be the time required to produce five of them: and the longer the time is, which is required to produce every ſingle pulſe, the longer will be the time required to produce five of them. But as the time required to produce five pulſes is greater, the velocity of ſound is leſs; and as that time is leſs, the velocity of ſound is greater. Therefore as the time of producing every ſingle pulſe is greater, the velocity of ſound will be leſs, and as that time is leſs, the velocity will be greater; that is, the velocity of ſound will be inverſely as the time in which each pulſe is produced. The caſe will be the ſame univerſally. The leſs the time, that is requiſite to produce any ſingle pulſe, the leſs time likewiſe will be requiſite to produce any given number of pulſes: and where the thickneſs of the pulſes is given, a given number of them will propagate the ſound to a given diſtance. Therefore the leſs the time, that is requiſite to produce any ſingle pulſe, the leſs the time, that is requiſite to propagate the ſound to a given diſtance; and conſequently the greater is the velocity with which the ſound moves. In like manner we might ſhew univerſally, that the longer the time is, in which a ſingle pulſe is produced, the leſs will be the velocity of the ſound. Therefore, univerſally, the velocity of ſound is inverſely as the time in which each pulſe is produced, upon ſuppoſition that all the pulſes, in whatever time they are produced, have the ſame thickneſs.

But ſince, upon ſuppoſition that the times of producing each pulſe are equal, the velocity of ſound will be directly as the thickneſs of the pulſes; and, upon ſuppoſition that the thickneſs of the pulſes is equal, the velocity of ſound will be inverſely as the time, in which each pulſe is produced; it follows, that, when neither the thickneſs of the pulſes nor the times of producing each pulſe are equal, the velocity of ſound will be in

a ratio compounded of the direct one of the thickneſs of the pulſes and the inverſe one of the time in which each pulſe is produced.

Thus calling the thickneſs of a pulſe B, and the time in which it is produced T, the velocity of ſound will be as B directly and as T inverſely, or as B divided by T, or as $\frac{B}{T}$, for the fraction $\frac{B}{T}$, like all other fractions, is as its numerator B directly, and as its denominator T inverſely, and expreſſes a quotient ariſing from the diviſion of its numerator B by its denominator T.

77. Sound moves with the ſame velocity at all diſtances from the ſounding body.

The ſound of a cannon, or of a bell moves at the rate of 1142 feet in a ſecond at all diſtances from the gun or the bell: if it moves at this rate for the firſt mile, it would move juſt at the ſame rate for the ſecond mile: ſo that a perſon who is within one mile of the cannon, when it is diſcharged, will hear the report juſt as ſoon again as another who is at the diſtance of two miles. The velocity of the ſound does not decreaſe, as it is propagated forwards, but continues the ſame from the firſt to the laſt.

Since all the particles of air, which are put in motion by the ſame ſounding body, at whatever diſtance they are from this body, perform their vibrations in equal times, by propoſition 68, each pulſe will be produced in an equal time, by propoſition 66. And the pulſes at all diſtances from the ſounding body have the ſame thickneſs, by propoſition 70. Now the velocity of ſound is directly as the thickneſs of the pulſes and inverſely as the time in which each pulſe is produced, by propoſition 76. But at all diſtances from the ſame ſounding body the thickneſs of the pulſes and time in which each pulſe is produced are the ſame. Therefore at all diſtances from the ſounding body the velocity of ſound is the ſame.

If in the time that a certain number of pulſes, five for inſtance, are produced, the ſound will be carryed through a certain ſpace from *a* to E, Plat. XIX. fig. 7, then in the time of producing an equal number of pulſes, each of the ſame thickneſs with each of thoſe, the ſound will be carryed to an equal diſtance beyond E. But the next five pulſes will be produced in an equal time, by propoſitions 68, 66, and each of them will have the ſame thickneſs with each of the firſt five, by propoſition 70. Therefore in whatever time the ſound was propagated from *a* to E through the ſpace

aE, it will be carryed forwards from E through an equal ſpace in an equal time: and conſequently the velocity of the ſound in this latter part of its motion is the ſame that it was in the former part.

If the time of producing a pulſe is called T, and the thickneſs of the pulſe is called B, the velocity of ſound will be as $\frac{B}{T}$, by propoſition 76. But at all diſtances from the ſounding body, B is a given quantity, by propoſition 70, and T is likewiſe a given quantity, by propoſitions 68, 66. Therefore at all diſtances from the ſame ſounding body $\frac{B}{T}$ or the velocity of the ſound will be a given or invariable quantity, that is, the ſound will have the ſame velocity from the firſt to the laſt.

But here perhaps the reader may think, that, if ſound continues to move with the ſame velocity at all diſtances from the ſounding body, it ought never to ſtop: for why ſhould not it continue to move on, even to an infinite diſtance, if the velocity of it does not decay? In order to clear up this difficulty we muſt diſtinguiſh between the velocity, with which the particles of air move in performing their vibrations, and the velocity, with which ſound moves in conſequence of thoſe vibrations. We have ſeen, in propoſition 73, that, as the vibrations ſpread themſelves from the ſounding body, the velocity, with which each particle vibrates, will decreaſe, till it is quite deſtroyed: and when this motion ceaſes, the ſound will be propagated no farther. But though the velocity, with which the particles vibrate, decreaſes, as the ſound ſpreads from the ſounding body, yet the times of their vibration will continue the ſame, by prop. 68. And if the times of vibration continue the ſame, the time, in which each pulſe is produced, and the thickneſs of the pulſes will likewiſe continue the ſame, by propoſitions 66, 69. Now the velocity of ſound is not altered, if each pulſe continues to be produced in an equal time, and, when produced, has the ſame thickneſs; by propoſition 77. Therefore, though the velocity, with which the particles vibrate, decreaſes, as the ſound ſpreads, and conſequently the ſound decays, till it ceaſes entirely; yet the velocity, with which the ſound itſelf is propagated, continues the ſame to the laſt, or the velocity, with which it is moving at the time that it ſtops, is the ſame as the velocity, with which it began to move.

78. *Sounds of different tones move with the ſame velocity.*

The fact is evident: for a peal of bells are heard in the ſame order, in which they are rung, whether we are near to them or at any diſtance from them: whereas if the grave tones had moved either faſter or ſlower

than

than the acute ones, the order would be changed in the progreſs of the ſound, ſo that when the ear is at any great diſtance from the bells, in whatever order they were rung, thoſe would be heard the firſt, the ſounds of which move with the greateſt velocities.

Suppoſe there were two ſtrings, whoſe lengths were to each other as 2 to 1; if theſe ſtrings have equal diameters and are ſtretched with equal forces, the times of their vibrations will be to each other as 1 to 2, by propoſition 55, that is, whilſt the longer ſtring is performing 1 vibration, the ſhorter will perform 2. From hence it follows, by propoſitions 59, 60, 66, that whilſt the longer ſtring is producing 1 pulſe, the ſhorter will produce 2; and conſequently, whilſt the longer is producing 5 pulſes the ſhorter will produce 10. Now if the ſound of the longer ſtring is propagated from *a* to E, Plat. XIX. fig. 7, in the time that it produces 5 pulſes; though the ſhorter ſtring in the mean time will produce 10 pulſes, yet the ſound of the ſhorter ſtring will only be propagated to an equal diſtance. For the thickneſs of the pulſes produced by the longer ſtring is to the thickneſs of thoſe produced by the ſhorter as 2 to 1, by propoſition 69. Therefore, if the thickneſs of 5 pulſes produced by the longer ſtring will reach from *a* to E, the thickneſs of 10 pulſes produced by the ſhorter will only reach from *a* to E. But, ſince whilſt the longer ſtring produces 5 pulſes, the ſhorter produces 10, the ſound of each ſtring will be propagated through the ſame ſpace *a*E in an equal time; and conſequently the ſound of each moves with the ſame velocity.

The quicker any ſtring vibrates, the more pulſes it will produce in a given time. But the leſs the time is, in which a pulſe is produced, the leſs will be its thickneſs, by propoſition 69. Conſequently the more pulſes any ſtring produces in a given time, the leſs will be their thickneſs. Now the velocity of ſound is inverſely as the time, in which a pulſe is produced, and directly as the thickneſs of the pulſes, by propoſition 76. Therefore the faſter or quicker any ſtring vibrates the greater will be the velocity of its ſound; becauſe the pulſes are produced in a leſs time; and the quicker it vibrates the leſs will be the velocity of its ſound, becauſe the thickneſs of the pulſes is leſs. But ſince, as the times of vibration are ſhorter, the velocity of ſound encreaſes upon one account and decreaſes juſt as much upon the other account, an alteration in the times of vibration or in the tones of the ſtring will make no alteration in the velocity, with which its ſound will be propagated.

A difference in the time of vibration is what makes a difference in the tones of ſound, as has been obſerved already. Now if we call the time of vi-

vibration, or time, in which a pulſe is produced, T, and call the thickneſs of a pulſe B, the velocity of ſound will be as $\frac{B}{T}$, by propoſition 76. But ſince B is directly as T, by propoſition 69, $\frac{B}{T}$ will be an invariable quantity.

79. *Sounds of different intenſities are propagated with the ſame velocity.*

A low ſound cannot indeed be heard ſo far as a loud one may. But either low ſounds or loud ones will be conveyed in an equal time to any equal diſtances, at which either of them can be heard. The report of a cannon does not move faſter or paſs over a given ſpace ſooner than the ſound of an elaſtic ſtring.

This has in effect been proved already. For, in propoſition 73, we have ſhewn that the intenſity of ſound is different at different diſtances from the ſounding body. And, in propoſition 77, we have ſhewn that ſound at different diſtances from the ſounding body is propagated with the ſame velocity. Therefore ſounds of different intenſities are propagated with the ſame velocity.

Indeed in this inſtance the ſound ariſes from one and the ſame body, and the intenſity differs only upon account of the different diſtances of the ear. But however, when ſounds ariſe from different cauſes, they can differ from one another only in reſpect either of their tone or their intenſity. In the laſt propoſition we have proved, that the difference of tone makes no difference of velocity: and this inſtance is ſufficient to prove all, which this propoſition requires, that a difference of intenſity makes no difference of velocity. And conſequently all ſounds, however they are produced, move equally faſt.

80. *The velocity of ſound is directly as the ſquare root of the airs elaſticity, and inverſely as the ſquare root of its denſity.*

What we have ſaid hitherto concerning the velocity of ſound ſuppoſes the air to be always in the ſame ſtate both as to its elaſticity and as to its denſity. But ſince both the elaſticity and denſity are variable, we muſt conſider what alterations may be produced in the velocity of ſound, by a change of the air in either of theſe particulars.

Firſt, if the airs elaſticity could be changed without changing its denſity, the velocity of ſound would vary in the ſame proportion with the ſquare root of its elaſticity. Thus if the elaſticity of the air at different times was changed in the proportion of 1, 4, 9, the ſquare roots of theſe elaſticities are 1, 2, 3: and in theſe different ſtates of its elaſticity the velo-

locity of ſound would not be the ſame: but if we call the velocity 1, when the airs elaſticity is 1; the velocity would be doubled or would be 2, when the elaſticity is 4, and would be tripled or would be 3, when the elaſticity is 9; that is, the velocity would be as the ſquare root of the airs denſity.

When ſound is conveyed to any given diſtance *a*E, Plat. XIX. fig. 7, all the particles from *a* to E are made to vibrate ſucceſſively, by propoſition 59. Now the elaſticity of the air is the moving force by which theſe vibrations are produced; and ſince the denſity is ſuppoſed to be given, the number of particles or quantity of matter between *a* and E will be the ſame, whatever the elaſticity is. But, where the quantity of matter is given, the accelerating force will be proportional to the moving force; as appears from what has been ſaid under propoſition 54 concerning the vibration of elaſtic ſtrings. Therefore the accelerating force is proportional to the elaſticity. But each particle vibrates by the ſame law with a pendulum in a cycloid, by propoſition 67, and conſequently the ſquares of the times of vibration, or the ſquares of the times in which the ſound will be conveyed from *a* to E, being inverſely as the accelerating forces, by propoſition 138 of mechanics, will be inverſely as the airs elaſticity. From whence it follows that the times, in which the ſound will be conveyed through the given ſpace *a*E, will be inverſely as the ſquare roots of this elaſticity. But the times, in which ſound is conveyed through a given ſpace, are inverſely as its velocity. Therefore ſince the times, in which ſound is propagated to a given diſtance, are inverſely as its velocity, and are likewiſe inverſely as the ſquare roots of the airs elaſticity, it follows that the velocity of ſound muſt be varyed in the ſame proportion as the ſquare roots of the airs elaſticity; for unleſs they were varyed in the ſame proportion or were directly as each other, the times in which ſound is conveyed through a given ſpace could not be inverſely as either of them.

Secondly, if the airs denſity could be changed without changing its elaſticity, the velocity of ſound would vary in the inverſe proportion of the ſquare root of the airs denſity. Thus if the airs denſity was changed in the proportion of 1, 4, 9, the ſquare roots of theſe denſities are 1, 2, 3, and in theſe different ſtates of its denſity the velocity of ſound would not be the ſame: but would be as theſe ſquare roots inverted, that is, if, when the denſity of the air is 1, we call the velocity of ſound 3; the velocity will be 2, when the denſity is 4; and will be 1, when the denſity is 9.

When ſound is conveyed to any given diſtance *a*E, all the particles, that are between *a* and E vibrate ſucceſſively. The elaſticity of the air pro-

produces this vibratory motion, or is the moving force; and this force is given, by the ſuppoſition, for we ſuppoſe the elaſticity invariable. But when the moving force is given, the accelerating force is inverſely as the quantity of matter; as appears from what has been ſaid in a ſimilar caſe, under propoſition 56, concerning the vibration of elaſtic ſtrings: and ſince the quantity of matter between *a* and E is proportional to the number of particles in that ſpace, or to the denſity of the air, it follows that the accelerating force is inverſely as the airs denſity. Now, by propoſition 138 of mechanics, the ſquares of the times, in which theſe particles will vibrate, or in which the ſound by their ſucceſſive vibrations will be conveyed through this given ſpace *a*E, are inverſely as the accelerating forces. But, if the accelerating forces are inverſely as the ſquares of the times, in which the ſound is propagated through the given ſpace *a*E, and likewiſe are inverſely as the airs denſity; we may conclude that the ſquares of the times, in which ſound is propagated through a given ſpace, is proportional to the airs denſity: for unleſs the ſquares of the times, in which ſound is propagated through a given ſpace, were varyed in the ſame proportion with the airs denſity, it would be impoſſible for the accelerating forces to be inverſely as either of them. But if the ſquares of the times, in which ſound is propagated through a given ſpace, are directly as the airs denſity, then the times themſelves muſt be directly as the ſquare roots of the denſity. Now the velocity of ſound is inverſely as the times taken up to propagate it to a given diſtance. Therefore the velocity is inverſely as the ſquare roots of the airs denſity.

But ſince the velocity of ſound is directly as the ſquare root of the airs elaſticity, when the denſity is given, and inverſely as the ſquare root of its denſity, when the elaſticity is given; it follows, that, when both the elaſticity and denſity are varyed, the velocity of ſound will be in a ratio compounded of the direct one of the ſquare root of the elaſticity, and the inverſe one of the ſquare root of the denſity. Thus if we call the airs elaſticity E and the ſquare root of that elaſticity $\sqrt{E}$, and if we call the denſity D, and the ſquare root of the denſity $\sqrt{D}$, the velocity of ſound will always be as $\sqrt{E}$ directly and as $\sqrt{D}$ inverſely, or as $\frac{\sqrt{E}}{\sqrt{D}}$, for this fraction is directly as $\sqrt{E}$ its numerator, and inverſely as $\sqrt{D}$ its denominator. This is what is meant, when the velocity of ſound is ſaid to be in a ratio compounded of the direct ſubduplicate ratio of the airs elaſticity and an inverſe ſubduplicate ratio of its denſity.

81. *Sound moves with the ſame velocity, at whatever height the mercury ſtands in the barometer.*

When

When the mercury ſtands high, the atmoſphere is heavier, than when it ſtands low; as appears from propoſition 46. But the compreſſing force upon the lower air is the weight of the atmoſphere: and conſequently the denſity of the air, being always proportional to the compreſſing force, muſt be as the weight of the atmoſphere, or as the height of the mercury in the barometer, by propoſition 50. Now if the airs denſity encreaſes, when the mercury riſes, the velocity of ſound ought to decreaſe, in the ſame proportion that the ſquare root of the denſity encreaſes, by prop. 80; and conſequently ſound ought to move ſlower, when the mercury is high, than when it is low. This would be the caſe, if a change in the weight of the atmoſphere made no alteration in the air, except in its denſity. But the elaſticity of the air is proportional to the force that compreſſes it, or to the weight of the incumbent atmoſphere, by propoſition 48. Therefore the elaſticity encreaſes, when the mercury riſes in the barometer. Now if the airs elaſticity encreaſes, when the mercury riſes, the velocity of ſound ought to encreaſe, in the ſame proportion that the ſquare root of the elaſticity encreaſes, by prop. 80; and conſequently ſound ought to move faſter, when the mercury is high, than when it is low. We have ſeen therefore that, when the mercury riſes, the velocity of ſound decreaſes as the ſquare root of the airs denſity encreaſes, and encreaſes as the ſquare root of the airs elaſticity encreaſes. But the elaſticity is always proportional to the denſity, for either of them are proportional to the compreſſing force, by propoſitions 48, 50: and conſequently the ſquare root of the denſity is proportional to the ſquare root of the elaſticity. Therefore, ſince the velocity of ſound decreaſes as the ſquare root of the denſity encreaſes, and encreaſes as the ſquare root of the elaſticity encreaſes, and ſince the ſquare root of the denſity is always changed in the ſame proportion with the ſquare root of the elaſticity, it follows, that the velocity of ſound, as the mercury riſes in the barometer, will decreaſe and encreaſe in the ſame proportion; and conſequently at all heights of the mercury this velocity will continue the ſame. This may be expreſſed in a much ſhorter manner. If we call the denſity of the air D and its elaſticity E; then, whenever the denſity is changed by the compreſſing force being changed, D is directly as E, by propoſitions 48, 50, and conſequently $\sqrt{D}$ is directly as $\sqrt{E}$. Therefore $\frac{\sqrt{E}}{\sqrt{D}}$ is a given quantity at all heights of the barometer, or in all ſtates of the incumbent atmoſpheres weight. But the velocity of ſound is as $\frac{\sqrt{E}}{\sqrt{D}}$, by prop. 80. Therefore the velocity of ſound is a given or invariable quantity at all heights of the mercury in the barometer.

The method of trying with what velocity ſounds are propagated, is by having a canon fired at a diſtance. Now, though light is not propagated in an inſtant to any diſtance however remote, but takes up ſome time to paſs from one place to another, by propoſition 33 of optics, yet the velocity of it is ſo great, that the time taken up by a ray to paſs over ſuch diſtances, as a canon can be placed at, will be infinitely ſhort and may be neglected. So that the flaſh or light will be ſeen at the ſame inſtant that the canon is diſcharged. But the ſound will be found to move much ſlower than the light; for there will be ſome interval of time between ſeeing the flaſh and hearing the report. When this interval of time is eſtimated, and the diſtance of the canon is known, the time that ſound takes up to deſcribe that diſtance is determined. And by ſuch experiments as theſe ſounds have been found to move at the rate of 1142 feet in a ſecond. By ſuch experiments it has likewiſe been found, that ſounds of different ſorts are propagated with the ſame velocity; a ſound produced by the ſtroke of a hammer moves at the ſame rate with the report of a gun. Theſe experiments may be made upon a ſound produced by the ſtroke of a hammer, as well as upon thoſe produced by a gun: for at the inſtant the ſtroke is given, it will be ſeen by the eye at a diſtance; becauſe the ſtroke is ſeen by means of the light, which comes to the eye of the ſpectator from the hammer and from the body that is ſtruck; by propoſition 113 of optics: and the velocity of light is ſo great, that the time taken up by a ray to paſs over the ſpace, which lies between the hammer and the ſpectators eye, may be neglected. The ſound of the ſtroke will not be heard by the ear at the ſame diſtance, till ſome time after the ſtroke is given: and this interval of time being found, the time that is taken up by the ſound to paſs over the ſpace, which lies between the hammer and the ear, is determined.

82. *The velocity of ſound in ſummer is ſomething greater than in winter.*

The denſity of the air is leſs in ſummer than in winter, on account of the heat; but if the mercury ſtands at the ſame height in the barometer the elaſticity will be the ſame, by propoſition 48. But the velocity of ſound is inverſely as the ſquare root of the airs denſity, when the elaſticity is given, by propoſition 80. Therefore the velocity of ſound will be greater in ſummer than in winter, in the ſame proportion that the ſquare root of the airs denſity is leſs in ſummer than in winter. Indeed this difference of velocity may eſcape very accurate obſervers: for the denſity of the air does not differ very much in ſummer and in winter; and conſequently the ſquare roots of its denſity and the velocity of ſound will differ much leſs.

83. *The wind will alter both the intenſity and the velocity of ſound.*

When the atmoſphere is ſtill, or no wind blows, the pulſes are propagated equally in all directions from a ſounding body, by propoſition 71. But as the wind is a current of air moving forwards with a direct progreſſive motion, the pulſes, whilſt they are excited in the air, will partake of this motion. Therefore ſince the pulſes are carryed in that direction in which the current of air moves, they will extend to a greater diſtance from the ſounding body on that ſide towards which the wind blows, than on the contrary ſide. Upon this account the ſound will be heard farther with a fair wind, than with a contrary wind. And conſequently, if two perſons were at equal diſtances from the ſounding body, on contrary ſides, he that has the wind fair to bring the ſound will hear it much ſtronger, than he that is on the other ſide: becauſe, if the ſound can be heard to a greater diſtance on one ſide than on the other, its intenſity at equal diſtances muſt be greater on one ſide than on the other.

From hence it is evident, that, if the wind moved with the ſame velocity that ſound does, the pulſes, as ſoon as they are produced, would be all carryed off in the direction of the wind, and the ſound would not be heard at all againſt the wind. But the air in very ſtrong winds is found to move only 66 feet in a ſecond, and ſound moves 1142 feet in the ſame time. Therefore as ſound moves with a greater velocity than wind, the pulſes will be propagated and the ſound will be heard againſt the wind: though it is weaker and reaches to a ſmaller diſtance in this direction, than it does on the other ſide of the ſounding body, where it has a fair wind to convey it.

Now if the current of air is moving the ſame way with the pulſes of ſound that are excited in it, they will partake of its motion: but if the current is moving the contrary way, the pulſes will be hindered in their progreſs by its motion. Conſequently in a fair wind the pulſes will be accelerated, and ſound will be propagated with a greater velocity: but in a contrary wind the pulſes will be retarded, and the ſound will be propagated with a leſs velocity.

84. *If two elaſtic ſtrings are uniſons; by ſtriking one of them ſo as to make it ſound, the other will have the motion communicated to it, and will be made to ſound too.*

Strings are called uniſons, when they perform their reſpective vibrations in equal times. The motion, which is raiſed in the air by the firſt

ſtring, or by the ſtring which is ſuppoſed to be ſtruck, will impell the ſecond ſtring, whether it is an uniſon to the former or not. But in order to encreaſe the motion communicated, ſo as to make the ſecond ſtring vibrate ſtrong enough to ſound, it is neceſſary that this ſecond ſtring, after it has begun to move, ſhould be always moving in the ſame direction with the firſt. For then, as the firſt ſtring in each vibration impells the air forwards, the ſecond ſtring will likewiſe be moving forwards; and conſequently each impulſe of the air will accelerate the ſecond ſtring, till its vibrations become ſtrong enough to produce a ſound. But if the ſecond ſtring is an uniſon to the firſt, or performs all its vibrations in the ſame time with the firſt, it will be always moving forwards, when the firſt is moving forwards, and backwards, when the firſt is moving backwards. Therefore by ſtriking any elaſtic ſtring, ſo as to make it ſound, any other uniſon ſtring, which is near to it, will be made to ſound too.

This method of communicating motion is not peculiar to elaſtic ſtrings: for a ſound raiſed by any other means will make ſuch bodies tremble, as are uniſons with the note of that ſound. If a perſon, after he has found the tone of a drinking-glaſs, holds his mouth near the edge of it, and ſounds the ſame note with his voice; he will make the glaſs tremble: and by raiſing his voice, without altering the note, he may encreaſe this trembling motion ſo as to break the glaſs.

Upon theſe principles we may ſee the reaſon why the panes of glaſs in a window will be made to tremble by ſome ſounds, and yet other ſounds, which are louder, will not always produce this effect. A weak clap of thunder will ſometimes ſhake the windows very much, and yet a louder clap will not ſhake them at all.

85. *When ſound ſtrikes againſt an obſtacle, it will be reflected.*

By ſound we here mean the pulſes in the air, which are, properly ſpeaking, the cauſes of ſound. If theſe pulſes in their progreſs ſtrike upon any obſtacle, ſuch as a rock, a thick wood, or the ſide of a building, the air, which is condenſed at the obſtacle, is prevented from expanding itſelf forwards, or from propagating the ſound beyond the obſtacle. Therefore in expanding itſelf the motion, which would otherwiſe have been propagated forwards, will be returned from the obſtacle; and a perſon, who is placed ſo as to receive the pulſes in their return, will hear the ſound by reflection. Such a reflected ſound is called an echo.

The number of ſyllables, which an echo repeats diſtinctly, depends upon the diſtance of the obſtacle, from whence the ſound is reflected.

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The ſyllables, that we hear diſtinctly repeated, are theſe, which are returned after we have done ſpeaking. Therefore if the obſtacle is ſo near to us, that the firſt ſyllable we ſpeak will be returned before we can ſpeak a ſecond, no reflected ſound at all can be heard diſtinctly; becauſe the direct and reflected ſound, or the voice and the echo, will be confounded with one another. If the obſtacle is at ſuch a diſtance, that five ſyllables may be ſpoken before one will be returned; then if we ſpeak, a ſentence conſiſting of ten ſyllables, the firſt five will be reflected, whilſt we are ſpeaking the five laſt, ſo that in ſpeaking the five laſt ſyllables, the voice will be confounded with the echo of the five firſt, and we ſhall hear the echo of only the five laſt ſyllables diſtinctly; becauſe theſe only will be returned after we have done ſpeaking. But if the obſtacle is at ſuch a diſtance, that we may ſpeak ten ſyllables, before the firſt of them will return to the ſpeaker; then if we ſpeak a ſentence of only ten ſyllables, we ſhall have done ſpeaking before the echo begins, and conſequently we may hear the whole ſentence diſtinctly repeated by the echo.

Sometimes the ſame ſound is frequently repeated by an echo. This happens when there are ſeveral obſtacles at different diſtances. For though there are ſeveral obſtacles, yet if all of them are at the ſame diſtance, the ſound will be returned from them all at one and the ſame time: and conſequently the ſeveral reflected ſounds will be heard together, and will make but one echo. But if the obſtacles are at different diſtances, each will return the ſound at a different time, and as many echos will be heard as there are obſtacles, that produce them.

CHAP. VII.

Of ſome effects produced by the preſſure and by the elaſticity of the air.

86. *Liquors are raiſed into a ſyringe by the preſſure of the air.*

THE general word *liquor* is here uſed for any unelaſtic fluid, ſuch as mercury, water, &c. If the pipe of a ſyringe is dipped below the ſurface of any ſuch fluid, of water for inſtance, in an open veſſel or baſon, and then the piſton is drawn up, the water will riſe out of the baſon into the barrel or ſhaft of the ſyringe. The ſhaft or barrel is a hollow cylinder, and the piſton conſiſts of a ſtem and of a ſhort cylinder, which ſo exactly fits the cavity of the barrel, that no air can paſs between them. Conſequently, when the piſton is drawn up, it muſt drive the air before it out of the barrel, and make a vacuum there. For no freſh air could poſſibly enter

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but through the pipe; and its paſſage this way is prevented by the water in the baſon, becauſe the orifice of the pipe is below the ſurface. From hence it follows, by what has been ſaid under propoſition 46, that the preſſure of the air upon the ſurface of the water in the baſon will make the water riſe into the barrel.

But then we muſt obſerve, by propoſition 46, that there is a certain and determinate height, beyond which water cannot be raiſed into a ſyringe. For though the preſſure of the air will ſupport a column of water 35 feet high, yet it will not ſupport one of any greater perpendicular height. Conſequently, if the perpendicular height of the ſyringe is more than 35 feet, the water will riſe, till the piſton is got to the height of 35 feet, and then it will ſtop: ſo that if the piſton is drawn ſtill higher, a vacuum will be left in the upper part of the ſyringe, and no water will riſe into it. In the ſame manner, by propoſition 46, we might ſhew, that a ſyringe cannot raiſe a column of mercury, which is more than 30 inches high. Indeed as the weight of the atmoſphere is variable, the heights, to which a ſyringe will raiſe either water or mercury at different times, may be ſomething greater or ſomething leſs than thoſe here aſſigned.

87. *Liquors are made to run through a ſiphon by the preſſure of the air.*

Let the leg IH of the ſiphon HIK, Plat. XIX. fig. 8, be put into water or any other liquor contained in an open veſſel FG, and let a perſon ſuck with his mouth at K the end of the other leg, till he has drawn the water thither, ſo as to fill the ſiphon from H to K: then, if he takes his mouth away, the water will begin to run out at K, and will keep running out of the veſſel through the ſiphon, till it ſinks in the veſſel below the orifice H; or till it is all run out if the orifice H touches the bottom of the veſſel.

When the ſiphon has been filled with the liquor by ſucking at K, then ſuppoſing the leg IK to be longer than the leg IH, the moment of the liquor, as in the inverted ſiphon propoſition 12, in the leg IK will be greater than in the ſhorter. Now the preſſure of the air upon the orifice H balances its preſſure upon the orifice K: ſo that theſe two forces, one of which by acting upon H would drive the liquor from H towards K, and the other by acting upon K would drive it from K towards H, being equal and in contrary directions, will deſtroy one another. Conſequently the liquor will begin to move towards that ſide, on which there is the greateſt moment, that is, it will begin to run out of the longer leg at K. But as the liquor runs out of the ſiphon through the orifice K, it leaves a vacuum behind it, and the preſſure of the atmoſphere upon the ſur-

ſurface FG will raiſe the liquor out of the veſſel into that vacuum, by propoſition 46. Thus the greater moment of the liquor in the longer leg of the ſiphon will continue to make it run out at the orifice K; and the preſſure of the atmoſphere upon the ſurface FG will continue to raiſe the liquor out of the veſſel into the ſiphon; till ſo much is run out of the veſſel as to ſink the level or ſurface below the orifice H.

Since the liquor begins to run out of the longer leg firſt, becauſe the moment on that ſide is greater than on the other; it follows, that if the leg IK, which hangs over the ſide of the veſſel, was ſhorter than the leg IH, which is dipped into the liquor, upon filling the ſiphon, by ſucking at K, the liquor inſtead of running out at K, after we had done ſucking, would fall back again out of the ſiphon into the veſſel.

88. *The water is raiſed into a common ſucking pump by the preſſure of the air.*

The handle AC of a pump, Plat. XIX. fig. 9, is a lever of the firſt kind, and turns round upon a fulcrum or bolt at B. The uſe of this handle is to thruſt down and to draw up the piſton, ſucker, or bucket D, which is faſtened to an iron rod AD, and this rod is joyned to the handle by a bolt at A. LM is the ſhaft or barrel of the pump: and towards the bottom of the ſhaft there is a plug E, which has a valve of leather loaded with lead in the middle of it, that opens upwards, as repreſented in the figure. This plug divides the ſhaft into two parts LE and EM; and the upper part LE communicates with the lower part EM through the valve in the middle of the plug. In the middle of the ſucker D there is a hole, which is covered by ſuch another valve as that at E, which likewiſe opens upwards: the valve of the ſucker, as repreſented in the figure, is ſhut down ſo as to cover the hole; and that in the plug is open. If the ſucker D is thruſt down towards the plug E, the air contained in the ſpace DE, which is between them, will be forced down againſt the valve E, and will ſhut it; ſo that none of this air will paſs into the lower part of the pump. But in the mean time, as the ſucker is preſſed down upon the air ED, the air ſo preſſed will force open the valve at D, and by this means all of it will riſe through the ſucker. When the ſucker, after having been thruſt down, is drawn back again, it will leave a vacuum in the ſpace DE. For in drawing it back again the preſſure of the outward air downwards will ſhut down the valve D, ſo that the ſucker, like the piſton of a ſyringe, having no other orifice than what is ſhut by the valve D, and being exactly fitted to the cavity of the pump, will drive the air before it, as it is drawn up from E to D. But indeed the firſt time D is drawn up, the

the ſpace DE will not quite be emptyed of all air: becauſe the air in the lower part of the pump EM, having the preſſure taken off, as the ſpace DE is emptyed, will expand itſelf, by propoſition 49: and, as the air EM expands, it will puſh open the valve E, and part of it will riſe into the ſpace DE. But what air riſes in this manner will be pumped out by the ſucker, the next time it is thruſt down and drawn up; in the ſame manner that the air, which was at firſt between D and E, was pumped out before. And as the preſſure of the atmoſphere upon the water at M, which is under the pump, is by this means conſtantly made leſs, than upon any other part of the ſurface of the water in the well FG, the water will riſe into the pump, and will fill the vacuum, which the ſucker makes, by propoſition 46. When the ſhaft is thus filled with water as high as D, if the ſucker is thruſt downwards, it will force the water in the ſpace DE againſt the lower valve E and will ſhut it; ſo that, whilſt the ſucker is moving downwards, none of this water can return into the well. In the mean time, as the ſucker is thruſt down upon the water DE, the reaction of the water upwards will force open the valve of the ſucker D; and conſequently this water will riſe through the hole in the ſucker. But whatever water gets above the ſucker will be drawn up with it: for its weight will ſhut the valve D, as the ſucker riſes; and when this valve is ſhut down, there is no way for the water to fall back again. Thus, every time the ſucker is thruſt down, a column of water DE gets above it, which column is afterwards drawn up along with the ſucker, and the preſſure of the atmoſphere, as the ſucker, when it is drawn up, leaves a vacuum below it, will conſtantly keep the ſpace DE full of water from the well. When water enough has been raiſed above the ſucker to make it ſtand at the level H in the barrel, it is then above the ſpout I, and conſequently will run out there.

Whatever water is drawn up by the ſucker in pumping, is firſt raiſed into the barrel of the pump DM: and, ſince it is raiſed into the barrel, by the preſſure of the atmoſphere, the ſurface of the water FG muſt ſome way or other communicate with the external air. Therefore, if the veſſel FG was exactly cloſe every where, except juſt at the place where the pump is applyed, it would be impoſſible by pumping to draw any water out of that veſſel. For no water can be drawn out, unleſs it is raiſed into the barrel: but the preſſure of the atmoſphere is excluded from the ſurface FG, if the veſſel is exactly cloſe and does not communicate with the outward air; and conſequently, though a vacuum is made in the ſpace DE, no water will riſe out of the veſſel into that vacuum.

It is a common miſtake that water cannot be drawn out of a well by a ſucking pump, if the well is above 30 or 35 feet deep. Indeed the

preſſure

preſſure of the air can raiſe the water no higher, by propoſition 46: but when the preſſure of the atmoſphere has raiſed the water to this height, if the ſucker receives it there, it may be lifted afterwards by the ſucker to any height whatever. Thus if MH or the diſtance of the water in the well from the ſpout is 70 feet, the water may be raiſed from M to H by a ſucking pump, provided the plug E is placed low enough, and the ſtem AD of the ſucker is made long enough. Suppoſe the plug E to be 30 feet from the ſurface of the water M, and the diſtance between D and E to be 5 feet, ſo that MD is 35 feet; then ſince MH is 70 feet, DH muſt be 35 feet too. Now the preſſure of the atmoſphere will raiſe the water to D, when there is a vacuum made in the ſpace MD. And then every time the ſucker is thruſt down from D to E it will bring up all the water between D and E or 5 feet of water along with it. Conſequently after the water is raiſed to D, if the ſucker has been thruſt down and drawn up 7 times, there will be $7\times5=35$ feet of water above the ſucker. Therefore the water will then reach to H and will run out of the ſpout at I. But here it muſt be obſerved that every time the ſucker is drawn up aftewards, the perſon who pumps has a column of water to lift, that has the ſucker D for its baſe, and is 35 feet high: this weight is ſo great as to make ſuch a pump uſeleſs upon account of the great ſtrength that is requiſite to work it.

When a pump takes air at the lower valve, the water will fall out of the pump into the well, for the ſame reaſon that, by propoſition 46, the mercury would fall out of the tube of a barometer, if the air was let into the top of the tube. After the water has been ſunk out of the pump for ſome time, the leathers of the two valves grow dry, ſo that they will not open and ſhut freely, and a leather, which goes round the ſucker, grows dry too, and then the ſucker will not exactly fill the cavity of the barrel. The way of fetching the water again, as it is called, is by pouring a pail full of water down the pump, which moiſtens the valves and makes them both move freely and ſhut cloſe, and at the ſame time ſwells the leather, which is round the ſucker, and by this means makes the ſucker fill the cavity of the barrel. When this is done the pump is fit to work, and will draw the water out of the well as before.

89. *Fire-engines are either forcing pumps or lifting pumps.*

The fire-engine OP, Plat. XIX. fig. 10, is ſuch an one as ſpouts with a continued ſtream. Theſe engines conſiſt of two forcing pumps RG, and QK. We will conſider each of theſe pumps ſeparately. F is the piſton or forcer of the pump RG, it is one entire cylinder like the piſton of a

ſyringe. This pump has one valve at G, which opens upwards; and another at H, which opens into the veſſel MN; when this valve H is open, it ſtands as the valve I belonging to the other pump is repreſented in the figure. The water is poured into the tub or body of the engine through a grate that ſtrains it from the ſoil, ſuch as ſtraws or ſmall pieces of wood or ſtones, which would hinder the valves from moving freely and ſhutting cloſe, if it was not thus ſtrained. The veſſel MN is ſtrong and cloſe every where, except at H and I where it communicates with the two pumps. S is a braſs pipe, at the top of which the leathern trunk or pipe PT is faſtened, and V is a ſhort braſs pipe at the end of the leathern one. When the piſton F is riſing from G towards R it leaves a vacuum below it: and conſequently the valve G will open as there is no preſſure upon it, and the water will riſe out of the veſſel into this vacuum, as it does into the barrel of a ſyringe, by propoſition 86. In the mean time the air from the veſſel MN cannot get into this vacuum; becauſe by its preſſure againſt the valve H, this valve will be ſhut, and its paſſage this way will be prevented. This is the manner in which the water will be raiſed into either of the pumps. In order to ſee what becomes of the water, after it is raiſed, we will next conſider the pump QK. Suppoſe therefore that this pump had been filled with water in the manner already deſcribed: then if the forcer L is puſhed downwards from Q towards K, the water will be preſſed againſt the valve K and will ſhut it: but at the ſame time by being preſſed likewiſe againſt the valve I it will open this valve and will be driven into the veſſel MN. Now each of theſe forcers are joyned by means of an iron rod to the bar AB: each of theſe rods turns upon a bolt, one at D, the other at E. This bar turns round upon a center of motion at C, ſo that CA and CB are two levers of the ſecond kind. The men, who work the engine, act at A and B; C is the fulcrum of eachlever; D and E, which are reſpectively between the men that work or the powers that are applyed at A and B, are the places where the piſtons are joyned to the bar. It is plane from an inſpection of the figure that when one end A of the bar riſes the other end B will ſink, and conſequently if the ends of the bar are alternately lifted up and pulled down, the two piſtons will be alternately lifted up and puſhed down. But when the piſton riſes it draws the water into the pump, or however it makes a vacuum and the preſſure of the atmoſphere makes the water follow it. And when the piſton is puſhed down it forces the water out of the pump into the veſſel MN. Therefore whilſt one pump is drawing, the other is forcing. As the water is thus forced into the veſſel MN, it will firſt drive out the air through the pipe S, till the ſurface

of

of the water comes to be even with S: but afterwards as no air can paſs through S, the force, which drives the water into this veſſel and makes it riſe above S, will condenſe the air or ſqueeze it into a ſmall compaſs at the top M. When ſo much water is got into MN that the water and condenſed air quite fill the veſſel, and the moment or impulſe of the forcers can condenſe the air no farther; then if either of the forcers is driven down in theſe circumſtances, it will puſh the water out of the pump into the veſſel MN, as before. But now the veſſel will hold no more, and conſequently whatever is the quantity of water thus puſhed into the veſſel, the ſame quantity will be puſhed with an equal force through the pipe PTV.

From what has been ſaid it appears that this ſtream can only run at V, whilſt one of the forcers or piſtons is deſcending. And conſequently if there is the leaſt interval between the alternate deſcent of the forcers, the ſtream might be interrupted or might not be a continued one. Thus for inſtance, when the men at B have drawn their end of the bar down, the ſtream, as it is driven out by the forcer L, will ſtop, and ſince the next part of the ſtream is to be driven out by the other forcer F, it will not begin, till F or till the end of the bar A begins to be pulled down. Here will be ſome ſhort interval of time, let the men work as quick as they can: and if the ſtream is not interrupted in this interval, it muſt be driven out by ſome other means beſides the deſcent of the forcers. Now we have ſeen already that the air is greatly compreſſed at the top of the veſſel MN: and during the interval between the alternate deſcent of the forcers, this air will endeavour to expand itſelf by its own elaſticity or ſpring, by propoſition 47. But the compreſſed air at M will preſs upon the ſurface of the water in the veſſel MN more forcibly, than the external air through the pipe VTP preſſes upon that part of the ſame ſurface, which is within the pipe S. Therefore the ſpring of this air at M will drive the water out through the pipe PTV and will continue the ſtream during the ſhort interval between the alternate deſcent of the forcers.

The fire-engine FE, Plat. XIX. fig. 11, is the common ſquirting one. Such engines conſiſt of one lifting pump LTS. L is a valve that opens upwards. M is a moveable valve that opens upwards too, and is faſtened by means of the iron rod PM to a ſtrong frame HKNO. This frame is joyned by bolts at H and K to two levers of the firſt kind HGC and KID. G and I are the reſpective centers of motion of theſe levers, and the men work or the powers are applyed at C and D. The water is poured into the tub FE through grates that ſtrain out the ſoil. And the pump is

open at TS that it may communicate with the water in the tub. When the ends C and D of the two levers are raiſed, the frame HNOK will deſcend, and the piſton or lifter MP will deſcend with it. As the piſton deſcends, the outward air will cloſe the valve L, and the air that is in the pump below M will open the valve M, and will riſe through it. When the ends C and D are pulled down the piſton will be raiſed; and the valve of the piſton being forced againſt the air that had riſen above it will be ſhut down; ſo that this air cannot return into the lower part of the pump, but will be forced out through the valve L. Thus there will be a vacuum left in the bottom of the pump, and conſequently, by propoſition 46, the preſſure of the atmoſphere upon the ſurface FE will raiſe the water out of the tub into the pump. When the piſton is forced down again, the outward air will cloſe the valve L as before, and as M is forced down againſt the water below it, the reaction of the water upwards will open the valve M, and the water will keep riſing through it, till M is ſunk as low as it will go. Whatever water thus gets above M will all be lifted up by this lifter or piſton the next time that it is raiſed, and will be forced out through the pipe AB with the moment equal to what the piſton is raiſed with: for the weight of this water will keep the valve M ſhut, as the piſton riſes, ſo that none of it will fall back again; and the force with which the water is puſhed upwards will open the valve L.

The ſtream of ſuch an engine will not be a continued one: for the water only ſpouts out, whilſt the lifter is riſing; and then the ſtream is interrupted, whilſt the lifter is deſcending.

90. *The air, becauſe it is an elaſtic fluid, may be exhauſted out of a cloſe veſſel by an air-pump.*

The ſtructure of an air-pump and the manner of its working is very like that of the common ſucking pump. And we may explane it upon Plat. XIX. fig. 9. Suppoſe therefore FG to be a veſſel ſo cloſe all round that no air can either go out of it or come into it except at the ſingle orifice M, where the pump is ſcrewed on. Now if the piſton or ſucker D was thruſt down, till it came cloſe to the lower valve E, it is plane from what has been ſaid, in propoſition 88, that upon drawing up the piſton there will be a vacuum made in the ſpace DE: for no air can come through the ſucker into that ſpace, becauſe the preſſure of the atmoſphere from above will ſhut the valveD, and will ſtop up the orifice in the middle of the ſucker. But as the ſpace DE is thus emptyed, there will be no preſſure at all upon the valve E; and conſequently no preſſure at all

upon

upon the air below this valve. Therefore, ſince the compreſſing force is thus taken off from the air in the veſſel FG, it will expand itſelf, by its own ſpring, will force open the valve, and will riſe through it, ſo as to fill not only the cavity of the veſſel, but the whole ſpace DFG. By this means as much of the air as fills the ſpace DE is drawn out of the veſſel. In order to draw more air out, the piſton is thruſt down again. As the piſton deſcends, the air, which had riſen before out of the veſſel into the ſpace DE, will not return into the veſſel: for the piſton by thruſting it downwards makes it preſs upon and ſhut the valve E. But this air by preſſing upwards upon the valve D as the piſton deſcends will open this valve and will riſe through it. When the piſton or ſucker is drawn up the ſecond time, a vacuum will be made, as before, in the ſpace DE; and the air in FG will again expand itſelf by its elaſticity, when the compreſſing force is thus removed, and the force, with which it expands itſelf, will open the valve E, ſo that the veſſel FG will communicate with the ſpace DE, and the air, which before filled only the veſſel, will dilate itſelf and fill both the veſſel and the ſpace DE too, and conſequently there will be leſs of it in the veſſel than there was before. Thus the piſton, when it was drawn up the firſt time, exhauſted as much air out of the veſſel as would fill the ſpace DE; and when it is drawn up a ſecond time, it exhauſts as much of the remaining air as will fill the ſame ſpace DE. And in like manner whatever air remained in the veſſel, as much of that remainder will be exhauſted by drawing up the piſton as will fill this ſpace DE.

By this time the reader may ſee that no fluid, unleſs it was elaſtic, could be exhauſted in this manner out of a cloſe veſſel FG. If this veſſel was filled with water and there was no orifice but at M, where the pump is ſcrewed on, though a vacuum was made in DE by drawing up the ſucker, yet no water would riſe out of the veſſel into that vacuum. For the water is raiſed into a vacuum, that is made over any part of its ſurface, by the preſſure of the atmoſphere upon the other parts of its ſurface. Therefore ſince, if the veſſel is a cloſe one, the external air cannot preſs upon any part of the water contained in it, no water will riſe out of the veſſel into the vacuum DE or into the barrel of the pump. Now no water can be pumped out, unleſs it firſt riſes into the pump; and conſequently no water could be pumped out of a veſſel, that had no communication with the outward air. But if the veſſel FG is filled with an elaſtic fluid, ſuch as air is, then though it is a cloſe one yet upon making a vacuum in DE the air will riſe into that vacuum. For it does not want the preſſure of the outward air to raiſe it, but will expand itſelf by its own elaſticity; ſo that the ſame air, which at firſt filled only the veſſel FG, will, after it

is

is thus expanded, fill both the veſſel and the ſpace DE too. And whatever part of the air riſes into the barrel of the pump upon drawing up the piſton, it will be carryed off at the next return of the piſton.

The air may by this means be in a great meaſure exhauſted out of any veſſel. But yet after pumping ever ſo long there will be a ſmall quantity left, and a vacuum made by an air-pump will be an imperfect one, as has already been obſerved under propoſition 46.

This may perhaps be made evident from one particular inſtance. Let us ſuppoſe the capacity of DE to be equal to the capacity of the veſſel FG. Then the firſt time the ſucker is drawn up, the air in the veſſel FG will expand itſelf ſo as to fill both the ſpace DE and the veſſel FG; but DE and FG together is double the ſpace that this quantity of air uſually fills: and conſequently $\frac{1}{2}$ the air, that was in the veſſel, will riſe into the pump DE, where the ſucker left the vacuum for it to riſe into; ſo that only $\frac{1}{2}$ the original quantity of air will be left in the veſſel. When the ſucker is thruſt down again, what air had riſen into DE will riſe through the valve D, as has been already deſcribed. The ſecond time the ſucker is drawn up; the remaining air in the veſſel, which is now only $\frac{1}{2}$ the original quantity, will expand itſelf ſo as to fill both the veſſel and the ſpace DE. But the veſſel FG, and the ſpace DE together, make a ſpace double to FG alone. Therefore this remainder by the ſecond return of the ſucker is expanded into double the ſpace FG, and conſequently $\frac{1}{2}$ of it is left in the veſſel and $\frac{1}{2}$ of it riſes into the pump DE. But ſince $\frac{1}{2}$ the whole quantity was drawn out of the veſſel by the firſt aſcent of the ſucker, and only $\frac{1}{2}$ the remainder is drawn out by the ſecond aſcent, it follows that only $\frac{1}{4}$ of the original quantity is drawn out by this ſecond aſcent. Conſequently after the ſecond aſcent the remainder will be likewiſe $\frac{1}{4}$ of the original quantity. The $\frac{1}{2}$ of this remainder will in like manner be drawn out by the third aſcent; and the other half will ſtill be left in the veſſel. But the $\frac{1}{2}$ of $\frac{1}{4}$, or of this remainder, is $\frac{1}{8}$ of the whole. Therefore by the third aſcent of the ſucker $\frac{1}{8}$ of the whole is drawn out, and $\frac{1}{8}$ is left behind. The $\frac{1}{2}$ of this remainder or $\frac{1}{16}$ of the whole will be drawn out of the veſſel by the fourth aſcent of the ſucker; and the other $\frac{1}{2}$ or $\frac{1}{16}$ of the whole will be left behind. And thus we might continue pumping for ever without exhauſting the veſſel ſo as to make it a perfect vacuum. Becauſe whatever air remained in the veſſel before any aſcent of the ſucker, there would be always $\frac{1}{2}$ that quantity left in it after the aſcent.

The ſame would be equally true in all other inſtances. For the air, which the pump exhauſts at every aſcent of the ſucker, is not a certain

part

part of the whole quantity, but a certain part of the remainder. And conſequently, if there was any air remaining in the veſſel, when the ſucker is thruſt down, there will be a part of it ſtill remaining after the aſcent of the ſucker. Conſequently there can be no particular aſcent of the ſucker aſſigned, which will quite empty the veſſel. For, ſince by that particular aſcent the veſſel is ſuppoſed to be exhauſted, there muſt be air in the veſſel before it. And if there was air in the veſſel before that aſcent, there will be ſome part remain after it. Therefore the aſcent, which is aſſigned, will not abſolutely exhauſt the veſſel.

The moſt convenient way of exhauſting the air out of a receiver is by uſing two air-pumps together, as in Plat. XX. fig. 1. The two pumps are *aa* and *aa*; they communicate at the bottom with the veſſel *dd*: a tube *bb* runs up from this veſſel into the receiver *oooo*. The ſtems of the piſtons or ſuckers are *c* and *c*: each of theſe is a rack, or has teeth on the ſide: and at *f* there is a pinion or ſmall wheel, the teeth of which catch in the teeth of theſe racks: ſo that by turning the winch *fb* firſt one way and then the other, the pinion by catching in the racks will alternately thruſt down and then raiſe each of the ſuckers. By this means the air will be exhauſted out of the veſſel *dd*, the tube *bb*, and the receiver *oooo*, which, as they communicate with one another, may all together be conſidered but as one veſſel. The receiver is open at the bottom, and this open part is placed upon a wet leather at *ii*, which is ſomething bigger than the bottom. If the receiver is held down pretty cloſe to this leather none of the external air can get in round the edges: and after a little air is exhauſted the preſſure of the external air will keep it down cloſe enough without holding. The principal advantage of uſing two pumps in this form, beſides that of doing the work ſooner with two than it could be done with one, is as follows. When the ſucker of an air-pump is raiſed, the perſon, who works it, muſt lift a column of the atmoſphere, the baſe of which is the ſucker. This will not be very difficult at firſt, becauſe the ſpring of the included air will, by propoſition 48, be nearly equal to this weight, and conſequently he will feel but little of the weight. But when the veſſel is almoſt exhauſted, there will be but little ſpring in the remaining air, ſo that he will have the weight to lift with little or no aſſiſtance. Now by uſing two pumps, in which the piſton of one is deſcending, whilſt that of the other is aſcending, the weight of the atmoſphere, which preſſes upon the deſcending one, will in ſome meaſure balance the weight upon the aſcending one: and conſequently he will find it much leſs difficult to work two pumps thus contrived, than to work one pump alone.

91.

91. *As the air is an elaſtic fluid, a great quantity of it may be forced into a veſſel by a condenſing engine.*

The manner in which an additional quantity of air is forced into a veſſel by means of a condenſer may be explaned upon Plat. XIX. fig. 9, if we only ſuppoſe the valve E to open downwards inſtead of upwards, and the piſton D to be a ſolid one like that of a ſyringe without any valve in it. By thruſting down this piſton the air in the ſpace DE will be forced againſt the valve E, and, ſince this valve by the ſuppoſition opens downwards, the air thus thruſt againſt the valve will open it, and will be thruſt through the paſſage at E into the veſſel FG. When the piſton is drawn up again it will leave a vacuum in the ſpace DE: but none of the air will eſcape out of the veſſel FG into this vacuum; becauſe the ſpring of the air included in the veſſel will preſs upwards upon the valve E; and ſince this valve opens downwards, ſuch a preſſure upwards will keep it ſhut, and thus the paſſage at E will be ſtopped. Now there muſt be a ſmall hole at K, ſo that, when the piſton has been drawn up above K, the external air may come through this hole into the vacuum DE. And then, by thruſting down the piſton again, this hole will be ſtopped, and the air, which came through it into the ſpace DE, will be forced through the valve E into the veſſel FG, as before. Thus by alternately drawing up the piſton and thruſting it down again, there will be a conſtant ſupply of new air through the hole K, which will be forced into the veſſel FG. And as an unuſual quantity of air is thus forced into the veſſel FG, the air in that veſſel muſt be condenſed.

92. *The fleſh is ſwelled under a cupping glaſs or the blood is raiſed into it partly by the weight of the air, and partly by its elaſticity.*

When a cupping glaſs is applyed to the fleſh, an imperfect vacuum is made in it, or great part of the air is exhauſted out of it. There are two ways of doing this. In ſome cupping glaſſes, ſuch as have a ſmall orifice at the top covered with a valve, the vacuum is made, after the glaſs is applyed to the fleſh, by an exhauſting ſyringe, which is nothing but a ſmall air-pump. In other cupping glaſſes the vacuum is made by burning a piece of tow in it, and applying it to the fleſh, whilſt the tow is light. The fire will go out the very inſtant that the glaſs is thus applyed: and ſince the heat, whilſt the tow was burning, rarefyed the air and by that means drove moſt of it out of the glaſs, when this rarefyed air cools again, there will not be enough of it to fill the glaſs.

As

As there is a vacuum in the cupping glaſs, any other part of the body, which ſtill continues to be preſſed as uſual by the weight of the atmoſphere, muſt be more preſſed than that particular part, which is under the glaſs. By this greater preſſure upon the other parts, the fluids contained in the body will be driven towards the part under the glaſs; and conſequently this part will be ſtretched out by an unuſual quantity of thoſe fluids or will ſwell. But there is another cauſe, beſides this inequality of preſſure, which contributes to the ſwelling, and that is the elaſticity of the air contained in the fleſh: for the ſpring of this air will make it expand ſo as to diſtend or ſwell the fleſh, when the external preſſure is removed, by propoſition 49. For, if an animal is put into the receiver of an air-pump; when the air is exhauſted, no one part of the animal is preſſed more than the reſt, for no part is preſſed at all. But in theſe circumſtances the body of the animal, from the ſpring of the air contained in it, will ſwell. And the ſame cauſe, which produces a ſwelling of the whole body, when it is in a vacuum, will likewiſe produce a ſwelling in any particular part of the body that is in a vacuum.

When the glaſs has thus ſwelled the part, where it was applyed, it is taken off; and the fleſh is ſcarifyed or wounded with ſmall lancets in ſeveral places; and then the glaſs is applyed again in the ſame manner as before. In theſe circumſtances, as the blood or other fluids of the body are driven towards the cupping glaſs, they will riſe through the wounds into the glaſs, till they fill the vacuum.

93. *Suction and reſpiration in animals are effects of the airs preſſure.*

We have already ſeen in propoſition 88, how a common pump ſucks up the water, and in propoſition 90, how the receiver of an air-pump, when the air is exhauſted, is fixed by ſuction to the leather upon which it ſtands. When a vacuum is made in a pump, the preſſure of the atmoſphere makes the water riſe into that vacuum. And when a vacuum is made in a receiver, the preſſure of the external air fixes the receiver down to the leather upon which it ſtands. The ſuction of animals is performed in much the ſame manner. If a perſon through the ſiphon KIH, Plat. XIX. fig. 8, ſucks liquor out of a veſſel; he firſt by the help of his tongue makes a vacuum in his mouth, which is applyed at the orifice K. The air in the ſiphon expands itſelf into this vacuum; and as faſt as the air comes into his mouth, he carryes it off, ſtill keeping a vacuum at K. But whilſt the air is thus exhauſted out of the ſiphon, the liquor will be continually raiſed into it, from the veſſel FG, by the preſſure of the atmoſphere, by propoſition 46, till at laſt this liquor will come to his

mouth at R. And as long as he carryes off the liquor, and keeps a vacuum in his mouth, by the help of his tongue, as at firſt, the liquor will continue to run into his mouth. This is the method in which he ſucks the liquor through the ſiphon or through any other pipe of any other ſhape.

If a vacuum is made in a cupping glaſs the preſſure of the atmoſphere will drive the fluids of the body into the glaſs, by propoſition 92. And in like manner, if a vacuum is made in the mouth of an animal, when it is applyed to the breaſt, the preſſure of the atmoſphere will continue to drive the milk into the mouth of the animal, as long as it continues to carry off the milk as it comes into its mouth, and to preſerve the vacuum by the help of its tongue.

We muſt conſider reſpiration as conſiſting of two parts, the firſt is called inſpiration or drawing the breath into the lungs, the ſecond is expiration or driving the breath out of the lungs.

Reſpiration is performed by means of the muſcles, which raiſe or depreſs the ribs, and of the diaphragm or midriff. The midriff is a muſcular ſubſtance which divides the body into two parts. That part, which is above it, is the thorax or cheſt, and contains the heart and lungs. The part, which is below it, is the abdomen or belly, and contains the reſt of the inteſtines. Juſt before we draw in our breath, our ribs are in a depreſſed ſtate, and the midriff is convex towards the thorax, or riſes up into it, ſo that the capacity of the thorax is as ſmall as it can be: and the lungs at this time are flaccid. In order to draw in our breath, by a voluntary action of the muſcles, the ribs are raiſed and the midriff, inſtead of being convex towards the thorax, is contracted and becomes flat: by theſe means the capacity of the thorax is enlarged. The additional ſpace in the thorax, when it is thus enlarged, would be a vacuum, if no air was to come in to fill it up: but the preſſure of the atmoſphere forces the air through the mouth or noſtrils down the wind-pipe, where it is catched by the lungs and diſtends them, till they fill the thorax. By the contrary action of the muſcles, this air will be breathed out again. The ribs are made to fall, and the diaphragm is made convex towards the thorax ſo as to riſe into it; and thus the capacity of the thorax is again diminiſhed, ſo as to preſs the lungs till the air, which was taken in before, is driven out through the mouth or noſtrils.

CHAP.

CHAP. VIII.

Of winds particularly the trade-winds.

IT was neceſſary for the ſake of method to treat of this ſubject in this place. But the reader will not be able to underſtand the terms and reaſoning that are made uſe of in this chapter, till he is a little acquainted with the principles of aſtronomy and geography, that are to be explaned hereafter.

95. *When the atmoſphere is more rare in ſome places than it is in others, a wind or current of air may be produced, which will blow from the more denſe parts towards thoſe that are rarer.*

When the air in a room is heated, the external air, which is cooler, will force itſelf into the room through any creviſe, and by that means a ſtream or current of air is produced. For ſince the outward or cool air is more denſe than that in the room, which has been rarefyed by being heated, the preſſure from without will be greater than the preſſure from within, and conſequently the air will begin and continue to move in the direction of the ſtronger preſſure, till the equilibrium is reſtored. In like manner, when any part of the atmoſphere is more rarefyed than the other parts, which are near it, the denſer air will preſs more towards the rarefyed place than the rarefyed air preſſes the contrary way, and conſequently a current of air, or wind blowing towards the place of rarefaction, will be produced; and this wind will continue till the equilibrium is reſtored.

A like ſort of reaſoning would ſerve to ſhew us, that, if one part of the air is more denſe than the others that are near it, the preſſure from this place, where the condenſation happens, will be greater than the preſſures of the neighbouring air, which is not condenſed: and conſequently a wind will blow from this place of condenſation towards any other place where the air is leſs denſe.

96. *The ſun in its daily motion will produce a conſtant eaſt-wind.*

The hotteſt time of the day is about two or three hours after the ſun has paſſed the meridian, that is, about two or three o'clock in the afternoon. Therefore at this time of the day in any particular place the air is moſt rarefyed. Now the ſun moves from the eaſt of any place to the meridian of that place, and then from the meridian it moves towards the weſt: ſo that by three o'clock in the afternoon, when the air is moſt

rarefyed at any place, the ſun will have got far to the weſt of the place, and will have left the parts to the eaſt of it a conſiderable time. From whence it follows that, when the air at any place is in the ſtate of greateſt rarefaction, the air to the eaſt of that place will be cooled and condenſed; becauſe the ſun has left it for ſome time: and the air to the weſt of it will be rarefyed, upon account of the ſuns near approach to the weſtern parts. Thus at the time when the greateſt rarefaction happens at any particular place, as the eaſtern air keeps continually cooling and the weſtern air is continually more and more heated, the current will ſet in from the eaſt, by propoſition 95. But as the ſun apparently moves round the earth every day from eaſt to weſt, this rarefaction will conſtantly follow the ſun, and each place round the earths ſurface will in its turn become the place of greateſt rarefaction. And conſequently, by this daily motion of the ſun, an eaſtern current of air will be produced, or an eaſt-wind will be made to blow, all round the earths ſurface.

97. *As the ſun in its annual motion is never far from the equator; it will produce a conſtant north wind in north latitude, and a conſtant ſouth wind in ſouth latitude.*

The ſun in its apparent annual motion is twice vertical to the equator, and is never more than 23 $\frac{1}{2}$ degrees diſtant from it. So that the air at the equator will be conſtantly heated by the nearneſs of the ſun. Indeed the ſun is vertical once every year at each tropic, and continues ſo for ſome time; but then, at the oppoſite time of the year, it is removed to the diſtance of 47 degrees and is in the other tropic: by this means the air, which is heated at either tropic, whilſt the ſun is vertical, will cool again, whilſt it is in the other tropic. Thus the greateſt annual heat will be at the equator: and conſequently we may conclude that the atmoſphere all round the earth at the equator will be more rarefyed than it is any where elſe. Therefore, by propoſition 95, a conſtant current will ſet in towards the equator both from the north ſide and from the ſouth ſide. And a current ſetting in towards the equator on the north ſide or in north latitude is a north wind, and ſuch a current in ſouth latitude is a ſouth wind.

98. *The general trade-wind is a north-eaſt wind in north latitude, and a ſouth-eaſt wind in ſouth latitude.*

The ſuns daily motion produces an eaſt wind, by propoſition 96; and its nearneſs to the equator in its annual motion produces a north wind in north latitude, by propoſition 97. But theſe two winds compounded to-

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gether will produce an intermediate or north-eaſt wind, by propoſition 16 of mechanics. In like manner the ſouth wind in ſouth latitude compounded with the daily eaſt wind will produce a ſouth-eaſt wind. Theſe winds, from their great uſe in ſailing or trade are called trade-winds.

99. *The general winds both in north and ſouth latitude are liable to variations; but theſe variations will be the feweſt and will be made in the moſt regular manner near to the equator.*

It is impoſſible that the trade-winds ſhould blow in the regular manner already deſcribed all over the world, unleſs the whole ſurface of the globe was uniformly covered with water; ſo that there might not be any extraordinary rarefaction produced in particular places by other cauſes beſides the ſuns direct heat. But as land is longer in heating than water, ſo when once heated it retains the heat longer than water does. Therefore, as the ſun paſſes near iſlands or continents, the atmoſphere over thoſe iſlands or continents will be more rarefyed, than over the open ocean; and conſequently, by propoſition 95, the general trade-wind may frequently vary and ſet in towards the land, when by the rule already eſtabliſhed, in propoſition 95, it ought to have blown ſome other way. And indeed the winds at all times and in all places are upon this account much more irregular near the ſhore than out at ſea.

But as ſome variation in the winds may be produced by land in general, ſo the nature of the ſoil may make thoſe variations very conſiderable. A flat country of a ſandy ſoil will be more remarkably heated by the near approach of the ſun than any other: for this reaſon the rarefaction of the air over ſuch places will be greater than ordinary: and conſequently near ſuch lands the variation of the winds from their regular courſe will be moſt conſiderable.

There are many other cauſes, by which the wind may be varyed from its regular courſe, beſides the accidental poſition of land. Mountains may reflect the wind and change its current: or by their particular ſituation the air may have a ſort of channel between them, and may flow in the direction of this channel. The ſudden fall of hail or ſnow in any place may condenſe the air; and then the wind, which, before they fell, might blow towards the place, will afterwards blow from it, by propoſition 95. When the ſnow lies upon the tops of mountains, the air will be condenſed; but as it melts, the air will be rarefyed again: and conſequently the wind may blow one way, as long as the ſnow continues, and the contrary way, as as it is melted.

Now any of theſe cauſes and many more, that cannot be enumerated, may vary the courſe of the wind. But where the ſuns direct heat, which, by

by propoſition 98, is the cauſe of the general trade-winds, is the ſtrongeſt, theſe accidental cauſes will vary the courſe of the wind but little. Therefore about the torrid zone the winds will be more regular than they are in higher latitudes. It is indeed impoſſible, from knowing the cauſes of the trade-winds, to ſhew demonſtrably, that theſe winds can blow only to the limits of 30 degrees latitude on each ſide of the equator. In fact theſe are found to be the limits: and ſince the cauſe of the trade-winds, which we have here aſſigned, is the ſtrongeſt near the equator, it is reaſonable to imagine, that this cauſe will produce its effects in the moſt regular manner near the equator, and that in higher latitudes the regular trade-winds, where the ſuns direct heat is much leſs, ſhould not blow: becauſe in theſe higher latitudes accidental rerefactions or condenſations of the air may change the direction of the current much more, than they can where the cauſe of a conſtant regular current of air is more forcible.

Theſe winds are called tropical winds, becauſe their limits are near the tropics. In the following propoſitions we will conſider what variations the trade-winds themſelves are liable to within the limits of 30 degrees latitude. And, that the reader may underſtand the deſcription of the winds the better, there is a map inſerted at the end of this chapter, in which the ſhaded parts are deſigned to repreſent the limits of the trade-winds, and the directions, in which the little darts fly, repreſent the directions in which the winds blow.

We will conſider the whole ocean as divided into three parts. The firſt part is the Atlantic and Ethiopic ocean, which lies between America to the weſt, and Africa to the eaſt. The ſecond is the Indian ocean, which lies between Africa to the weſt, and the Indian iſlands to the eaſt. The third is the Pacific ocean or ſouth ſea, which lies between the Indian iſlands to the weſt and America to the eaſt. All theſe parts communicate with one another at the ſouth: but the land between the tropics divides them from one another in thoſe places where the trade-winds blow.

100. *In the Atlantic ocean upon the coaſt of Guinea there is a conſtant ſouth-weſt wind.*

This place is near the equator, and conſequently the direct heat of the ſun is very great, and is nearly the ſame all the year round. The ſoil is flat and ſandy, ſo that the rarefaction over the land will be much greater than it is over the ocean, which is cloſe to it; by what has been ſaid under propoſition 99. Therefore the wind all the year round will

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ſet in from the ocean towards the land: and a wind blowing in this direction is a ſouth-weſt wind.

101. *The equator is not the limit between the north-eaſt and ſouth-eaſt trade-winds in the Atlantic and Ethiopic ocean.*

This is an exception to the general rule already laid down, in propoſition 98. For by that rule the north-eaſt wind ought to blow from 30 degrees north latitude to the equator, and the ſouth-eaſt wind from 30 degrees ſouth latitude to the equator; ſo that the equator ought to be the limit between them. But in the Atlantic and Ethiopic ocean the north-eaſt wind reaches only from 30 degrees to 4 degrees north latitude; and the ſouth-eaſt wind reaches from 30 degrees ſouth latitude to 4 degrees north latitude, ſo that a circle of latitude 4 degrees to the north of the equator is the limit between them.

Now, in propoſition 97, we ſhewed, that the air would be more rarefyed round the equator than it is any where elſe, and from hence conſidering the equator as the circle of greateſt rarefaction we concluded, in propoſition 98, that the equator is the limit between the north-eaſt and ſouth-eaſt trade-winds. But, in propoſition 99, we obſerved, that though the rarefactions would be made in this manner, if the whole ſurface of the globe was uniformly covered with water, yet ſome variations may be occaſioned by the accidental poſition of iſlands or continents. And in the Atlantic ocean we may obſerve that there is more land a little to the north of the equator than at the equator, eſpecially on the African ſide. Therefore the equator will not be the circle of greateſt rarefaction: but this circle, and conſequently the limit between the north-eaſt and ſouth-eaſt trade-winds, will be a little to the north of the equator.

102. *In one part of the Atlantic ocean there are almoſt conſtant calms.*

This place is between 4 and 10 degrees north latitude, and is between 20 and 30 degrees to the weſt of the meridian of London. From the great quantity of rain, that falls there, it is called *the rains*, and from the calms, which are ſometimes interrupted by ſudden and violent guſts from all points of the compaſs, it is called *the calms and tornados*. Now the regular trade-wind, which is a north-eaſt one, blows from this place on one ſide, and the ſouth-weſt wind on the coaſt of Guinea, by propoſition 100, blows from it on the other ſide. Conſequently the air at the place being acted upon both ways by theſe equal forces will be in equilibrio, and for that reaſon will not move one way more than the other, ſo that there will be little or no wind at all.

And

And here it may be aſked, if the air is carryed from this place by two contrary winds blowing from it, whence is freſh air ſupplyed? To this we anſwer, that, as the air by the winds in the lower parts of the atmoſphere is carryed off, freſh air muſt be ſupplyed by the winds in the upper parts of the atmoſphere. Or ſince the air is continually carryed off by the two lower currents, which blow from the place, the preſſure of the air at this place will be diminiſhed, and conſequently, by propoſition 95, the neighbouring air will be carryed to the place by two upper currents in the higher parts of the atmoſphere. Now theſe two oppoſite currents, which in the upper parts of the atmoſphere blow to the place, will form an eddy where they meet, and if this motion is not ceaſed before the air ſubſides into the lower parts of the atmoſphere, it may occaſion thoſe ſudden and violent guſts from all points of the compaſs, which are called tornados.

103. *The trade-winds in the Atlantic and Ethiopic ſeas are more eaſterly near the coaſts of America, than they are near the coaſts of Africa.*

The trade-wind in north latitude is compounded of a north and an eaſt wind; and in ſouth latitude it is compounded of a ſouth and an eaſt wind, by propoſition 98. Now either of theſe compound winds will be more eaſterly as the eaſt wind, which is a part of them, is ſtronger. But the eaſt wind in any place will be ſtronger in proportion as the rarefaction to the weſtward of that place is greater, by propoſition 95. The continent of America lies to the weſtward of the coaſt of America; and the ocean lies to the weſtward of the coaſt of Africa. And, by what was ſaid under propoſition 99, the air over the continent will be more rarefyed, than over the ocean. Therefore the atmoſphere is more rarefyed to the weſtward of the coaſt of America, than to the weſtward of the coaſt of Africa: conſequently the eaſt wind in the Atlantic and Ethiopic ſeas will be ſtronger, and the trade-winds will be more eaſterly on the American ſide than on the African ſide.

104. *In the Atlantic and Ethiopic ſeas, when the ſun is in the tropic of cancer, the trade-wind in north latitude will be more eaſterly, and in ſouth latitude more ſoutherly than uſual: and when the ſun is in the tropic of capricorn, the trade-wind in ſouth latitude will be more eaſterly, and in north latitude more northerly than uſual.*

When the ſun is in the tropic of cancer, the atmoſphere about that tropic is as much rarefyed as it is about the equator. But the north wind in north latitude will be weak, unleſs the air at the equator is more rarefyed than it is at the tropic of cancer, by propoſition 97. Therefore when the

the ſun is in this tropic the north wind will be very weak, and conſequently the trade-wind in north latitude, which, by propoſition 98, is compounded of this and an eaſt wind, will be more eaſterly than it is at other times of the year.

But when the ſun is in the tropic of cancer, the air to the ſouth of the equator will be cooled in the ſuns abſence, whilſt the air to the north of the equator will be rarefyed by the vertical rays of the ſun. But ſince the denſity to the ſouth of the equator is encreaſed and to the north of the equator is diminiſhed; it follows, by propoſition 97, that in ſouth latitude the current of air from ſouth to north will ſet in more ſtrongly than uſual. But the trade-wind in ſouth latitude is compounded of this ſouth wind and an eaſt one. Therefore, in proportion as the ſouth wind is ſtronger, the direction of the compound wind will be more ſoutherly.

The like changes are made when the ſun is in the tropic of capricorn. For as the air is then equally rarefyed from the equator to this tropic, there will be ſcarce any ſouth wind in ſouth latitude: and conſequently the trade-wind in this latitude will be more eaſterly than uſual. But at the ſame time that the air in ſouth latitude is much rarefyed by the ſun in the ſouthern tropic, the air in north latitude will be condenſed in the abſence of the ſun. Therefore in north latitude the airs motion from north to ſouth will be encreaſed, or the north wind will become ſtronger: and upon this account the trade-wind, which is compounded of a north and an eaſt wind, will become more northerly.

105. *In the northern parts of the Indian ocean, the wind is north-eaſt from october to april, and ſouth-weſt from april to october.*

We have already ſeen, in propoſition 98, why a conſtant north-eaſt wind might blow here: or why, inſtead of this north-eaſt wind, there might be a conſtant ſouth-weſt one occaſioned by the rarefaction of the air over the continent of China and India in the ſame manner as in the Atlantic ocean, by propoſition 100, ſuch a ſouth-weſt wind blows upon the coaſt of Guinea. The only difficulty is to ſhew why theſe winds ſhould blow periodically in the Indian ocean, one of them from october to april or during the winter months, the other from april to october or during the ſummer months.

To remove this difficulty and ſhew the cauſe of theſe monſoons or changeable winds in the Indian ocean, we muſt obſerve that the land at the coaſt of Guinea is near the equator, and conſequently the air over that land will be heated equally all the year round: ſo that whatever effect the rarefaction of this air produces in the regular courſe of the trade-

wind, this effect will be conſtant. But the land at the continent of China and India is under the tropic of cancer; and conſequently the extraordinary rarefaction of the air over that land will continue only whilſt the ſun is to the north of the equator or during the ſummer months; and then the air will cool and condenſe again, whilſt the ſun is to the ſouth of the equator or during the winter months. Therefore in ſummer the wind will ſet in from the Indian ocean towards the land or will blow from the ſouth-weſt. But then in winter the wind in the Indian ocean, whilſt the air over the land becomes more denſe than it is at the equator, will blow from the north-eaſt, by propoſition 98.

106. *To the eaſt of Sumatra and Java the north-eaſt wind in winter is more northerly, and the ſouth-weſt wind in ſummer is more ſoutherly, than they are to the weſt of thoſe iſlands.*

The part of the Indian ocean that lies to the eaſt of Sumatra and Java is ſubject to the ſame monſoons with the part that lies to the weſt of thoſe iſlands, which have been deſcribed and accounted for, in propoſition 105. The only difference is what we have deſcribed in the propoſition now before us.

Now the north-eaſt wind being compounded of a north and an eaſt wind will be more northerly in proportion as the north wind is ſtronger. And the north wind in any place will be ſtronger in proportion as the rarefaction to the ſouth of that place is greater, by propoſition 95. But the rarefaction at the equator will be greater on the eaſt ſide of Sumatra and Java, than it is on the weſt ſide of them, by propoſition 99: becauſe on the eaſt ſide there is land at the equator, for the Indian iſlands are moſt of them either at the equator or near it; but on the weſt ſide there is no land at the equator. Therefore, on the eaſt ſide, the north wind will be ſtronger, and conſequently the north-eaſt wind will be more northerly than on the weſt ſide.

The ſouth-weſt wind, which blows for the other half of the year, is occaſioned by the extraordinary rarefaction of the air over the continent of China, &c. in the ſummer months, by propoſition 105. Now this land on the weſt ſide of Sumatra and Java lies to the north-eaſt of the ocean, and conſequently, by propoſition 95, it occaſions a ſouth-weſt wind. But on the eaſt ſide of Sumatra and Java this land lies rather to the north of the ocean, and conſequently the current of air ſetting in towards it, or the wind occaſioned by it, will be more ſoutherly than it is on the weſt ſide of theſe iſlands.

107. *In the ſouthern parts of the Indian ocean between the latitudes of three and ten degrees, a ſouth-eaſt wind blows from the end of may to the beginning of december; and then from december to may the wind is changed into a north-weſt one.*

We have already ſeen a ſimilar inſtance of a monſoon or changeable trade-wind, in propoſition 105, where the land, towards which the wind blows one half of the year, was ſhewn to be the occaſion of the change. So, in propoſition 100, we found that the ſituation of the land would make the wind conſtantly ſet in towards the coaſt for the whole year together. Now we may reaſonably conclude that the effect deſcribed in this propoſition is produced by a ſimilar cauſe. The ſouth-eaſt wind, which blows during one half of the year, is the regular trade-wind of ſouth latitude, by propoſition 98. And it is probable the wind here is changed into a north-weſt one during the other half year by the land about this part of the ocean where the change happens. New Holland lies to the north-weſt of this ocean, and when the ſun is to the ſouth of the equator, the air over this tract of land may be ſo rarefyed as to cauſe the wind to ſet in towards the coaſt. Indeed upon this ſuppoſition it would be very difficult to ſhew any reaſon why this north-eaſt wind ſhould reach only to 10 degrees ſouth latitude, and not as far as the coaſt of new Holland. All that we can well ſay about it ſeems to be that ſuch changes planely are produced in other places by the extraordinary rarefaction of the atmoſphere over the land: and therefore it is probable that the ſame effect in this place is produced by the ſame cauſe.

108. *The trade-winds do not vary in the Atlantic and Ethiopic ocean, as they do in the Indian ocean.*

The monſoons in the Indian ocean have been deſcribed, in propoſitions 105, 106, 107. But in the Atlantic and Ethiopic oceans the winds blow the ſame way all the year without any ſuch conſiderable variations. We have ſeen already how far we can explane theſe variations, and the only queſtion remaining is why the ſame ſhould not happen in the Atlantic and Ethiopic oceans. Now the cauſe of the winds varying in the Indian ocean, by propoſitions 105, 106, 107, is the land, which lies to the north of that ocean in north latitude, and to the ſouth-weſt of it in ſouth latitude. But the Atlantic ocean is open to the north and has very little land under the tropic of capricorn to the ſouth weſt: conſequently the ſame changes will not be made here that are made in the Indian ocean.

109. *The trade-winds are moſt regular in the Pacific ocean.*

The irregularities, which happen in the other parts of the ocean are principally occaſioned by the neighbouring land. But the Pacific ocean is a vaſt tract of ſea with little or no land interſperſed. And conſequently in this part of the ocean the north-eaſt wind will blow regularly in north latitude, and the ſouth-eaſt wind in ſouth latitude. Near the coaſts indeed there will be more irregularities in this ocean as well as in the reſt, by propoſition 99, than there are out at ſea.

110. *The rotation of the earth round its axis does not at all contribute to the trade-winds.*

In one reſpect indeed the rotation of the earth is the remote cauſe of the trade-winds; becauſe theſe winds, by propoſition 96, are owing to the ſuns apparent daily motion, and this apparent motion is owing to the earths rotation. But this rotation does not at all contribute to the trade-winds as an immediate cauſe. It has been imagined that the trade-winds will be produced as the earth turns round from weſt to eaſt, if the atmoſphere, which encompaſſes the earth either is at reſt or moves the ſame way with the earth but only more ſlowly: for whether the earth is relatively at reſt in the atmoſphere, till the atmoſphere is made by ſome cauſe to move from eaſt to weſt, or whether the atmoſphere is at reſt and the earth moves in it from weſt to eaſt, either motion will make the air apparently move from eaſt to weſt or will produce an eaſt wind.

This opinion is liable to many objections, ſome of which will be obvious from what has been already ſaid, and thoſe are the only ones that we will take notice of; except merely to obſerve that the foundation of the opinion is falſe; becauſe, as the earth turns round, the motion of it, if not originally, yet in length of time has been communicated to the atmoſphere, ſo that the atmoſphere turns round as faſt as the earth does. But if it were otherwiſe, nothing would follow from the atmoſpheres reſting, whilſt the earth moves, except an eaſt wind; whereas the regular trade-wind is north-eaſt in north latitude, and ſouth-eaſt in ſouth latitude, by propoſition 98. Beſides, in the Indian ocean for half the year there is a ſouth-weſt wind in ſouth latitude, and a north-weſt wind in north latitude, which cannot poſſibly ariſe from the rotation of the earth round its axis, ſince this rotation, if it could produce any wind at all, could only produce an eaſterly one. And indeed, if the rotation of the earth contributed at all to the eaſterly trade-winds, we muſt neceſſarily find that thoſe winds would be more violent than the weſterly trade-winds, to which

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WHOLE WORLD
Shewing the Variations *of y^e*
COMPASS
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with a View of the Generall
and Coasting Trade Winds
and Monsoons *or shifting*
Trade Winds
by the Direction of
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EASTERN TARTARY

ASIA

EMPIRE of CHINA

EMPIRE of PERSIA

INDOSTAN

THE INDIAN OR EASTERN OCEAN

HOLLANDIA NOVA

AFRICA

GUINEA

THE SOUTHERN OCEAN

West Variation

East Variation

Degrees of Variation

WEST

Line of no Variation

Van Diemens Land

New Zeland

J. Harris delin. & Sculp.

we are ſure this rotation can contribute nothing. Whereas juſt the reverſe is true: for in the Indian ocean where the wind, by propoſition 105, blows half the year from the north-eaſt, and the other half from the ſouth-weſt, the ſouth-weſt wind has rather more force than the north-eaſt one. And conſequently the rotation of the earth round its axis does not contribute at all to the production of the trade-winds.

CHAP. IX.

Of the reſiſtance of fluids.

III. *The reſiſtance, which a body meets with in moving through a fluid, is owing either to the force of inactivity in the particles of the fluid, or to the tenacity of the fluid.*

A body, ſuch as a ſtone or a ball of lead, cannot move forwards in water, or in any other fluid, till it has removed the parts of the fluid, which lie before it, out of the way; and theſe particles, ſince they have that general property of matter called the force of inactivity, will reſiſt being moved. This is one cauſe of the reſiſtance, which fluids make to bodies that are moving in them. And this reſiſtance will be made by the moſt perfect fluids as well as by the moſt imperfect ones, by air as well as by liquid honey. For however freely the parts of a perfect fluid may move amongſt one another, yet they are ſtill matter, and conſequently from their force of inactivity they will reſiſt bodies that are moving in them.

But there is another cauſe of reſiſtance, which is principally to be found in imperfect fluids, the reſiſtance ariſing from the tenacity of the parts. For ſince a body cannot move forwards in a fluid, till the particles of the fluid, that lie before it, are removed out of the way; if theſe particles adhere at all to one another, this adheſion or tenacity will hinder or reſiſt the motion of the body. This cauſe of reſiſtance is very inconſiderable in moſt fluids: for by the definition of a fluid the parts of it move freely amongſt one another. Thoſe fluids, where this ſort of reſiſtance is conſiderable, ſuch as turpentine or liquid honey, are very imperfect ones, and to diſtinguiſh them from others, which are more perfect, they are called tenacious fluids.

There is indeed a third cauſe of the reſiſtance, which fluids make to the motion of bodies, and that is the friction of the body againſt the particles of the fluid. But no notice was taken of this reſiſtance in the propoſition, becauſe from the nature of a fluid it is very inconſiderable, and becauſe there will be no particular uſe made of it in the propoſitions that follow.

112. *If equal ſpherical bodies are moving in a given fluid, the reſiſtance, which ariſes from the force of inactivity in the particles of the fluid, is as the ſquare of the velocities with which theſe bodies move.*

If two equal ſpheres of lead move in water, and their velocities are to one another as 2 to 3, the reſiſtances, which they ſuffer from the force of inactivity in the particles of the water, will be as the ſquares of their velocities, or as 4 to 9. For firſt, in an equal time the number of particles, which theſe two bodies meet with, will be as their reſpective velocities, or as 2 to 3: for the ſpaces which they will deſcribe in an equal time, will be as their velocities, by propoſition 10 of mechanics, and the number of particles, which they meet with in the ſpaces that they deſcribe, will be as the ſpaces themſelves, that is, as the reſpective velocities of the two bodies. But the force of inactivity, and conſequently the reſiſtance ariſing from it, is proportional to the quantity of matter, or to the number of particles, by which this reſiſtance is made. Therefore the reſiſtance ariſing from the force of inactivity in the particles of the water will be as the velocities of theſe balls of lead, or as 2 to 3. Secondly, if a given quantity of matter is to be moved, the moving force muſt be proportional to the velocity, which is communicated, by propoſition 13 of mechanics; and the reſiſtance, which that given quantity of matter makes to being moved, is proportional to the force that is requiſite to move it. Therefore the reſiſtance ariſing from the force of inactivity in a given number of particles of the water will be proportional to the reſpective velocities, with which they are driven out of the way by the bodies in motion. But the velocities, with which the particles of water are driven out of the way, muſt be the ſame as the velocities with which the bodies move. Therefore if the velocities of the bodies are as 2 to 3, the reſiſtance of the water to their reſpective motion will be as 2 to 3.

Now ſince the reſiſtance of a fluid is as the velocity of the body moving in it upon a double account: firſt becauſe the number of particles that are moved in a given time is as the bodys velocity; and ſecondly becauſe the reſiſtance ariſing from a given number of particles will likewiſe be as the bodys velocity; it follows upon the whole that the reſiſtance will be as the velocity and as the velocity, or as the velocity multiplyed into itſelf, or as the ſquare of the velocity.

Thus in the inſtance before us, if the velocities of two equal ſpherical bodies moving in the ſame fluid are to one another as 2 to 3; the reſiſtance of that fluid to the bodies will be as 2 to 3 reſpectively upon account of the different number of particles of the fluid, which each body

will

will remove out of the way, or communicate motion to, in a given time; and this reſiſtance will likewiſe be as 2 to 3 reſpectively, upon account of the different velocities with which thoſe particles of the fluid will be removed. Therefore upon both accounts together the reſiſtance to each body reſpectively will be as 2 to 3 and as 2 to 3, or as 4 to 9, that is, as the ſquares of their reſpective velocities.

Or otherwiſe. When a ſpherical body is moving in any fluid below the ſurface of it, as in water for inſtance, it cannot move on, without removing a column of water out of the way: this column is a cylinder, the baſe of which is the body in motion, and the height or length of which is the line deſcribed by the body. But the ſize of the body and conſequently the baſe of the column is given, by the ſuppoſition. Therefore the quantity of matter in the column is proportional to its height or to the ſpace deſcribed by the body in a given time. Euc. b. XII. prop. 11. Now as the body, in removing this column of the fluid communicates motion to it, the reſiſtance of the fluid is proportional to the quantity of motion ſo communicated. For ſince the reſiſtancce of the fluid ariſing from its inactivity is nothing elſe but its reſiſtance to being moved, this reſiſtance will neceſſarily be greater or leſs in proportion to the moment or quantity of motion, which is to be communicated to the fluid. Now the moment communicated to the fluid will be as the quantity of matter that is moved, and as the velocity of its motion, by propoſition 13 of mechanics. The quantity of matter is as the ſpace deſcribed by the body in a given time, as has juſt been proved. And the ſpace deſcribed in a given time is as the velocity of the body, by propoſition 10 of mechanics. Therefore the quantity of the fluid, that is moved in a given time, will be as the bodys velocity. But it is plane that the velocity, with which this column of the fluid is removed, muſt be proportional to the velocity, with which the body moves forwards; for as faſt as it moves forwards, juſt ſo faſt it muſt neceſſarily remove the fluid out of the way. Therefore the moment, that is communicated to the fluid, will be as the velocity of the body upon a double account; firſt, becauſe the quantity of matter, to which motion is communicated, will be as the velocity of the body; and ſecondly, becauſe the velocity, with which that quantity is moved, will likewiſe be as the velocity of the body: conſequently the moment of the fluid, that is removed, or the reſiſtance of the fluid, being as the velocity and as the velocity, will be as the velocity multiplyed into itſelf, or as the ſquare of the velocity of the body.

It is by means of the waters reſiſtance that a ſhip under ſail is turned by its rudder, ſo as not to ſail directly before the wind. If the wind blows

in

in the direction CD, Plat. XX. fig. 6, the ſhip would move in that direction, unleſs the rudder was to make it move in a different one. Suppoſe the rudder was put into the poſition EC: then, as the ſhip moves along, the rudder will be reſiſted by the water, that lies before it; and this reſiſtance, by the ſecond law of motion, will be made in lines perpendicular to the rudders ſurface. Now a reſiſtance puſhing againſt the fore part of the rudder will have the ſame effect as any force pulling behind it. Therefore CA perpendicular to CE may be put for the force, with which the reſiſtance of the water acts. Thus the end of the ſhip at C is acted upon by two forces, one of which is the force of the wind in the direction CD, the other is the reſiſtance in the direction CA. By theſe two forces acting together the end C of the ſhip will, by propoſition 16 of mechanics, be turned out of the line CD into the line CB, which is the diagonal of a parallelogram whoſe two ſides are CD and CA. But, when the end C of the ſhip, where the rudder is faſtened, is turned one way, the ſhip will turn round upon its center of gravity, and conſequently the other end or head of the ſhip will be turned the contrary way. Therefore by turning the end C, where the rudder is faſtened, into the line CB, the head of the ſhip will be turned into the line CF, and conſequently this line CF is the direction, in which the ſhip will ſail.

113. *When unequal ſpherical bodies move with a given velocity in the ſame fluid, the reſiſtance of the fluid ariſing from the inactivity of it will be as the ſquares of their diameters.*

When a ſphere of lead, or of any other ſubſtance, moves along in water, or in any other fluid, it removes a column of this water out of the way. The column is a cylinder; the height of it is the ſpace, which the ſphere deſcribes and the baſe of it is equal to the ſphere or rather to a great circle of the ſphere. Now where the velocity is given, the ſpace deſcribed in a given time and conſequently the height of that column, which the ſphere removes, will likewiſe be given. Therefore the contents, or quantity of matter contained in that column, will be as its baſe or as a great circle of the ſphere. Euc. b. XII. prop. 11. But ſince the velocity of the ſphere, conſequently of the column of fluid removed by the motion of the ſphere, is given, the reſiſtance, which the fluid makes to the motion of the ſphere by its inactivity, will be proportional to the quantity of matter in this column, to which the ſphere is to communicate motion in order to remove it out of the way. Therefore the reſiſtance which the ſphere ſuffers, will be as the baſe of the column, or as a great circle of the ſphere. But the great circles of ſpheres are as the ſquares of their di-

diameters. Euc. b. XII. prop. 2. Therefore the reſiſtance, which a given fluid makes to a ſpherical body moving with a given velocity, will be as the ſquare of its diameter.

114. *If two unequal ſpheres conſiſting of the ſame ſubſtance are moving in the ſame fluid with equal velocities, the greater ſphere will be leſs reſiſted in proportion to its weight than the leſſer ſphere.*

The weights of ſpheres or their ſolid contents are as the cubes of their diameters. Euc. b. XII. prop. 18. Thus if the diameters of two leaden ſpheres are to each other as 3 to 2, their ſolid contents, or weights, will be to each other as 27 to 8. The reſiſtances, which they ſuffer in moving through the ſame fluid, are as the ſquares of their diameters or as 9 to 4, by propoſition 113. But 27 bears a greater proportion to 8, than 9 bears to 4. Therefore the weights of the bodies differ more, than the reſiſtances, which they would ſuffer in a given fluid; or the weight of the greater ſphere exceeds the weight of the leſſer ſphere more than the reſiſtance of a given fluid againſt the former exceeds the reſiſtance againſt the latter, when they are both moving with equal velocities.

The weight of all ſpheres encreaſes as the cube of their diameter encreaſes. But the reſiſtance made to them by a given fluid, when they are moving with a given velocity, encreaſes as the ſquare of their diameter encreaſes, by propoſition 113. Now the cube of any quantity encreaſes faſter than the ſquare of that quantity. Therefore the weights of ſpheres encreaſe faſter than the reſiſtances, that are made to their motion, when their own velocity and the denſity of the fluid is given. Thus if any very ſmall ſpherical drops of water are floating in the atmoſphere; their weight may be ſo ſmall that the reſiſtance of the air will keep them from falling. But if any number of theſe ſmall drops were to unite and to form one larger ſpherical drop; the reſiſtance of the air, which could ſupport each of them, when they were ſeparate, may be inſufficient to ſupport them, when they are united into a large maſs. When the drop encreaſes the quantity of matter or weight of it encreaſes, and the reſiſtance, which the air makes to it, by propoſition 113, encreaſes likewiſe. Now if the weight did not encreaſe faſter than the reſiſtance; it is evident that the reſiſtance of the air, if it was ſufficient to ſupport the drop, when it was ſmall, would be ſufficient likewiſe to ſupport it, when it was ever ſo large. For though the weight of a large drop is greater than the weight of a ſmall one, yet, upon this ſuppoſition, the reſiſtance of the air againſt a large drop would be juſt as much greater than the reſiſtance againſt a ſmall one. But if, as we have juſt proved, the weight of

the drop encreaſes faſter than the reſiſtance, which the air makes to its motion; then though the reſiſtance was in equilibrio with the weight of a ſmall drop, yet, when the drop has encreaſed, its weight may become too great for the reſiſtance to ſupport it. The weight and reſiſtance may be in equilibrio when the drop is ſmall, and ſo they would be in all ſtates of the drop, if the weight encreaſed no faſter than the reſiſtance. But the weight does encreaſe faſter than the reſiſtance. Therefore, when the drop is large, the weight of the drop will over-balance the reſiſtance of the air, and the drop will fall.

115. *When a body is moving in a fluid, if all other circumſtances are equal, the reſiſtance of the fluid ariſing from the inactivity of its particles, is proportional to the ſide of the body, which is perpendicularly oppoſed to that reſiſtance.*

This is proved in the ſame manner with propoſition 113. For if a body is moving in any given fluid, as water for inſtance, and is moving with a given velocity; then, ſuppoſing the ſhape of that ſide of the body which moves foremoſt to be given, the reſiſtance will be proportional to the quantity of matter contained in that column of water, which the body removes in a given time. Now the height or rather the length of this column will be proportional to the ſpace, which the body deſcribes in a given time: but this ſpace is given, by propoſition 10 of mechanics, becauſe the body moves with a given velocity. Therefore the height of the column, which the body removes out of the way or to which it communicates motion, is given: and conſequently the quantity of matter contained in it will be as its baſe. But the baſe of the column ſo removed is equal to the ſide of the body, which moves foremoſt, or to the ſide which is perpendicularly oppoſed to the reſiſtance of the fluid. Therefore the reſiſtance of the fluid will be proportional to this ſide of the body.

116. *When equal ſpheres move with the ſame velocity in different fluids, the reſiſtances will be as the denſities of thoſe fluids.*

If two equal ſpheres of lead were to move with the ſame velocity, one in water and the other in mercury, the reſiſtances of theſe fluids to the motion of the ſpheres would be as the reſpective denſities of water and mercury, or as 1 to 14. The reſiſtances ariſing from the inactivity of the fluid are as the moments communicated to the fluid in a given time by the bodies in motion. Theſe moments will be as the quantities of matter that are moved, becauſe the bodies move with equal velocities; and con-

conſequently the column of each fluid will be removed with equal velocities. But the bodies are equal, by the ſuppoſition. Therefore the baſes of the columns of water and of mercury, to which they reſpectively communicate motion, will be equal. The bodies likewiſe move with equal velocities or deſcribe equal ſpaces in equal times, by the ſuppoſition. Therefore the heights or lengths of the columns of water and mercury, to which they reſpectively communicate motion, will be equal. Now if the baſes and heights of the columns, to which motion is communicated, are equal, the bulks of theſe columns muſt be equal: and conſequently the quantities of matter contained in them will be as their denſities, by propoſition 1. But their moments and conſequently their reſiſtances are as their quantities of matter. Therefore their reſiſtances are as their reſpective denſities: ſo that the reſiſtance, which one of the bodies ſuffers in water will be as much leſs than what the other ſuffers in mercury, as the denſity of water is leſs than the denſity of mercury.

If any drops of water are ſo large that the reſiſtance of the air in the upper parts of the atmoſphere cannot ſupport them; yet as thoſe drops fall, they come into ſuch parts of the atmoſphere as have a greater denſity, by propoſition 50. But the reſiſtance of the air againſt thoſe drops will be proportional to the denſity of the air. Therefore in falling they may come to ſuch parts of the atmoſphere as are denſe enough to ſupport them: and conſequently the drops may ſtop there and fall no farther.

117. *The retardation of bodies, in a reſiſting fluid, will be proportional to the reſiſtance of the fluid, where the weights of the bodies are given.*

The immediate effect of the reſiſtance, which a body ſuffers whilſt it moves in a fluid, is to deſtroy a part of the bodys moment. The body cannot move forwards without communicating motion to the fluid. But the reſiſtance of the fluid to the bodys motion is the reſiſtance, which the fluid makes to being moved. Therefore the reſiſtance of the fluid is proportional to the motion, which the body communicates to it. Now whatever motion the body communicates to the fluid, it muſt loſe itſelf, by propoſition 15 of mechanics. Therefore the motion loſt by the body is proportional to the motion, which it communicates to the fluid, or to the reſiſtance of the fluid.

The retardation of a body, whilſt it is reſiſted, is the decreaſe of the bodys velocity. Now the moment deſtroyed is proportional to the reſiſtance, by what has been ſhewn already: and when the quantity of matter or weight of the body is given, the velocity is proportional to the

moment, by propoſition 13 of mechanics. Therefore the velocity deſtroyed or the retardation will be proportional to the reſiſtance.

118. *The retardation of bodies, when the reſiſtance is given, will be inverſely as their weights.*

If two bodies, that have unequal weights, meet with an equal reſiſtance in moving through a fluid, thoſe bodies will not be retarded equally: but the heavier body will be leſs retarded or will loſe leſs velocity than the lighter; ſo that in all caſes, the greater the weight of the body is, ſo much the leſs it will be retarded by a given reſiſtance. The ſame reſiſtance will produce an equal change in the motion of a body, or will deſtroy an equal moment, whatever is the weight of the body: becauſe the ſame cauſe will always produce an equal effect. But in equal moments the velocity is inverſely as the quantity of matter, by propoſition 14 of mechanics. Therefore when equal moments are deſtroyed, the velocity deſtroyed or the retardation will be inverſely as the quantity of matter in the body, in which the given moment is deſtroyed. But the weight is proportional to the quantity of matter, by propoſition 27 of mechanics; and conſequently the retardation from a given reſiſtance will be inverſely as the weight of the body againſt which that given reſiſtance is made.

119. *The retardation of ſpherical bodies, when they are moving with equal velocities in the ſame fluid, will be inverſely as their diameters.*

The reſiſtance, which ſpherical bodies ſuffer in a given fluid, is as the ſquare of their diameter, by propoſition 113. and the retardation, when the weight is given, is proportional to the reſiſtance, by propoſition 117. But the weight of unequal ſpheres conſiſting of the ſame ſubſtance is as the cube of their diameter. Euc. b. XII. prop. 18. And the retardation, when the reſiſtance is given, is inverſely as the weight, by propoſition 118, or inverſely as the cube of the diameter. Now when unequal ſpheres are moving with the ſame velocity in the ſame fluid, the retardation will be unequal both becauſe the reſiſtances are unequal and becauſe the weights are unequal. Therefore the retardations will be directly as the ſquares of the diameters, and inverſely as the cubes of the diameters. And theſe two ratios compounded give the inverſe one of the diameters.

Thus ſuppoſe the diameters of two ſpheres were to each other as 2 to 3, the ſquares of the diameters are as 4 to 9, and the cubes are as 8 to 27. The retardations therefore will be as 4 to 9 directly and as 8 to

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27 inverſely, or as $\frac{4}{8}$ to $\frac{9}{27}$, for theſe two fractions are to each other directly as their numerators 4 and 9, and inverſely as their denominators 8 and 27. But $\frac{4}{8}=\frac{1}{2}$ and $\frac{9}{27}=\frac{1}{3}$. Therefore the retardations are either as $\frac{4}{8}$ to $\frac{9}{27}$, or as $\frac{1}{2}$ to $\frac{1}{3}$. But $\frac{1}{2}$ and $\frac{1}{3}$ are the reciprocals of the diameters 2 and 3. Therefore the retardations are as the reciprocals of the diameters or inverſely as the diameters.

Now as the fluid retards the motion or deſtroys the velocity of a body that is moving in it, ſo the reader will eaſily underſtand, that if the body is at reſt and the fluid in motion, the fluid will communicate velocity to the body. For ſince as the body, when it moves and ſtrikes upon the fluid, is retarded by the reaction of the fluid, ſo when the body is at reſt and the fluid moves againſt it or ſtrikes upon it, the body will be accelerated or will have velocity communicated to it by the action of the fluid. Now action and reaction being equal, by propoſition 15 of mechanics, a fluid will communicate velocity in one caſe in the ſame manner and by the ſame laws that it deſtroys it in the other caſe. Therefore, when ſpherical bodies are ſtruck upon by a ſtream of any fluid, the velocities acquired by them will be inverſely as their diameters. Thus we ſee that the wind will move leſſer bodies, though it is unable to move greater bodies of the ſame ſort; it will drive ſmall ſand or duſt before it, but cannot move looſe ſtones or clods of earth. Small drops of rain ſeldom fall perpendicular, becauſe a very little wind will carry them in the ſame direction with itſelf and make them fall obliquely. But heavy ſhowers or large drops, fall either quite or nearly perpendicular. If a ſmall ſtone is dropped into a ſtrong ſtream of water, it will be carryed a little forwards by the ſtream and will ſink obliquely: but a large ſtone dropped into the ſame ſtream would ſink downright without being carryed forwards at all by the ſtream.

120. *Heavy bodies, when they fall through the air, move with unequal velocities.*

Though bodies falling by their own weight would move with equal velocities, if they were to meet with no reſiſtance, by propoſition 26 of mechanics; yet in falling through the air, which does reſiſt them, it will be otherwiſe. A bullet and a feather will ſet out with and continue to move with equal velocities, when they fall in an exhauſted receiver: but the feather will fall ſlower than the bullet, if they fall through the air. The ſide of the feather is bigger than that of the bullet, and conſequently the feather will be more reſiſted, by propoſition 115, and will be more retarded, by propoſition 117, than the bullet. The feather is lighter than

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the bullet, and conſequently the feather will be more retarded than the bullet, by propoſition 118. But if the feather upon theſe two accounts is more retarded than the bullet, it is evident that the feather muſt fall ſlower than the bullet.

A ball of cork will fall ſlower than a bullet of the ſame ſize. For though, by propoſition 26 of mechanics, they will begin to fall with the ſame velocity; and though, by propoſition 113, they will be equally reſiſted, yet as the cork is lighter it will be more retarded, and conſequently will fall ſlower than the bullet.

If a ſheet of lead was to fall with one of its flat ſides downwards, it would fall ſlower, than if it was to fall with one of its edges downwards. For though the weight is the ſame in both caſes, yet as the ſide is broader than the edge, it will be more reſiſted and more retarded, by propoſition 115, and conſequently will fall ſlower in one caſe than in the other.

And univerſally, though the ſides of two bodies oppoſed to the reſiſtance of the air are equal, yet they will not fall with equal velocities unleſs their weights are equal, by propoſition 118; and though their weights are equal, yet they will not fall with equal velocities, unleſs their ſides oppoſed to the reſiſtance are equal, by propoſitions 113, 117.

121. *When a body moves in a tenacious fluid, the reſiſtance of every infinitely thin lamina of the fluid ariſing from its tenacity is inverſely as the bodys velocity.*

Liquid honey, ſoft tallow, turpentine, or ſoft clay, are tenacious fluids. Now if a body is moving in ſoft clay, and we conceive the clay to be divided into very thin ſurfaces, each ſurface or lamina by its tenacity or the coheſion of its parts will reſiſt the body more in proportion as its velocity is leſs, and will reſiſt it leſs in proportion as its velocity is greater. The tenacity of each lamina is the ſame whether the body moves faſt or ſlow. And this tenacity will continue to reſiſt the body, till the body has broken through the lamina and has got clear of it. Now the ſlower the body moves, the longer it will be in breaking through the lamina, and the faſter it moves the ſooner it will break through: ſo that the time, during which the coheſion or tenacity of the lamina acts, is greater as the bodys velocity is leſs, and leſs as the bodys velocity is greater, that is, the time, during which the tenacity of the lamina reſiſts the body, is inverſely as the bodys velocity. But ſince the reſiſtance of each lamina is the ſame in itſelf, it can only be greater or leſs in proportion as it acts for a longer or a ſhorter time, that is, the reſiſtance is proportional to

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the time of its action. But the time of its action is inverſely as the bodys velocity. Therefore the reſiſtance of every infinitely thin lamina will be inverſely as the bodys velocity.

122. *When a body moves in a tenacious fluid, the reſiſtance of any given number of laminas ariſing from their tenacity is inverſely as the bodys velocity.*

The reſiſtance of each lamina is inverſely as the bodys velocity, by propoſition 121: and conſequently the reſiſtance of a given number of them will be in the ſame proportion. This may be proved diſtinctly, as the laſt propoſition was. For the force of coheſion or tenacity in any certain number of laminas is the ſame, whatever the bodys velocity is, that breaks that coheſion. Thus the force, with which the particles of 10 laminas are united, is the ſame, whether the body moves faſt or ſlow. But the longer time this force acts upon the body ſo much the more of its motion the body will loſe, and the leſs time it acts ſo much leſs of the bodys motion will be loſt; that is, the reſiſtance ariſing from theſe 10 laminas will be directly as the time that the body takes up to break through them. But ſince each lamina is ſuppoſed to have the ſame thickneſs, any given number of them will likewiſe have a given thickneſs, ſo that the thickneſs of 10 laminas, or of any other given number, will be a given quantity or a given ſpace. Conſequently the time that the body takes up to break through any given number of lamina or to paſs over a given ſpace will be inverſely as its velocity, by propoſition 9 of mechanics. Now the reſiſtance is directly as the time, and the time is inverſely as the velocity. Therefore the reſiſtance is inverſely as the velocity.

From hence we may obſerve, that if two balls of lead, that fall upon a maſs of ſoft clay, have unequal velocities and yet make pits or holes of equal depths in the clay and then ſtop, the reſiſtance, which they ſuffer, muſt be inverſely as their velocities. Each ball ſinks to an equal depth in the clay, by the ſuppoſition. Therefore each ball breaks through an equal number of laminas or thin plates of the clay; and conſequently the reſiſtance of the clay upon theſe balls will be inverſely as their velocities, by the propoſition now before us.

123. *When a body moves in a tenacious fluid with a given velocity, the reſiſtance ariſing from the fluids tenacity is directly as the number of laminas, through which the body paſſes.*

Since the velocity of the body is given, the reſiſtance of each lamina will be given, by propoſition 121: and conſequently the reſiſtance, which the

the body ſuffers, can only be greater or leſs in proportion as the number of laminas is greater or leſs; that is, the reſiſtance will be directly as the number of laminas.

124. *The reſiſtance ariſing from the tenacity of a fluid is uniform, or is always proportional to the time during which a body moves in that fluid.*

In order to prove this we will ſhew, that, whatever velocity a given body moves with, the reſiſtance, which it ſuffers in an equal time, will always be equal. The ſpace through which a body paſſes or the number of laminas through which it breaks in a given time is directly as its velocity, by propoſition 10 of mechanics. But the reſiſtance is proportional to the number of laminas, through which the body paſſes, by propoſition 123. Therefore the reſiſtance is directly as the bodys velocity. This is true, when each lamina reſiſts equally. But by propoſition 121, the reſiſtance of each lamina will be inverſely as the bodys velocity. Conſequently, when the body is moving for a given time, the reſiſtance, which it ſuffers, will be directly as its velocity and inverſely as its velocity; directly as its velocity, by propoſition 123, becauſe the number of laminas is in this proportion; and inverſely as its velocity, by propoſition 121, becauſe the reſiſtance of each lamina, through which the body paſſes, is in this proportion. So that if we call the velocity of the body V, the reſiſtance, which it ſuffers in a given time, will be as V directly and as V inverſely, or as $\frac{V}{V}$. But $\frac{V}{V}=1$ or is an invariable quantity. Therefore the reſiſtance, which the body ſuffers in a given time, will always be the ſame, whatever is the velocity with which it moves.

An inſtance will make this planer. If two equal balls of lead A and B are moving in liquid honey, let the velocity of A be to that of B as 2 to 4, or let B's velocity be double A's velocity: then ſuppoſe A in a ſecond of time breaks through 10 very thin plates, ſurfaces, or laminas of the honey, it is evident that B, which moves with double the velocity, will in an equal time move twice as far, and conſequently will break through 20 ſuch laminas. Now the reſiſtance, which each body ſuffers in this ſecond of time will be the ſame. For firſt, the reſiſtance, which A ſuffers, is directly as the number of laminas it breaks through, by propoſition 123, and is inverſely as its velocity, by propoſition 121. Therefore the reſiſtance, which A ſuffers, is as 10 directly and as 2 inverſely, or as the improper fraction $\frac{10}{2}=5$. Secondly for the ſame reaſons the reſiſtance which B ſuffers will be as $\frac{20}{4}=5$. And conſequently they ſuffer equal reſiſtances.

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But if the reſiſtance ariſing from the tenacity of a fluid is equal in equal times, then in unequal times it will be proportional to the times. So that the reſiſtance, which a body ſuffers by moving in ſuch a fluid, will be greater, as the time of its motion is longer, and leſs, as the time of its motion is ſhorter; or the reſiſtance will encreaſe uniformly, that is, it will encreaſe in the ſame proportion with the time of the bodys motion.

125. *The moments of falling bodies are as their quantity of matter and as their velocity conjoyntly.*

The foreign philoſophers are of a different opinion: they maintain that the moments of falling bodies are as their quantity of matter and as the ſquare of their velocity conjoyntly. But the experiments, which they make uſe of upon this occaſion, will be found upon examination to eſtabliſh our opinion, and conſequently to confute theirs.

The principal experiments made uſe of by them are of this ſort. Two balls of lead may have equal bulks, but if one is ſolid and the other is hollow they will have different weights. Now if two ſuch balls are let fall from heights, which are inverſely as their weights, they will each of them make a pit or hole of the ſame depth in any maſs of ſoft clay or tallow, on which they fall. From hence theſe philoſophers reaſon in this manner. The balls make equal holes or equal impreſſions in the clay; and conſequently they muſt have equal moments. But the quantities of matter contained in the balls are inverſely as the heights, from which they fall, by the ſuppoſition, and the heights, from which they fall, are as the ſquares of the velocities, which they acquire in falling, and with which they ſtrike upon the clay, by propoſition 106 of mechanics. Therefore the quantities of matter are inverſely as the ſquares of the velocities, when the moments are equal. But if the moments are equal, when the quantities of matter multiplyed into the ſquares of the velocities are equal, the moments muſt be as the quantities of matter and as the ſquares of the velocities conjoyntly.

Now here we muſt obſerve that the moments of theſe balls will not be equal, though they make equal pits in the clay, upon which they fall. For the clay is a tenacious fluid: and when equal pits are made in it, the balls muſt break through an equal number of laminas: conſequently the reſiſtance, which the clay makes, will be inverſely as the velocities of the two balls, by propoſition 122. But ſince each ball ſtops or loſes all its moment, when it has thus made a pit of a given depth, the reſiſtance, which it ſuffers in making ſuch a pit muſt be proportional to its moment. For as the moment of a body is greater or leſs, a proportionably greater

or leſs reſiſtance will be neceſſary to ſtop it. Therefore the moments are inverſely as the velocities. The conſequence is that the experiment will prove that theory of the moment to be the true one, which will make the moments of theſe balls to be inverſely as their velocities. But we have ſhewn, under propoſition 107 of mechanics, in one particular inſtance, that the moments of thoſe balls will be inverſely as their velocities, if their moments are as their quantities of matter and their velocities conjoyntly. Therefore this theory of the moment is the true one.

What was there ſhewn in one particular inſtance may be proved univerſally. If the moments of bodies are as their quantities of matter and their velocities conjoyntly, then, wherever the ſquares of the velocities are inverſe as the quantities of matter, the moments will be inverſely as the velocities. Call the quantity of matter M, and the ſquare of the velocity VV: then, by the ſuppoſition, M is inverſely as VV, or as the reciprocal of VV; that is, M is as $\frac{1}{VV}$. Now if the moment is as the quantity of matter multiplyed into the velocity it will be either as $V \times M$, or as $V \times \frac{1}{VV}$, becauſe M is as $\frac{1}{VV}$. But $V \times \frac{1}{VV} = \frac{V}{VV} = \frac{1}{V}$, which is the reciprocal of the velocity. Therefore when M is inverſely as VV, or when the quantity of matter is inverſely as the ſquare of the velocity, the moment will be as $\frac{1}{V}$ or inverſely as the velocity. But when heavy bodies are let fall from unequal heights, the ſquares of the velocities acquired in falling are as thoſe heights, by propoſition 106 of mechanics: and if the quantities of matter are inverſely as the heights, they will likewiſe be inverſely as the ſquares of the velocities acquired in falling: conſequently the moments of ſuch bodies, if our theory of the moments is true, will be inverſely as the velocities. But when bodies fall upon ſoft clay with moments, which are inverſely as their velocities, they will break through an equal number of laminas of the clay, or will make pits of equal depths, by propoſition 122. And when bodies are let fall from heights which are inverſely as their quantities of matter, they are found by experiment to make equal pits. Therefore their moments are inverſely as their velocities, and conſequently our theory of the moment is the true one.

CHAP.

CHAP. X.

Of rain and the barometer.

126. *Vapours are raiſed by the motion of the air, and are ſupported by its reſiſtance.*

WHEN a hard body, ſuch as wood or paper, is on fire, the particles of the body, that keep conſtantly aſcending from it, are called ſmoke; till thoſe particles are heated red-hot, and then they are called flame; for flame is nothing more than red-hot ſmoke. When a fluid is heated, the particles of it, that keep conſtantly aſcending from it, are called vapours: and if theſe particles are capable of being heated ſo as to be red-hot, which is the caſe in ſpirits of wine and all other inflammable ſpirits, then the aſcending vapour, whilſt it is heated to this degree, is likewiſe called flame.

It is moſt probable that ſmoke, from one ſort of bodies, and vapours, from the other ſort, are made to aſcend in the ſame manner and by the ſame cauſe. We will therefore conſider, firſt, by what cauſe ſmoke is made to aſcend from a hard body that is on fire. Now heat ſeparates the parts of the body; and when the parts are thus made very ſmall, they will be eaſily moved by a ſtream or current of air, as has been ſhewn under propoſition 119. The ſame heat, which ſeparates the particles of the body on fire, will rarefy the air that encompaſſes it. And as the air, when it is rarefyed, becomes ſpecifically lighter than the reſt of the air at the ſame diſtance from the top of the atmoſphere, this rarefyed air will aſcend, by propoſitions 33, 50, till it comes to a part of the atmoſphere of the ſame denſity or ſpecific gravity with itſelf, and there it will ſtop, by propoſition 34. Thus, when a body is on fire, there is a conſtant ſtream or current of air riſing from it: the ſmall particles of the body will move along with this current: and the ſmoke conſiſts of theſe particles riſing in this manner.

The particles of fluids are more eaſily ſeparated from one another than thoſe of hard bodies; and conſequently a leſs degree of heat will be ſufficient to make them ſmall enough to aſcend in ſuch a ſtream or current of air. This current does not conſiſt of ſuch air only as is on the outſide of the water, or other fluid, from whence the vapours are raiſed: for there is air in the interſtices between the particles of water, or of any other fluid, and, as the heat expands this air, it will become lighter than it was, and as it riſes out of the water, it will carry up the particles of the water

along with it. That there is air mingled with water, may be made evident to ſenſe: for if water is either heated till it boils, or is placed in an exhauſted receiver, the air contained in it will expand itſelf, and will riſe to the ſurface of the water in large bubbles: theſe bubbles will expand themſelves at the ſurface, till they break, and thus the included air eſcapes. When, by boiling water and by ſetting it to bubble in the receiver of an air-pump, moſt of the air is exhauſted out of it, fewer vapours, as we are told, will riſe from it afterwards, than would have riſen by the ſame degree of heat, if it had not been thus purged of the air that was contained in it. This ſhews that vapours are raiſed by the ſtream of air, which riſes out of a fluid, when it is heated; becauſe as there is leſs air in the fluid, and conſequently leſs muſt riſe out of it in any given degree of heat, the vapours are leſs plentifully raiſed from it.

Now here it is obvious to remark, that the heat, which is ſufficient to raiſe the ſteam juſt above the ſurface of the water, would ſcarce be ſufficient to carry the vapours to any great height in the atmoſphere: becauſe the rarefyed air, which raiſed them at firſt, would ſoon cool, and, before it had aſcended far, would recover its former denſity. But to this we may reply, that the air, which encompaſſes any particle of vapour, is not cooled ſo ſoon as is imagined; for the rays of light produce heat principally by being refracted and reflected; and conſequently, when any particle of water is floating in the air, by refracting and reflecting the rays it will warm the air that ſurrounds it: and as this warmed or rarefyed air, which is next the particle, will be lighter than the air that is farther from the particle, it will continue to aſcend and will ſtill carry the particle along with it. Indeed as the vapours aſcend, they will be attenuated, or will be ſeparated into ſmaller particles; for as the heat at firſt ſeparated the vapours from the maſs of fluid from whence thoſe vapours aroſe, ſo the ſame cauſe, ſtill acting as the vapours riſe, will ſeparate the parts of which they conſiſt. And the leſs the particles are the more readily they will move the ſame way that the air is moving, by propoſition 119. But now by removing one difficulty we have brought on another. For as before it might be aſked, why the vapours ſhould riſe to any conſiderable diſtance above the ſurface of the water, ſo from the reaſon here aſſigned it may be farther aſked, why they ſhould ever ſtop, till they have got up to the top of the atmoſphere. Since as long as they are expoſed to the rays of light they will be heated, and will rarefy the air, which is next to them; and this rarefyed air becoming ſpecifically lighter than the reſt of the air, which is at the ſame diſtance from the top of the atmoſphere, ſhould ſtill continue to riſe, by propoſition 33. But this difficulty may likewiſe be removed

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by conſidering that the particles would not move the ſame way that the air moves, if the air was void of all reſiſtance. As a body by moving in a fluid would not be retarded, ſo neither could the fluid communicate motion to the body by ſtriking upon it, if the fluid was void of reſiſtance. Now as the particles riſe they find the atmoſphere leſs denſe, by propoſition 50; and conſequently, when they have riſen far, if the air about them is farther rarefyed by the heat, which they produce, its denſity, being but ſmall in itſelf, may by propoſition 116, become too ſmall after this rarefaction to make any ſenſible reſiſtance to the particles, if they were moving in it, and therefore too ſmall to communicate any motion to particles at reſt in it. And, whenever this happens, the vapours will ſtop, and can aſcend no farther.

When the vapours are thus raiſed at firſt, it is ſcarce neceſſary to explane the reaſon why they are afterwards ſupported. For it is obvious, that if the reſiſtance of the atmoſphere, when the air was in motion, could raiſe the particles, the ſame reſiſtance, when they are raiſed, will keep the particles from falling back again.

127. *Dew conſiſts of ſuch vapours as are condenſed, either whilſt they are riſing or after they have riſen to very ſmall heights.*

When the earth has been heated in the day-time, it will retain the heat, or the heat will continue for ſome time, after the ſun is down. The air, which is a leſs compact or leſs denſe ſubſtance, will retain the heat for a leſs time. So that in the evening the ſurface of the earth will be warmer than the air about it. And conſequently the vapours will continue to riſe from the earth: but as they come immediately into a cool air, they will riſe no higher, but as the rarefyed air, in which they begin to riſe, is condenſed, the ſmall particles of vapour will be brought nearer together or will be condenſed, till many ſuch particles, when thus united, form larger drops or dew. Such drops may likewiſe be produced, not only from the vapours as they are riſing in the evening, but from ſuch likewiſe as have riſen in the day time. For if the latter part of the day has been very warm, and has raiſed great plenty of vapours, which cannot have aſcended very high before the cool of the evening comes on; the air, in which theſe vapours float, will be condenſed; and as there are many vapours floating in it, they muſt be ſo near to one another, that this condenſation will bring ſome of them cloſe together; and when theſe little drops of water touch one another, they will run together and make a larger drop. The reader need not be told that hoar-froſt is frozen dew.

It is evident that dew does not fall as rain does. For if it was a falling vapour as rain is, we ſhould always find the dew upon the outſide and ne-

never upon the inſide of a garden-bell-glaſs. The manner of its ſticking to the panes of glaſs in a window ſhews that it is a vapour, which is condenſed or formed into drops by cold. The air in a room is commonly warmer than the air abroad, either when there is a number of people in the room, or when the weather changes of a ſudden from warm to cool, as it commonly does when the evening comes on, for the air in the room has not a free communication with the air abroad, and therefore the air in the room will not change its temperature ſo ſoon as the air abroad. In theſe caſes, thoſe vapours in the room, which are near to the windows, will be condenſed by the cold of the outward air, and the dew will ſtick on the inſide of the glaſs. But if the weather changes ſuddenly from cold to warm, as in a ſudden thaw, the air abroad will be warm, before the air in a room, which does not communicate freely with it, can be warmed to the ſame degree: and then the vapours, that float in the outward air near the windows, will be condenſed by the cold of the room, and the dew will ſtick upon the outſide of the glaſs.

128. *When the weight of the air is diminiſhed, the vapours, that float in it, are united either into ſmall drops, which form clouds, or elſe into ſuch large drops, as the atmoſphere cannot ſupport, and theſe are rain.*

When the weight of the air is diminiſhed the denſity of it will likewiſe be diminiſhed, by propoſition 50, and conſequently the vapours that float in it will be leſs reſiſted, by propoſition 116. As the vapours are leſs reſiſted, they will begin to fall, and, as they ſtrike upon one another in falling, they will unite and form ſmall drops. Now though theſe drops will require a greater reſiſtance to ſupport them than the ſmall particles of the vapour required, before they were united, by propoſition 114; yet it is not neceſſary that they ſhould fall at once to the bottom of the atmoſphere: becauſe, as they fall, they come into denſer parts of the atmoſphere, by propoſition 50, and ſo may meet with a reſiſtance in theſe denſer parts, which will be ſufficient to ſupport them, by propoſition 116. When the particles of vapour are thus united into ſmall drops, and are afterwards ſupported at any height in the atmoſphere, they make clouds. Or, if this was to happen nearer to the earths ſurface, we ſhould call ſuch clouds near the earths ſurface by another name: for clouds are only miſts, that hang in the air at ſome diſtance from the earth, and miſts are only clouds near the earths ſurface.

But when the ſmall drops, of which a cloud conſiſts, are united into ſuch large drops, that no part of the atmoſphere is denſe enough to produce a reſiſtance ſufficient to ſupport them, they will then fall to the bottom

tom of the atmoſphere. Theſe large drops are rain: and from the manner in which rain is produced we may ſee the reaſon, why the drops ſhould be much larger in ſome ſhowers than they are in others. The ſmalleſt drops of rain are too big for the atmoſphere to ſupport. But if theſe drops are formed in the higher parts of the atmoſphere, many of them will be united, before they come to the ground, and the drops of rain will be very large. But if the vapours are not united into thoſe ſmaller drops, till they get very near the earths ſurface, that is, if the rain is formed at no great height, the drops will not be large.

The weight of the air at any place will be diminiſhed, if in the lower parts of the atmoſphere there are two contrary winds blowing from that place. And this is the reaſon of thoſe almoſt perpetual rains which fall at a place deſcribed in propoſition 102.

129. *Rain may fall, though the weight of the atmoſphere is not changed.*

The reader might obſerve in propoſition 128, that the decreaſe of the airs weight is only the remote cauſe of rain. For the decreaſe of weight only makes the airs denſity leſs: and as it is the decreaſe of denſity which makes the atmoſphere unfit to ſupport the vapours, this is to be looked upon as the immediate cauſe of rain. But yet, though we here affirm that rain may fall, when the weight of the atmoſphere is not changed, we do not mean that if heat diminiſhes the airs denſity, it will rain. A decreaſe of denſity, when it is owing to a decreaſe of weight, will produce rain: but a decreaſe of denſity, when it is owing to heat only, is ſeldom or never attended with rain. Becauſe the ſame heat, which rarefyes the air, will attenuate the vapours and make them more eaſily ſupported, by propoſitions 126, 114.

However, rain may be produced without any change in the weight of the atmoſphere, not indeed by a change in the airs denſity, but by a change in the ſize of the vapours. Thus if the wind in the lower parts of the atmoſphere blows one way, and in the upper parts blows equally the contrary way; little or no change will be produced in the weight of the atmoſphere. But at the limit or ſurface between thoſe two currents, the vapours in one current will ſtrike againſt thoſe of the other; and by this means theſe vapours will unite into drops too large to be ſupported in the atmoſphere, by propoſitions 114, 128. Or if the wind is checked by a high hill or mountain, the vapours in a current of air ſo checked will be formed into drops without any great alteration in the weight of the atmoſphere. It is evident that rain is frequently produced by this latter cauſe: becauſe more rain is obſerved to fall in mountainous than in plane countries.

Snow

Snow conſiſts of ſuch vapours as are frozen, whilſt they are in ſmall particles: for if theſe ſtick together, after they are frozen, the maſs that is formed out of them will be of a looſe texture as ſnow is. Hail, which is a more compact maſs of frozen water, conſiſts of ſuch vapours as are united into drops, and are frozen, whilſt they are falling.

We may now reſume the ſubject of the barometer, and may account for ſome of the principal variations in the height of the mercury.

130. *The mercury will ſtand at the ſame height either in an inclined barometer or in an erect one.*

If the mercury at any time ſtands at the height of 30 inches in the barometer D, Plat. XIX. fig. 8, then by inclining this barometer into the poſition E the perpendicular height of the mercury will not be altered; for it will ſtill ſtand at the height of 30 inches: ſo that if the level AB is 30 inches from the ſurface FG, the mercury will ſtand at this level, either in the erect tube D, or in the inclined one E. Now here it is evident, that if NL is the height of the mercury, when the tube is erect, and NM is the height of the mercury in the ſame tube or an equal one, when it is inclined, there muſt be more mercury in the inclined tube than there is in the erect one. For we may conſider NL as the ſide, and NM as the diagonal of a right-angled parallelogram. But the diagonal of a right-angled parallelogram is longer than the ſide. Therefore, though either L or M are at the ſame perpendicular diſtance from FG, yet NM will be longer than NL. Since then the column of mercury is longer in the inclined tube than in the erect one, there will be more mercury in the inclined tube than in the erect one. The queſtion therefore is, how the preſſure of the atmoſphere can ſupport a greater quantity of matter in one ſituation of the tube than in the other. We cannot ſay that though in the inclined tube there is a greater quantity of mercury than in the erect one, yet a part of this greater weight will be ſupported by the ſide of the tube as by an inclined plane: for this account of the matter has been ſhewn already to be a falſe one, under propoſition 21. The true anſwer is that the column of air, which ſupports the mercury in the inclined tube, is greater than the column, which ſupports it in the erect one. The height of the column of air is indeed the ſame in both caſes; for in either caſe it is equal to the height of the atmoſphere. But the baſe of the column of air, and conſequently its weight, is greater when the tube is inclined than when it is erect. For the baſe of the column of air, which ſupports the mercury in the tube, is equal to as much of the ſtagnant mercurys ſurface as the baſe of the tube covers, as was ſhewn under

der propoſition 46. Now if the diameters of the tubes D and E are equal; the baſe of the inclined tube E will cover a greater part of the ſurface FG than the erect tube D covers, or the baſe of the tube E will be greater than the baſe of the tube D. For the contents of the inclined tube are greater than thoſe of the erect one, as has been ſhewn already. But the column of mercury in each of the tubes are cylinders of the ſame height. Therefore their baſes are as their contents. Euc. b. XII. prop. 11. Since then the contents or the weights of mercury in each tube are as their baſes, when their heights are equal; and the baſe of the column of air, which ſupports the mercury, and conſequently the weight of this column, is proportional to the baſe of the tube; it follows that the weight of the air will always be proportional to the weight of the mercury, when it ſtands at a given height whether the tube is inclined or erect.

Though we have here ſhewn, that the contents of the inclined tube are greater than thoſe of the erect one, and, conſequently, that their baſes muſt be unequal, ſince their perpendicular heights are the ſame; yet it will not be improper to ſhew what we mean by the baſe of the inclined tube or rather of the inclined column of mercury, and why this ſhould be greater than the baſe of the erect one. Now by the baſe of the inclined column we do not mean the bottom of the inclined tube but the loweſt horizontal ſection of it. Thus if we conſider the ſurface FG as a plane paſſing through the two tubes D and E; this plane will cut the tube D perpendicularly and the tube E obliquely. But a perpendicular ſection of a cylinder is a circle, and an oblique ſection of it is an ellipſis. Therefore the baſe of the erect column is circular, and the baſe of the inclined tube is elliptical. Now, by the ſuppoſition, the two tubes have equal diameters: and conſequently the ſhorter axis of the elliptical baſe will be equal to the diameter of the circular one. So that the two baſes will be ſuch as are repreſented in Plat. XX. fig. 7, the circular one will be EHDI, and the elliptical one will be EADB; where the elliptical baſe is planely bigger than the circular one: becauſe though they are equal one way, ſince ED, which is the diameter of the circle, is the ſhorter axis of the ellipſis; yet the other way AB is longer than HI.

There is another ſort of inclined barometer, ſuch an one as ABR, Plat. XIX. fig. 12, which is erect for 28 inches from A to B, and then is inclined from B to C. The mercury will ſtand at the ſame height in this barometer, as if it had been a ſtrait one AS: for the column of air preſſing at the baſe A would be the ſame in either caſe; and though there is more mercury in the tube ABR, than there would be in the tube ABS; yet

ſuppoſing the mercury to ſtand at the ſame level DC in either caſe; the preſſure of the mercury downwards will in either caſe be the ſame. For, by propoſition 22, the preſſure of fluids is as their baſe and perpendicular height: and here the baſe A is the ſame and the perpendicular height is the ſame, whether the tube is erect all the way up as AS or is inclined at the top as ABR.

The advantage, which is propoſed by theſe diagonal barometers, as they are called, is to make the variation of the mercury greater and conſequently more apparent, upon a given change in the weather. Thus ſuppoſe AB or 28 inches to be the leaſt height of the mercury and AD or 31 inches to be the greateſt height of it, then the whole variation will be within the compaſs BD or 3 inches. But if the barometer, inſtead of being erect at the top, is inclined into the poſition BC, then, as the mercury ſtands at the ſame perpendicular height in this diagonal barometer as in an erect one, AB will be the leaſt height, and ABC will be the greateſt height; ſince D and C are on the ſame level or at the ſame perpendicular diſtance from A. Now though BD one ſide of the parallelogram is but 3 inches long, yet BC may be 30 inches long, or more, and conſequently ſince AB is the leaſt height and ABC is the greateſt height; the variation of the mercury will be much greater than in an erect barometer; in particular, if BC is 30 inches long, the variation will be 30 inches inſtead of 3, or will be 10 times greater in the diagonal barometer than it would have been in an erect one.

131. *The barometer ſtands at the ſame perpendicular height, whether the tube is large or ſmall.*

If the mercury ſtands at the ſame height either in the large tube C or in the ſmall tube D, there muſt be more mercury in the large one than in the ſmall one. But ſince the heights are equal the quantities of mercury contained in theſe tubes will be as their baſes. Euc. b. XII. prop. 11. Now ſince the columns of air, by which the mercury is ſupported in theſe tubes, are as the reſpective baſes of the tubes, by propoſition 46, the columns of air will be proportional to the weights in each tube, when the perpendicular heights are equal.

But though the heights of the mercury would be the ſame in ſmall tubes as in large ones, if, as we muſt ſuppoſe in the propoſition, the mercury moved equally free in both; yet in fact upon any change of weather, the variation will be greater in a large tube than in a ſmall one. Becauſe in a large tube the weight of mercury is ſo great that the motion of of it will not be hindered by any attraction or repulſion of the glaſs upon it;

PLATE. XIX.

Fig. 1.

Fig. 2.

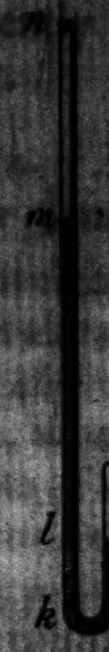


Fig. 3.

Fig. 4.

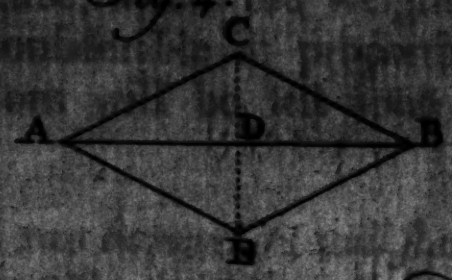


Fig. 5.

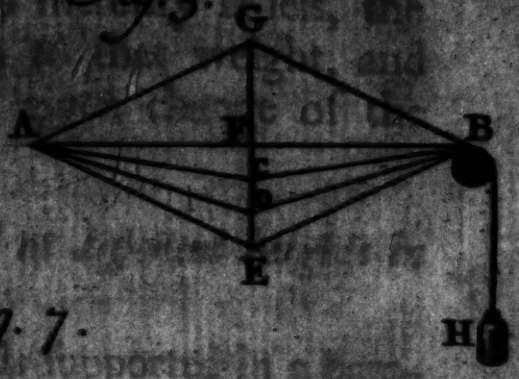


Fig. 6.

Fig. 7.

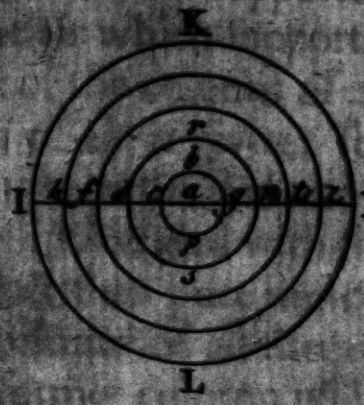


Fig. 8.

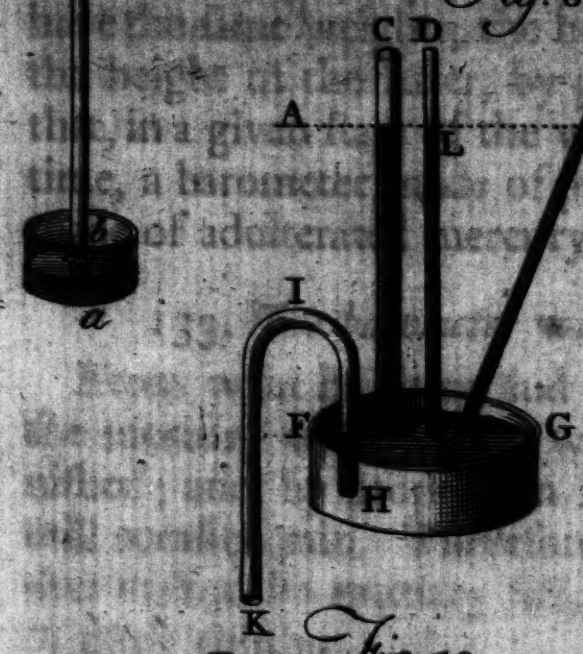


Fig. 9.

Fig. 12.

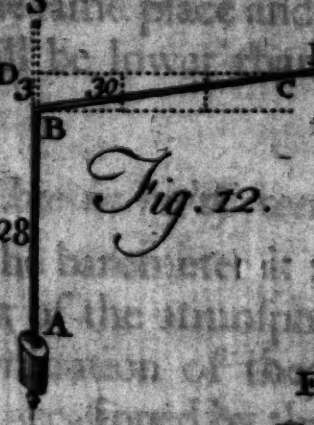


Fig. 10.

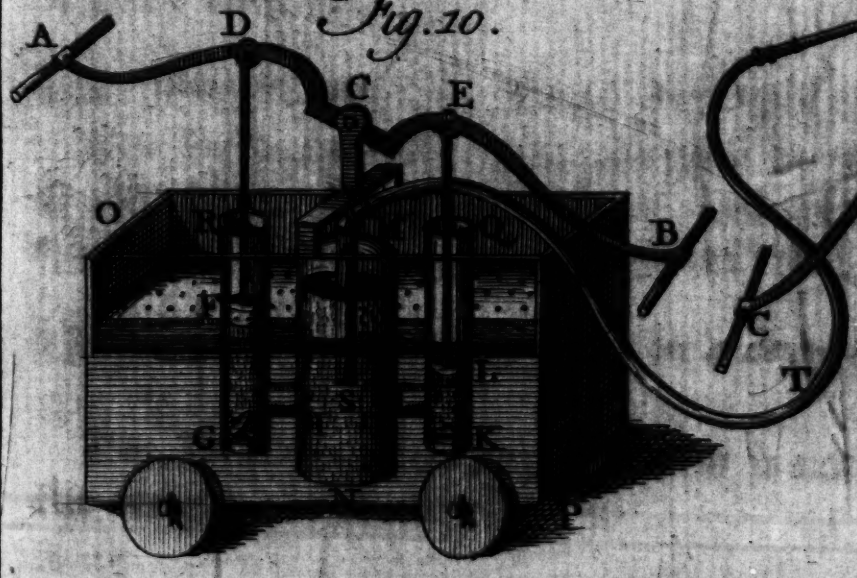
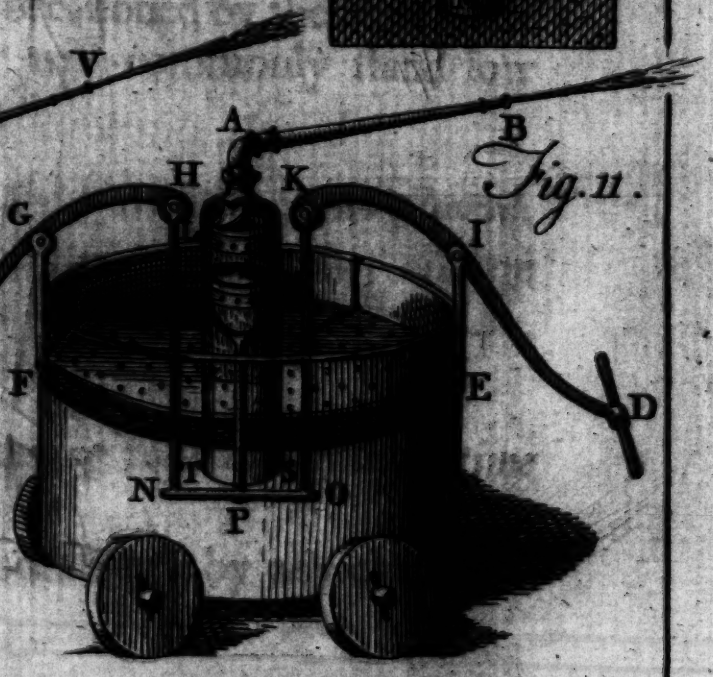
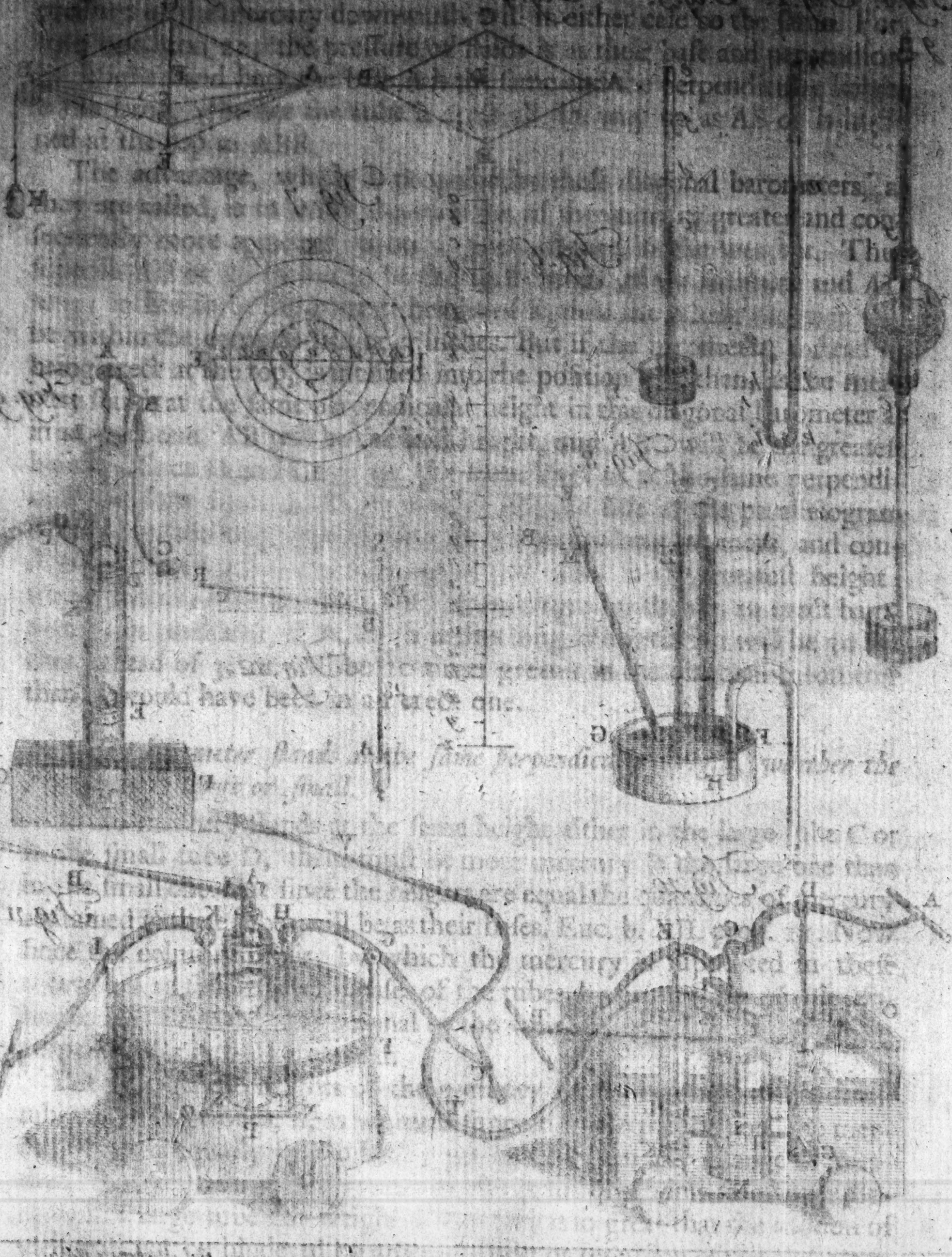


Fig. 11.



Gul. Stephens calavit



it; whereas in a ſmall tube, where the weight of mercury is leſs, the action of the glaſs is conſiderable in proportion to that weight, and conſequently the variations will be leſs upon a given change of the weather.

132. *It may happen that two barometers will ſtand at different heights in the ſame place and at the ſame time.*

The weight of the column of mercury, which is ſupported in a barometer, is always equal to the weight of the air which ſupports it, by propoſition 46: and conſequently the weight of the column of mercury will be given, when the weight of the column of air is given: ſo that in the ſame ſtate of the weather, the mercury ought to ſtand at the ſame height in all barometers. But mercury is frequently adulterated with tin, or lead, or biſmuth, and this makes it ſpecifically lighter than pure mercury: ſo that when a column of pure mercury and a column of adulterated mercury have the ſame weights, the height of the former column will be leſs than the height of the latter, by propoſition 5. From hence then it follows, that, in a given ſtate of the weather, or in the ſame place and at the ſame time, a barometer made of pure mercury will be lower than one that is made of adulterated mercury.

133. *The barometer will commonly be low in rainy weather.*

From what has been ſaid already about the barometer it appears that the mercury will be low, when the weight of the atmoſphere is diminiſhed; and, by propoſition 128, ſuch a diminution of the atmoſphere will occaſion rain. Therefore, ſince rain is occaſioned by the ſame cauſe that makes the mercury fall, the barometer will commonly ſtand low in rainy weather.

134. *Showers may fall, though the height of the barometer is not altered.*

For, by propoſition 129, rain may be produced wi hout any change in the weight of the atmoſphere, and conſequently, by propoſition 46, without any change in the height of the barometer.

135. *The barometer is not lower in a dewy evening than at any other time of the day.*

For, by propoſition 127, the dew is not occaſioned by any change in the weight of the atmoſphere.

136. *The barometer is the loweſt of all in violent ſtorms of wind.*

When the air moves horizontally with a great velocity, as it does in violent ſtorms of wind, by propoſition 95, its weight or rather its preſſure downwards occaſioned by its weight will be diminiſhed. For as any heavy body, by propoſition 152 of mechanics, may have ſuch a velocity given it, when it is thrown horizontally, as may either carry it quite off from the earths center, or ſuch a velocity as will make it move round the earth in a circle without either departing from the center or approaching to it; ſo every degree of velocity given to the air will make it tend or preſs leſs towards the center: and for this reaſon, as the mercury in the baſon will be leſs preſſed, the height of it in the tube will be leſs in ſtorms, than it is any other time.

137. *When a ſtorm of wind is over the mercury will riſe very faſt.*

Becauſe as the horizontal velocity of the air ceaſes, the preſſure downwards will be ſuddenly reſtored, by propoſition 136, and conſequently the mercury in the barometer will keep riſing as this preſſure is reſtored.

CHAP. XI.

Of the thermometer.

138. *The variations of different thermometers are ſeldom equal, upon equal variations of heat or cold.*

A thermometer is a well-known inſtrument for eſtimating different degrees of heat or cold. It conſiſts of a tube or ſtem with a hollow ball at one end of it. The cavity of the ball and part of the tube is filled with ſpirits of wine, or with linſeed oil, or with mercury. The upper end of the tube is commonly ſealed hermetically. But in ſealing this end the liquor in the thermometer is rarefyed by heating it, till it almoſt fills the tube; ſo that when it is ſealed, and the liquor contracts again as it cools, there will be a vacuum left in the upper part of the tube. Any of theſe fluids will rarefy by heat, and will contract again, when they cool: and conſequently in warm weather, the ſpirits, or the oil, or the mercury, whichever the thermometer is made of, will ſtand higher than in cold weather.

Thus far thermometers may be ſaid to vary alike: they will either riſe or ſink from the ſame cauſes. But then upon an equal encreaſe of heat they ſeldom vary equally; though they are made of the ſame liquor.

One

One thermometer made with ſpirits of wine may vary upon any equal encreaſe of heat much more than another that is made with the ſame ſort of ſpirits; ſo that if one riſes an inch, another may riſe but $\frac{1}{2}$ or $\frac{1}{4}$ inch.

The variation of a thermometer is directly as the capacity of the ball and inverſely as the baſe of the ſtem. Firſt, if the baſe of the ſtem or cylindrical tube is given, the variation, when the ſpirits are equally warmed, will be directly as the capacity of the ball. For when the ſpirits are equally warmed and conſequently are equally rarefyed in the balls of two different thermometers, whatever proportion the bulk of the ſpirits in one ball bears to the bulk of the ſpirits in the other ball, before they were rarefyed, the ſame proportion theſe bulks will ſtill bear to each other after they are rarefyed. Thus if one ball is double the other and conſequently the bulk of ſpirits in one is double the bulk of ſpirits in the other, before they are warmed; then upon being warmed equally their denſities will diminiſh equally. But if their denſities diminiſh equally, their bulks will ſtill have the ſame proportion to each other; or the bulk of ſpirits in one thermometer will ſtill be double the bulk in the other. But if the bulks continue in the ſame proportion to each other, after they are ſwelled as they were before; the ſpirits muſt ſwell in proportion to their reſpective bulks, or the ſpirits in one muſt ſwell twice as much as in the other. But if the ſpirits ſwell in this proportion and by ſwelling riſe into equal tubes in each, they muſt riſe twice as high in the tube of one of theſe thermometers as they do in the tube of the other. And ſo in all other inſtances, the ſpirits upon being equally warmed will ſwell in proportion to their bulk, that is, in proportion to the capacity of the ball that contains them. But the heights, to which they riſe in equal tubes, will be as the encreaſe of their bulk. Therefore the heights to which they riſe or the variations in equal degrees of heat will be as the capacity of the ball, when the tubes are equal. We have here ſuppoſed that the ſpirits in the balls of the thermometers are equally heated quite through. In ſudden changes of heat and cold, it will be otherwiſe: for the ſpirits in a ſmall ball will be ſooner heated quite through than in a large one. And conſequently if the heat does not laſt long enough to warm the ſpirits in a large ball as much as they are warmed in a ſmall one; the ſpirits will not be equally rarefyed in both, and will not ſwell in proportion to their reſpective bulks: but thoſe in the ſmall ball will ſwell more in proportion than thoſe in the large one. Secondly, if the balls are equal, the variations will be inverſely as the baſes of the ſtems. For if the balls are equal, then, upon being equally heated, the ſpirits contained in them will ſwell

equally;

equally; and conſequently equal quantities will riſe into the ſtems. Now the ſpirits, which riſe into a cylindrical ſtem, are a cylindrical column. But the heights of equal cylinders are inverſely as their baſes. Euc. b. XII. prop. 15. Therefore when the balls are equal and equal cylinders of ſpirits riſe into the ſtems, the heights, to which they riſe, or the variations, will be inverſely as the baſes of the ſtems.

If there are two thermometers and we call the capacities of the balls C and c, and the baſes of the ſtems B and b; then from what has been ſaid it appears, that the variations will be as C to c directly, and as B to b inverſely, or as $\frac{C}{B}$ to $\frac{c}{b}$; ſince theſe two fractions are to each other as their numerators C and c directly, and as their denominators B and b inverſely. Conſequently the variations will not be equal in thoſe thermometers, unleſs $\frac{C}{B}$ is equal to $\frac{c}{b}$. But $\frac{C}{B}$ will not be equal to $\frac{c}{b}$ unleſs C is to c as B to b. Therefore the variations in two thermometers will not be equal, unleſs the capacities of their balls are to each other as the baſes of their reſpective ſtems.

139. *An univerſal ſcale may be made, by which the variations of different thermometers may be compared with one another.*

Let the ball of a thermometer be put into water, when it is beginning to freeze, or, which is the ſame as to heat or cold, into ſnow, when it is beginning to melt; and let the place where the fluid in the thermometer ſtands be marked. The place where the fluid ſtands in ſuch a tryal is the freezing point. Let the ball of the ſame thermometer be put into water juſt hot enough to let wax, that ſwims upon it, begin to coagulate. This again is another determinate degree of heat, and is to be marked upon the thermometer. Divide the diſtance between theſe two points into 110 equal parts; and each of theſe parts we call a degree. Now a thermometer often ſinks lower than the freezing point; becauſe the cold is frequently more intenſe than what is juſt ſufficient to make water freeze: for this reaſon the ſcale muſt not begin from the freezing point. This point therefore ſhould not be marked 0, nor ſhould the point where melted wax begins to coagulate be marked 110. In this ſcale, which from the inventer is called Fahrenheits ſcale, the freezing point is marked 32, and then the point, where melted wax begins to coagulate, being 110 degrees above it, muſt be marked 142. When the length of a degree is thus found in one part of the ſcale, 32 degrees of the ſame length are ſet off below the freezing point, and as many ſuch degrees as we pleaſe are ſet off above the point, where melted wax begins to coa-

gu-

gulate. If the thermometer is made with ſpirits of wine, only 33 degrees need be ſet off or marked above 142: and then the ſcale will begin from 0, 32 degrees will be the freezing point, 142 will be the point where melted wax begins to coagulate, and 142+33=175 degrees will be the higheſt point marked in the ſcale. The reaſon why no higher degree need be marked, in a ſcale applyed to a thermometer made with ſpirits, is, that at this degree of heat the ſpirits will boil, and conſequently the thermometer would burſt. But if the thermometer is made with mercury, the ſcale ſhould contain at leaſt 212 degrees from the bottom to the top, or 32 degrees below the freezing point, and 180 above it. The heat of boiling water, at the middle height of the mercury in the barometer, or in the middle weight of the atmoſphere, will raiſe the mercury in the thermometer to 212 degrees, or 180 degrees above the freezing point. A thermometer made with mercury will not burſt in ſuch a degree of heat as this: for mercury requires a greater degree to make it boil.

In thermometers with ſuch a ſcale, or, as they are called, in Fahrenheits thermometers, the greateſt degree of heat in the external parts of the human body is commonly about 96. Boerhave imagined that air, if its heat exceeded 80 or 90 degrees at moſt, would be deſtructive to the life of animals. But in this he was miſtaken. For in the year 1732 the thermometer in Penſylvania was at the height of 96 or 97; and in the year 1734 the height of it at Petersburgh was 98 degrees. The thermometer in our own climate is ſcarce ever higher than 78 degrees and ſeldom lower than 18; ſo that we may reckon 48 degrees to be the middle temperature of our air.

The variations of different thermometers, though they are not equal, may be compared with one another by Fahrenheits ſcale. For each degree upon different thermometers is proportional to their reſpective variations, and conſequently, though in equal heats one may vary more than another, yet each will vary an equal number of degrees. Thus, if, upon any given encreaſe of heat, one thermometer will vary twice as much as another, then the diſtance between the freezing point, and the point where melted wax begins to coagulate will be twice as great, or 110 degrees will be twice as long in one as in the other. Therefore each degree will be twice as long in the former thermometer as in the latter. But by the ſuppoſition, one of theſe thermometers in a given degree of heat will vary twice as much as the other does: and conſequently whatever heat raiſes the former one degree will likewiſe raiſe the latter one degree.

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We obſerved above, that the heat of boiling water is 212 degrees, when the barometer ſtands at the middle height. Becauſe, though in the middle weight of the atmoſphere water will not admit of a greater degree of heat than this boiling heat, yet if the atmoſphere is lighter a leſs degree of heat will make water boil, or if the weight of the atmoſphere is encreaſed, water will require a greater degree of heat to make it boil. Hot water, that is not hot enough to boil in the open air, where it is preſſed by the weight of the atmoſphere, will boil in the receiver of an air-pump, as the air is exhauſted, and the preſſure upon the waters ſurface is diminiſhed. For if the boiling heat is ſuch a degree of heat as will make the air, that is contained in the water, expand itſelf enough to put the whole body of the water in motion and to riſe out of it in bubbles; it is plane, by propoſition 48, that the leſs the water is preſſed, the leſs degree of heat will be ſufficient for this purpoſe. On the contrary as the compreſſing force is greater, a greater degree of heat will be requiſite to make water boil. And for this reaſon when water is ſcrewed down cloſe in a ſtrong iron pot, ſuch as Papins digeſter is, it is capable of a much greater degree of heat than in the open air. For the water is there kept together by the ſides of the pot, or the included air, as it endeavours to expand itſelf, is compreſſed with a force much greater than the weight of the atmoſphere. Upon this account it is, that in ſuch a digeſter water may not only be made hot enough to diſſolve the hardeſt animal ſubſtances ſuch as horn or bones, but even, as ſome affirm, to melt metals.

140. *If the ball of a thermometer is dipped into hot water, the fluid in the thermometer will ſink a little, before it begins to riſe.*

Not only fluids, ſuch as ſpirits, oil, or mercury, but likewiſe glaſs, or iron, or almoſt any hard bodys will expand, when they are heated and will contract again, when they grow cold. Now when the ball of a thermometer is dipped into hot water, the heat will be communicated to the glaſs, of which the ball is made, before it is communicated to the fluid contained in the ball. By this means the ball will be expanded and the capacity of it will be encreaſed: ſo that ſome of the fluid will ſink out of the ſtem into it. But when the ball has been long enough in the water for the fluid within it to be heated, this fluid will be expanded, and then it will riſe into the ſtem, and will continue to riſe as the heat encreaſes.

CHAP.

CHAP. XII.

Of the motion of fluids.

141. *The force, with which a fluid is driven out of a given hole in the bottom or ſide of a veſſel, is proportional to the perpendicular depth of the hole below the ſurface of the fluid.*

THE preſſure of water againſt any given ſurface, as for inſtance againſt a ſquare inch, in the ſide or in the bottom of a veſſel is proportional to the perpendicular height of the water above that part, by propoſition 24. Now if this given ſurface was taken out of the ſide or the bottom of the veſſel, a hole would be made there of the ſame dimenſions with the piece ſo taken out. Conſequently the preſſure of the water againſt this hole will be the ſame, that it was before againſt the ſurface, which was taken away to make the hole. But the preſſure againſt the hole is the force that drives the water out at it. Therefore this force is proportional to the perpendicular height of the water above the hole, or, which is the ſame thing, to the perpendicular depth of the hole below the ſurface.

142. *The moment, with which any fluid runs out of a given hole in the bottom or ſide of a veſſel, is proportional to the perpendicular depth of the hole below the ſurface of the fluid.*

The force, with which water is driven out of ſuch a hole, is as the perpendicular depth of the hole. But ſince this force is what communicates motion to the water, and ſince, by propoſition 12 of mechanics, the moment is always as the moving force, it follows, that the moment, with which the water runs out, will be as the perpendicular depth of the hole.

Thus if one hole of given dimenſions, ſuppoſe of an inch ſquare, is made in the ſide of a tub at the depth of 9 inches, and another equal hole is made at the depth of 16 inches, the moments, with which the water would begin to run out of the tub at theſe holes, would be to each other as 9 to 16.

143. *The moment, with which any fluid runs out of a given hole in the bottom or ſide of a veſſel, is as the ſquare of the fluids velocity, or as the ſquare of the quantity of matter.*

The moment, with which water runs out of any hole in the bottom or ſide of the containing veſſel, is as the quantity of water that runs out

in a given time, and as the velocity, with which that water runs, conjoyntly, by propoſition 13 of mechanics. So that, if we call the quantity of water q and the velocity v, the moment will be as q×v. Now this product ariſing from the quantity of matter multiplyed into the velocity is proportional either to the velocity multiplyed into itſelf, or to the quantity of matter multiplyed into itſelf, that is, q×v is either as v×v, or as q×q.

Where the area of the hole is given, the quantity of matter or quantity of water diſcharged at that hole in a given time is as the velocity, with which it runs. If there are two equal holes at different depths in the ſide of the ſame veſſel; it is evident, becauſe the holes are equal, that one of them cannot diſcharge more water than the other diſcharges in the ſame time, unleſs the water runs faſter out of the former than out of the latter. Thus if one hole diſcharges 2 pints of water whilſt another equal hole diſcharges only 1 pint, it muſt be becauſe the velocity at the former hole is double the velocity at the latter: or if the quantities diſcharged are as 3 to 2 the velocities muſt be in the ſame proportion: or whatever elſe is the proportion of the quantities diſcharged they will always be directly as the velocities, or q is always directly as v, when the dimenſions of the hole are given. Now if q is proportional to v, q×v will be as v×v or as q×q. But q×v is the moment, v×v is the ſquare of the velocity, and q×q is the ſquare of the quantity of matter. Therefore the moment is either as the ſquare of the velocity, or as the ſquare of the quantity of matter.

144. *The velocity, with which any fluid runs out of a hole in the bottom or ſide of a veſſel, is as the ſquare root of the perpendicular depth of the hole from the ſurface of the fluid.*

If equal holes are made in the ſide of a tub, one 16 inches below the ſurface and the other 9 inches below the ſurface, the velocities, with which water would run out at theſe two holes, would be as the ſquare root of 16 to the ſquare root of 9, or as 4 to 3. The moments are as the perpendicular depths, by propoſition 142. And the moments are likewiſe as the ſquares of the velocities, by propoſition 143. But ſince the moments are proportional either to the ſquares of the velocities or to the perpendicular depths of the holes below the ſurface, it follows, that the ſquares of the velocities muſt be proportional to the perpendicular depths; and conſequently the velocities muſt be proportional to the ſquare roots of the perpendicular depths.

If we call the perpendicular depth of the hole d, the quantity of matter q, and the velocity v, then the moment will be q×v, by propoſition

13 of mechanics. But $v \times v$ is as $q \times v$, by propoſition 143. And d is as $q \times v$ by propoſition 142. Therefore $v \times v$ is as d. And conſequently $\sqrt{v \times v}$ is as $\sqrt{d}$. But $\sqrt{v \times v} = v$. Therefore v is as $\sqrt{d}$, or the velocity is as the ſquare root of the perpendicular depth.

145. *The quantities of fluid diſcharged in a given time through equal holes, in the bottom or ſide of a veſſel, are as the ſquare roots of the perpendicular depths of the holes from the ſurface of the fluid.*

This might be proved in the ſame manner with the foregoing propoſition: but we may ſave the reader ſome trouble by deducing it from that propoſition. The quantity of water diſcharged in a given time through a given hole is as the velocity, with which the water runs, as has been ſhewn under propoſition 143. And the velocity is as the ſquare root of the perpendicular depth, by propoſition 144. Therefore the quantity of water diſcharged in a given time through a given hole is as the ſquare root of the perpendicular depth.

By what has been ſaid under propoſition 143, if the quantity of water diſcharged in a given time is called q, the velocity v, and the depth d, q is as v, and by propoſition 144, v is as $\sqrt{d}$. Therefore q is as $\sqrt{d}$.

146. *The forces, with which any fluid is driven out of unequal holes at a given depth in the bottom or ſide of a veſſel, are as the areas of the holes.*

The preſſure of water againſt unequal parts of the bottom or ſide of a veſſel, at a given depth, is proportional to the areas of theſe parts, by propoſition 24. Now in making a hole in the bottom or ſide of a veſſel, a part of the bottom or ſide is taken out, and the area of the hole is equal to the area of the part ſo taken out. And the ſame force, which preſſed that part, before the hole was made, will drive the water out at it afterwards. Conſequently, when unequal holes are made by taking out unequal parts, the force, with which the water is driven out, will be proportional to the areas of the holes.

147. *The moments, with which any fluid is driven out through unequal holes at a given depth in the bottom or ſide of a veſſel, are as the areas of the holes.*

The moment of the fluid will be as the moving force, by propoſition 12 of mechanics. But the moving force is as the area of the hole, by propoſition 146. Therefore the moment will be as the area of the hole.

Thus if two holes were made in the ſide of a veſſel at the ſame depth, and the area of one was 5 ſquare inches, and that of the other 7 ſquare inches, the moments, with which water would run out at theſe holes, would be as their areas or as 5 to 7.

148. *The quantities of any fluid, that paſs in a given time through unequal holes at the ſame depth in the bottom or ſide of a veſſel, are as the areas of the holes.*

The holes, however unequal, are at the ſame depth by the ſuppoſition; and conſequently, by propoſition 144, water would run out of them with the ſame velocity. But where the velocity is given the quantity of matter is proportional to the moment, by propoſition 13 of mechanics: and the moments are as the areas of the holes, by propoſition 147. Therefore the quantities of matter running out, in any given time, will be as the areas of the holes.

149. *The quantities of any fluid, that paſs in a given time through unequal holes at unequal depths in the bottom or ſide of a veſſel, are as the ſquare roots of the perpendicular depths and as the areas of the holes conjoyntly.*

The quantities of water paſſing through holes at unequal depths are as the ſquare roots of thoſe depths, by propoſition 145. And the quantities paſſing in a given time through unequal holes are as the areas of the holes, by propoſition 148. Therefore when the depths and the areas are unequal the quantities of water diſcharged in a given time will be both as the ſquare roots of the depths directly, and as the areas of the holes directly, or as the ſquare roots of the depths multiplyed into the areas of the holes.

If we call the areas of the holes h, the depths d, their ſquare roots $\sqrt{d}$, and the quantities of water diſcharged in a given time q; then, by propoſition 148, q is as h directly; and by propoſition 145, q is as $\sqrt{d}$ directly. Therefore q is as $h\sqrt{d}$ or as a product ariſing from h multiplyed into $\sqrt{d}$; for ſuch a product is directly as the multiplicator h and directly as $\sqrt{d}$ the multiplicand. So that univerſally, if unequal holes are made at unequal depths, the quantities of water diſcharged at thoſe holes in the ſame time will be as the areas of the holes multiplyed into the ſquare roots of their perpendicular depths below the ſurface, or as the areas of the holes and ſquare roots of the perpendicular depths conjoyntly.

Thus if two holes are made in the ſide of a veſſel, one at the depth of 9 inches and the other at the depth of 16 inches; the ſquare roots of

of thoſe depths are as 3 to 4. Suppoſe the area of the upper hole to be 5 ſquare inches and the area of the lower hole 7 ſquare inches; then the quantities of water diſcharged in a given time through each of theſe holes will be as their areas multiplyed into the ſquare roots of their reſpective depths, or as $3\times5=15$ to $4\times7=28$.

150. *The quantities of any fluid, that paſs in a given time through unequal holes at unequal depths in the bottom or ſide of a veſſel, are as the velocities, with which the fluid runs, and as the areas of the holes conjoyntly.*

The quantities of water diſcharged, in a given time will be as the velocities, by propoſition 144, becauſe the holes are at unequal depths. And the quantities of water diſcharged, in a given time, will likewiſe be as the areas of the holes, by propoſition 148, becauſe thoſe areas are unequal. Therefore upon both accounts together the quantities diſcharged, in a given time, will be as the velocities directly and as the areas of the holes directly, or as the velocities multiplyed into the areas of the holes, or as the velocities and areas of the holes conjoyntly.

If we call the quantities of matter q, the velocities v, and the areas of the holes h; then q is as v directly, by propoſition 144, and q is as h directly, by propoſition 148. Therefore q is as vh or as a product ariſing from v multiplyed into h: for ſuch a product is directly as v the multiplicator and directly as h the multiplicand.

Or otherwiſe. The quantities of water diſcharged in a given time are as the ſquare roots of the perpendicular depths and as the areas of the holes conjoyntly, by propoſition 149. But the velocities are as the ſquare roots of the perpendicular depths, by propoſition 144. Therefore the quantities diſcharged in a given time are as the velocities and as the areas of the hole conjoyntly.

If we call the depths d and their ſquare roots $\sqrt{d}$; q is as $h\sqrt{d}$, by propoſition 149. But v is as $\sqrt{d}$, by propoſition 144. Therefore $h\sqrt{d}$ is as vh, and conſequently q is as vh.

151. *In a given time, the moments of any fluid running through unequal holes at unequal depths are as the areas of the holes and as the ſquares of the velocities conjoyntly.*

The moments are as the areas of the holes, becauſe the holes are unequal, by propoſition 147; and the moments are as the ſquares of the velocities, becauſe the depths are unequal, by propoſition 143. Therefore the moments upon both accounts together are as the areas of the holes and as the ſquares of the velocities conjoyntly.

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If we call the moments m, the quantities of matter q, the velocities v, the ſquares of the velocities vv, and the areas of the holes h; then m will be as h directly by propoſition 147, and as vv directly by propoſition 143, and conſequently upon both accounts m will be as h×vv=hvv. Or otherwiſe. The moments are as the quantities of matter and the velocities conjoyntly, or m will be as qv, by propoſition 13 of mechanics. But q is as hv, by propoſition 150. Therefore m is either as q×v=qv, or as hv×v=hvv.

152. *When the moments of water running out of holes in the ſide of a veſſel are as the ſquares of the velocities, the quantities of matter are not equal, but are as the velocities.*

This appears from propoſition 143. But as an argument is ſometimes drawn from this motion of fluids to prove that the moments of bodies are as the ſquares of the velocities, when the quantities of matter are equal, it may not be improper to ſay ſomething more upon this ſubject. Now we have ſhewn in propoſition 143, that when equal holes are made at unequal depths in the ſide of a veſſel, the moments, with which water would run out at thoſe holes, would be as the ſquares of the velocities. But then we muſt deſire the reader to obſerve that the quantities of matter are unequal. For the very reaſon why the moments are as the ſquares of the velocities is, becauſe the quantities of water running out in a given time are as the velocities: and conſequently the moments are either as the quantities of matter multiplyed into the velocities, or as the velocities multiplyed into themſelves.

Thus if equal holes are made one at the depth of 9 inches, and the other at the depth of 16 inches, the velocities are as 3 to 4, and the moments as the ſquares of theſe velocities or as 9 to 16. For ſince the velocities are as 3 to 4, the quantities of water paſſing through theſe holes in a given time will be as 3 to 4; and conſequently the velocities multiplyed into the reſpective quantities of matter will be as 3×3=9 to 4×4=16, or as the ſquares of the velocities. Therefore the moments are as the ſquares of the velocities, for no other reaſon but becauſe they are compounded of the quantities of matter and the velocities, and in this caſe the quantities of matter are as the velocities, ſo that the product will be the ſame whether we multiply the quantities of matter by the velocities, or multiply the velocities reſpectively into themſelves.

153. *When water runs out of unequal holes at unequal depths in the ſide of a veſſel, the moments are not as the quantities of matter and ſquares of the velocities conjoyntly.*

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The quantities of water, that run in a given time out of unequal holes at unequal depths in the ſide of a veſſel, are as the areas of the holes and the velocities conjoyntly, by propoſition 150; or, calling the quantities of water q, the areas of the holes h, and the velocities v, q is as hv. But the moments are as hv×v or as q×v, that is, as the quantities of matter multiplyed into the velocities. Indeed the moments are as hvv or as the areas of the holes multiplyed into the ſquares of the velocities, by propoſition 151. But then the quantities of matter are not as the areas of the holes, by propoſition 150. Therefore the moments are not as the quantities of matter multiplyed into the ſquares of the velocities.

154. *A fluid runs out of a hole in the bottom or ſide of a veſſel with the ſame velocity that a heavy body would acquire in falling freely through a ſpace equal to the perpendicular diſtance between the ſurface of the fluid and the hole.*

If the veſſel ABCD, Plat. XX. fig. 2, is filled with water to the level HI and a hole is made in the middle of the bottom at K, the velocity, with which the ſtream L runs out at that hole, will be the ſame that a heavy body would acquire in falling freely through the ſpace HC, which is equal to the perpendicular diſtance between the ſurface of the water HI and the hole K. Suppoſe that FGHI was a cylinder of ice, which exactly fitted the veſſel; ſuppoſe farther that there was no water in the lower part of the veſſel between HI and CD, but that the bottom of this ice was to melt, and that a ſtream of water HIMNK was to run down from it; and, whilſt the ice thus melts, imagine the cylinder to keep ſinking by degrees, ſo that though it decays by the water running off from it, yet the bottom of it may always be at HI. Now every drop of water is a heavy body, by propoſition 8, and conſequently every drop will acquire the ſame velocity in falling from HI to K, that any other heavy body would acquire in falling through an equal ſpace HC.

The ſtream would run in the ſhape here repreſented: becauſe the particles of water cohere enough to keep them together ſo as to form one continued ſtream. If this continued ſtream runs uniformly the ſame quantity of water that runs off from HI in any given time muſt paſs through K in an equal time. But if the quantity that paſſes by K in a given time is equal to the quantity that runs off from HI in the ſame time, then, ſince the water, by what has been ſhewn already, acquires velocity in falling, it will move faſter at K than it does at HI; and therefore a ſection of the ſtream at K will be leſs than a ſection at HI; for if the ſtream was as broad at K as it is at HI, ſince it runs faſter at K than it does

does at HI, more water muſt in a given time paſs through the hole than runs off in an equal time from the bottom of the ice.

Let us now ſuppoſe that the lower part of the veſſel from HI to CD is filled with ice, except that part where the ſtream HIMNK runs, ſo that the ſtream might run through this cavity in the lower ice as through a funnel. The water in theſe circumſtances, as it melts off from the cylinder FGHI, would continue to run in the ſame manner as when the lower part of the veſſel was empty. For the channel in the lower ice is, by the ſuppoſition, juſt of the natural ſhape of the ſtream; and conſequently this lower ice will not at all hinder the motion of the ſtream. Therefore in theſe circumſtances the velocity, with which the water would run out at K, would be the ſame as before, or the ſame that a heavy body would acquire in falling freely through the ſpace HC.

Let us next ſuppoſe the lower ice to be melted, ſo that the lower part of the veſſel ſhould be full of water from HI to CD. This would make no alteration in the velocity of the ſtream at L. When the lower ice is changed into water, this water preſſing from the ſides cannot make the ſtream run either faſter or ſlower. For though the lower water preſſing from the ſides and endeavouring to run through the hole K might puſh the ſtream forwards, yet this ſide-water cannot run out but it muſt hinder the ſtream from running at the ſame time; and conſequently juſt as much as the ſide-water would upon one account make the ſtream run faſter, juſt ſo much upon the other account it would make it run ſlower. Therefore when the lower part of the veſſel is filled with water, the velocity, with which the ſtream runs out at K, is the ſame as before, or the ſame that a heavy body would acquire in falling freely through the ſpace HC.

Laſtly let us ſuppoſe that the ſtream is ſupplyed, not from any ice FGHI, but from the water HICD contained in the veſſel. We have ſeen that if there was a ſtream HIMNK conceived to run through the lower water, the force, with which the ſide-preſſures of the lower water would act upon ſuch a ſtream, could neither make it move faſter nor ſlower through the hole K. Therefore if the ſtream at K was ſupplyed by the force of theſe ſide-preſſures of the lower water, it would run neither faſter nor ſlower than the ſtream HIMNK. But the ſtream HIMNK runs out with the ſame velocity that a heavy body would acquire in falling freely from H to C. Therefore, if the ſtream was ſupplyed from the lower water, as in fact it is when the veſſel is full of water, the velocity of ſuch a ſtream running out at K will ſtill be the ſame.

Having thus ſhewn the propoſition to be true, if the hole was made in the middle of the bottom, we may eaſily ſhew that it will be equally true,

true, if the hole was made any where elſe in the bottom. For the velocity, with which water runs out of a hole made either in the middle or in any other part of the bottom is as the ſquare root of the perpendicular depth, by propoſition 144. But either the middle or any other part of the bottom is at the ſame perpendicular depth from the ſurface HI. Therefore the ſquare root of the perpendicular depth, and conſequently the velocity, is the ſame, whether the hole is made in the middle or in any other part of the bottom.

The ſame may likewiſe be ſaid of any hole made in the ſide of a veſſel. The velocity is as the ſquare root of the perpendicular diſtance from the ſurface, whether the hole is made in the bottom or in the ſide of a veſſel by propoſition 144. Therefore, where the depth is given, the velocity is the ſame, whether the hole is made in the bottom or in the ſide. But if the hole is in the bottom, the velocity, with which water runs out of it, is the ſame that a heavy body would acquire in falling freely through a ſpace equal to the diſtance of the hole from the ſurface of the water in the veſſel. Therefore, if the hole is made in the ſide of a veſſel, the velocity with which water runs out at it, will be the ſame that a heavy body would acquire in falling through a ſpace equal to the perpendicular depth of the hole from the ſurface of the water in the veſſel.

The whole propoſition may be proved otherwiſe. If the veſſel ABCD is filled with water to the level HI; then, ſuppoſing a ſmall hole to be made at H cloſe to the ſurface of the water, any drop of water that comes out at H will have the ſame velocity, that any other heavy body would begin to fall with. For as there is no water above H to force the drop out, ſuch a drop can come out by no other force but its own gravity, and conſequently as it comes out its velocity will be the ſame with that of any other heavy body. Now if the hole is made lower than H, the velocity, with which water will come out at any hole in any part of the bottom or ſide of the veſſel below H, encreaſes as the ſquare root of the diſtance from H encreaſes, by propoſition 144. But as a heavy body falls from H, the velocity of the body will encreaſe as the ſquare root of the diſtance from H encreaſes, by propoſition 107 of mechanics. Therefore ſince the velocity of the water, when the hole is at H, is the ſame with the velocity of a heavy body beginning to fall from H, and ſince, as the hole is made lower, or the body falls lower, the velocity of the water coming out of the hole encreaſes in the ſame proportion with the falling bodys velocity; it follows, that at all diſtances from H theſe velocities will be equal.

CHAP. XIII.

Of the times, in which cylindrical veſſels empty themſelves through holes in their bottoms.

155. *If cylindrical veſſels contain equal quantities, the times, in which they empty themſelves through holes in their bottoms, will be inverſely as the quantity diſcharged by them in a given time.*

IF two cylindrical veſſels contain a gallon of water each, and one diſcharges a quart through any hole in the bottom in the ſame time that the other diſcharges only a pint; it is evident, that, if they continue to run at the ſame rate, the former veſſel will be emptyed as ſoon again as the latter, that is, if the quantities diſcharged from each in a given time are as 2 to 1, the former will be emptyed in half the time that the latter is, or the times, in which they empty themſelves will be as 1 to 2. And ſo in all other caſes, when there is the ſame quantity to be diſcharged, before the veſſels are emptyed, the more water runs out in a given time ſo much the leſs will be the time required to diſcharge the whole. Therefore, when the contents of veſſels are given, as the quantity diſcharged in a given time is greater, the time of emptying the whole will be leſs, or the time of emptying the whole will be inverſely as the quantity diſcharged in a given time.

156. *When cylindrical veſſels diſcharge equal quantities in equal times, the times that they take up to empty themſelves will be directly as the quantities contained in the veſſels.*

If one veſſel contains two gallons of water and another veſſel contains only one gallon, then, if in any given time a pint runs out of each veſſel, it is evident, that, upon ſuppoſition they continue to run at the ſame rate, the former veſſel will be twice as long in emptying itſelf as the latter will. And ſo in all other caſes, when equal quantities are diſcharged in a given time, it is evident that the more water is contained in a veſſel the longer time will be taken up before all of it can run out. Therefore, when the quantities diſcharged in a given time are equal, the time neceſſary to diſcharge the whole will be greater as the contents of the veſſel are greater, and leſs as theſe contents are leſs, or the time of diſcharging the whole, ſo as to empty the veſſel, will be directly as the quantity that the veſſel contains.

157. *When both the quantities discharged in a given time and likewise the quantities contained in different cylindrical vessels are unequal, the times, in which they will empty themselves through holes at their bottoms, will be as the quantities contained in the vessels directly, and as the quantities discharged in a given time inversely.*

If we call the quantities of water contained in any vessels C, and the quantities discharged in any given time G; then the times, in which the vessels will empty themselves, will be as C directly, by proposition 156, and as G inversely, by proposition 155. Therefore the times, in which the vessels will empty themselves, will be as $\frac{C}{G}$; for this fraction is directly as the numerator C and inversely as the denominator G.

Thus if one vessel contains 20 gallons of water and another contains 12 gallons, and if the former in any given time discharges 2 pints, and the latter in an equal time discharges 3 pints; then the contents are to each other as 20 to 12, and the quantities discharged in a given time are as 2 to 3. Therefore the times, in which these vessels empty themselves, will be as 20 to 12 directly, and as 2 to 3 inversely, or as $\frac{20}{3} = 10$ to $\frac{12}{3} = 4$. And universally, if the contents are made the numerator of a fraction, and the quantities discharged in a given time are made the denominator, the times taken up to empty the vessel will be as that fraction. Or, since every fraction expresses the quotient of the numerator divided by the denominator, if the contents of the vessel are divided by the quantities discharged in a given time, the times taken up by the vessels to empty themselves will be as the quotients.

158. *When cylindrical vessels have equal heights but unequal bottoms, and empty themselves through equal holes at their respective bottoms, the times taken up to empty them will be as the areas of their bottoms.*

The vessels have equal heights, by the supposition, therefore the square roots of their heights are equal. But the velocities, with which they empty themselves and consequently the quantities of water which they discharge in a given time will be equal, by propositions 144, 143. But when the quantities discharged in a given time are equal, the times, in which vessels empty themselves, will be as their contents, by proposition 156. And the contents of cylinders, whose height is given, are as their bases or as the areas of their bottoms. Euc. b. XII. prop. 11. Therefore the times, in which such vessels empty themselves, will be as the areas of their bottoms.

159. *When cylindrical veſſels have equal heights and equal bottoms, the times, that they take up to empty themſelves through unequal holes in their reſpective bottoms, will be as the areas of thoſe holes inverſely.*

Since the heights are equal, the ſquare roots of thoſe heights and conſequently the velocities, with which the water runs out at the bottoms of thoſe veſſels, will be equal, by propoſition 144. Therefore the quantities diſcharged in a given time through unequal holes, will be as the areas of thoſe holes, by propoſition 148. But ſince the heights of the cylinders and their baſes or bottoms are equal, the contents will be equal. Euc. b. XII. prop. 11. Therefore the times taken up to empty the veſſels will be inverſely as the quantities diſcharged in a given time, by propoſition 155, or inverſely as the areas of the holes.

160. *When cylindrical veſſels have equal bottoms but unequal heights, the times, that they take up to empty themſelves through equal holes at their reſpective bottoms, will be as the ſquare roots of their perpendicular heights.*

If two cylindrical veſſels, that are filled with water, have equal bottoms and empty themſelves through equal holes, then, ſuppoſing the height of one to be 25 inches and the height of the other 9 inches, the times, that they will take up to empty themſelves, will be as 5 to 3, which are as the ſquare roots of their perpendicular heights, ſo that if the veſſel, which is 25 inches high, was 5 ſeconds in emptying itſelf, the other veſſel, which is 9 inches high, would empty itſelf in 3 ſeconds.

The contents of cylindrical veſſels, that have equal bottoms, are as their perpendicular heights. Euc. b. XII. prop. 11. If cylindrical veſſels have unequal heights, and equal holes are made at their reſpective bottoms, the quantities diſcharged in a given time will be as the ſquare roots of the heights of the veſſels, by propoſition 145: for the heights of the veſſels are as the depths of the holes in the bottoms below the ſurface of the water.

Now ſince cylindrical veſſels with equal bottoms and unequal heights, that empty themſelves through equal holes, neither contain equal quantities nor diſcharge equal quantities in equal times, it follows, by propoſition 157, that the times they take up to empty themſelves will be as their contents directly, and as the quantities diſcharged in equal times inverſely. But their contents are as their heights, and the quantities diſcharged in equal times are as the ſquare roots of their heights. Therefore

fore the times, in which they empty themſelves, will be as their heights directly and as the ſquare roots of their heights inverſely; or as their heights divided by the ſquare roots of their heights. But the quotients in ſuch a diviſion will be the ſquare roots of the heights. Therefore the times, in which the veſſels empty themſelves, being as thoſe quotients, will be as the ſquare roots of their heights.

Call the perpendicular height of a cylindrical veſſel h, and the ſquare root of its height $\sqrt{h}$. Then the contents, if the baſe is given, will be as h. Euc. b. XII. prop. 11. And if the area of the holes is given, the quantities diſcharged in a given time will be as $\sqrt{h}$, by propoſition 145. But the times in which the veſſels empty themſelves are as the contents directly and as the quantities diſcharged in a given time inverſely. Therefore the times in which veſſels with equal bottoms empty themſelves through equal holes will be as $\frac{h}{\sqrt{h}} = \sqrt{h}$.

Thus, in the inſtance before given, if the height of one veſſel is 25 inches and the height of another veſſel is 9 inches, the contents of theſe cylinders will be as their heights or as 25 to 9, becauſe their baſes are given, by the ſuppoſition. But the quantities diſcharged in a given time will be inverſely as the ſquare roots of 25 and 9 or inverſely as 5 to 3. And conſequently, by propoſition 157, the times in which theſe veſſels will empty themſelves will be as 25 to 9 directly and as 5 to 3 inverſely, or $\frac{25}{5} = 5$ to $\frac{9}{3} = 3$. But theſe quotients 5 and 3 are to one another as the ſquare roots of the perpendicular heights of the veſſels. Therefore the times in which the veſſels empty themſelves will be as the ſquare roots of their perpendicular heights.

161. *A fluid runs out of a hole in the bottom of a cylindrical veſſel with an uniformly retarded velocity.*

The velocity is ſaid to be uniformly retarded, if it decreaſes in proportion as the fluid has been moving for a longer time; that is, if the velocity, with which the fluid moves, is leſs in the ſame proportion as the time from the beginning of its motion is greater. Therefore, in order to prove the truth of the propoſition, we are to ſhew, that, when water is running out through a hole in the bottom of a cylindrical veſſel, in proportion as it has been running longer it will run ſlower, or that the velocity of it is inverſely as the time that has paſſed from the inſtant when it firſt began to run.

Now when a given cylindrical veſſel CDAB, Plat. XX. fig. 3, is filled with water, and is to empty itſelf at a given hole F in the bottom; the veſſel

veſſel will empty itſelf in a certain and determinate time, as appears from propoſitions 156, 155, for the height AC is given, the baſe AB is given, and the area of the hole F is given. But it is evident that the more of this time has paſſed the leſs of it is to come, or the longer the time has been from the firſt inſtant, when the water began to run, ſo much ſhorter will be the time, that remains and is to paſs before all the water will be run out. So that, when any of the water has run out, we may ſay, that the time, which will be taken up for the remainder to run out, will be inverſely as the time that has already paſſed from the firſt inſtant when the water began to run.

As the water ſinks in the veſſel DCAB, it will be full to different perpendicular heights. The perpendicular height of the water at firſt is BD; when it has run ſome time the perpendicular height will be only BE; afterwards, as the water continues to run out, its height in the veſſel will be BG, and thus the height will keep decreaſing till all the water is diſcharged. Therefore at different parts of the time, whilſt the water is diſcharging itſelf, we may conſider this veſſel CD as a veſſel of a given baſe AB, that diſcharges itſelf at a given hole F, but has different perpendicular heights: for though indeed the perpendicular height of the veſſel itſelf continues the ſame; yet, the perpendicular height of the water contained in it varys. From hence then it follows by propoſition 160, that the time, which will be taken up to empty the remainder of the water, will be as the ſquare root of the perpendicular height of that remainder. Thus the time taken up to empty the whole is to the time taken up to empty the remainder, after it has ſunk to E, as the ſquare root of BD to the ſquare root of BE; the time, in which the remainder BE will empty itſelf, is to the time in which the remainder BG will empty itſelf, as the ſquare root of BE is to the ſquare root of BG; and the ſame may be ſaid of any other remainder in the veſſel. But the velocity, with which the water runs out at F, will likewiſe be as the ſquare root of the perpendicular height of the remainder, by propoſition 144. Therefore, ſince both the velocity and time, that will be taken up for the remainder to run out, are each of them as the ſquare root of the ſame perpendicular height, it follows that the velocity, with which the water runs, is proportional to the time that will be taken up for the remainder to run out. But the time, that will be taken up for the remainder to run out, is inverſely as the time, that has already paſſed from the firſt inſtant, when the water began to run. Therefore the velocity is likewiſe inverſely as the time, that has paſſed from that firſt inſtant, or the velocity decreaſes as the time that the water has been running encreaſes, or the

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velocity decreaſes uniformly, or the motion of the water is uniformly retarded.

162. *The ſurface of the water in a cylindrical veſſel ſinks with an uniformly retarded motion, as the water is running out through a hole at the bottom.*

The ſurface of the water in the veſſel will ſink, as the water runs out at the bottom. But the water runs out at the bottom with an uniformly retarded motion. Therefore the ſurface of the water in the veſſel will ſink with an uniformly retarded motion.

163. *The ſpaces deſcribed in equal times by the ſurface of the water in a cylindrical veſſel, whilſt it is emptying itſelf through a hole in the bottom, are as the uneven numbers* 1, 3, 5, 7, 9, *&c, taken backwards.*

The ſurface of the water ſinks with an uniformly retarded velocity, by propoſition 162. But the ſpaces deſcribed in equal times, when the velocity is uniformly retarded, are as the uneven numbers taken backwards, by propoſition 115 of mechanics. Therefore the ſpaces deſcribed in equal times by the ſurface of the water in a cylindrical veſſel, whilſt it is emptying itſelf through a hole in the bottom, will be as the uneven numbers taken backwards.

Thus if the veſſel CDAB, Plat. XX. fig. 3, was 25 inches high and would empty itſelf in 5 minutes through the hole F, the level or ſurface of it would ſink 9 inches in the firſt minute, or from CD to 1E; 7 inches in the ſecond minute, or from 1E to 2G; 5 inches in the third minute, or from 2G to 3I, 3 inches in the fourth minute, or from 3I to 4K; and 1 inch in the laſt minute, or from 4K to 5B the bottom of the veſſel.

164. *The ſpaces deſcribed in unequal times by the ſurface of the water in a cylindrical veſſel, whilſt it is emptying itſelf through a hole in the bottom, will be as the ſquares of the times.*

The ſurface of the water ſinks with an uniformly retarded velocity, by propoſition 162. But the ſpaces deſcribed in unequal times, when the velocity is uniformly retarded, if we reckon from the beginning of the motion to the end of it, will be as the ſquares of the times, as appears from what has been ſaid under propoſition 111 of mechanics. Therefore the ſpaces deſcribed in unequal times by the ſurface of the water in a cylindrical veſſel, whilſt it is emptying itſelf through a hole in the bottom, are as the ſquares of the times.

If in the veſſel CDBA, Plat. XX. fig. 3, the ſurface of the water would ſink from 4K to 5B in 1 minute, then the ſpace through which the ſurface would ſink in 2 minutes may be thus found; the times are to each other as 1 to 2, the ſquares of thoſe times are as 1 to 4: therefore the ſpace through which the ſurface will ſink in 1 minute is to the ſpace through which it will ſink in 2 minutes as 1 to 4. And conſequently, if IB is 4 times KB, ſince the ſurface, by the ſuppoſition would ſink through the ſpace KB in 1 minute, it will ſink through the ſpace IB in 2 minutes. In like manner we may determine how far the ſurface would ſink in 5 minutes. The ſurface would ſink through KB in 1 minute: therefore as the ſquare of 1 is to the ſquare of 5, or as 1 is to 25, ſo is the ſpace KB to the ſpace, through which the ſurface would ſink in 5 minutes. Conſequently if DB is 25 times KB, the whole veſſel DB would empty itſelf in 5 minutes.

At what rate the water would ſink in each of theſe minutes taken ſeparately has been already ſhewn in propoſition 163. Suppoſing the veſſel to be 25 inches high; the water would ſink, through 9 inches in the firſt minute, through 7 in the ſecond, through 5 in the third, through 3 in the fourth, and through 1 in the fifth. Therefore in all the 5 minutes taken together it would ſink through $9+7+5+3+1=25$ inches.

165. *Whilſt a cylindrical veſſel is emptying itſelf, the quantities diſcharged in equal times through a hole in the bottom are as the uneven numbers* 1, 3, 5, 7, 9, &c, *taken backwards.*

The height of the water in the veſſel CDAB, Plat. XX. fig. 3, decreaſes in proportion as the water runs out at F, or the ſpace, through which the ſurface of the water deſcends, is proportional to the quantity diſcharged. But the ſpaces, through which the ſurface deſcends in equal times, whilſt the veſſel is emptying itſelf, are as the uneven numbers taken backwards, by propoſition 163. Therefore the quantities diſcharged in equal times will likewiſe be as the uneven numbers taken backwards.

A clepſydra is a veſſel, that, when it is filled with water, will empty itſelf in ſuch a manner that by the motion of the water we may divide our time into equal parts; juſt as we divide our time into hours or other equal parts by the motion of a clock. From what has been ſhewn already, in this propoſition or in propoſition 163, we may eaſily ſee how ſuch a veſſel is to be divided. Thus if the veſſel CDAB, Plat. XX. fig. 3, would be 5 hours in emptying itſelf at the hole F, and we would divide the veſſel

veſſel in ſuch a manner that the water contained in each diviſion might be diſcharged in an hour; we muſt divide it from the bottom A in the proportion of the 5 firſt uneven numbers 1, 3, 5, 7, 9. In this diviſion, if we reckon downwards from D to B, the diviſions are as the uneven numbers taken backwards, and conſequently the ſurface of the water will deſcend down each diviſion in an equal time, by propoſition 163, or the water contained in each diviſion will be diſcharged in an equal time, by the propoſition now before us. Therefore by the motion of the water in the veſſel the whole time that the veſſel takes up to empty itſelf, or 5 hours, will be divided into 5 equal parts, and conſequently each of thoſe parts is 1 hour. Thus in the firſt hour the water will ſink from D to E, in the ſecond from E to G, in the third from G to I, in the fourth from I to K, and in the fifth from K to B.

166. *The quantities of water diſcharged in unequal times, whilſt a cylindrical veſſel is emptying itſelf through a given hole at the bottom, are as the ſquares of thoſe times.*

If the cylindrical veſſel, CDAB, Plat. XX. fig. 3, is divided into 25 equal parts, and the whole veſſel, or all thoſe 25 equal parts, will be emptyed through the hole F in 5 hours, then in the 4 laſt hours of the 5 the ſpace EB or 16 of thoſe parts will be emptyed, in the 3 laſt hours the ſpace GB or 9 of thoſe parts will be emptyed, in the 2 laſt hours the ſpace IB or 4 of thoſe parts will be emptyed, in the laſt hour the ſpace KB or 1 of thoſe parts will be emptyed. Thus when the times are as 5, 4, 3, 2, 1, the ſpaces emptyed in thoſe times or the quantities of water diſcharged in order to empty thoſe ſpaces are as 25, 16, 9, 4, 1, which are the ſquares of the times. That the water will be diſcharged at this rate is plane, from propoſition 165. For all the 25 parts of the veſſel are emptyed in 5 hours, by the ſuppoſition. But in the firſt hour of the 5, 9 of theſe parts from D to E will be emptyed, by propoſition 165, and conſequently there will be 25—9=16 remain to be emptyed in the 4 laſt hours. 7 of theſe 16 parts will be emptyed in the ſecond hour, by propoſition 165, and conſequently there will be 16—7=9 remain to be emptyed in the 3 laſt hours. 5 of theſe parts will be emptyed in the third hour, by propoſition 165, and conſequently there will be 9—5=4 remain to be emptyed in the 2 laſt hours. 3 of theſe parts will be emptyed in the fourth hour, by propoſition 165, and conſequently there will be 4—3=1 remain to be emptyed in the laſt hour of the 5.

This may be proved more univerſally from propoſition 164. The ſpaces deſcribed in unequal times by the ſurface of the water in the veſſel

are as the ſquares of the times, by propoſition 164, if we reckon from the firſt inſtant, when the water begins to run, to the laſt, when it has done running. But the ſurface of the water ſinks in the veſſel in proportion to the quantity that is diſcharged through the hole. Therefore the quantities diſcharged in unequal times are likewiſe as the ſquares of thoſe times.

When two cylindrical veſſels, of equal bottoms and with an equal hole in each of their bottoms, are to be ſo contrived as to take up unequal times to empty themſelves, their heights muſt be reſpectively as the ſquares of the times, which they are deſigned to take up in emptying themſelves. Thus if two clepſydras were to be made, and it was deſigned that one ſhould be 6 hours and the other 3 hours in emptying itſelf, then if their bottoms are equal, and the holes made in their reſpective bottoms are equal, their heights muſt be to each other as 36 to 9, or as the ſquares of the times, in which it is deſigned they ſhould reſpectively empty themſelves. The baſes or bottoms are equal, by the ſuppoſition, and conſequently the quantities contained in the veſſels will be as their heights. Euc. b. XII. prop. 11. But the quantities diſcharged in unequal times will be as the ſquares of the times, by the propoſition now before us. Therefore the heights of theſe veſſels muſt be as the ſquares of the times in which it is deſigned that they ſhould empty themſelves.

167. *When two cylindrical veſſels have equal heights, equal bottoms, and an equal hole in their reſpective bottoms; one of them will diſcharge double the quantity of water, that is diſcharged in the ſame time by the other; if the firſt is kept always full, whilſt the other runs itſelf empty.*

If there are two cylindrical veſſels each of the ſize CDAB, Plat. XX. fig. 3, with an equal hole F in their reſpective bottoms, then ſuppoſe that, as the water runs out of one veſſel at the bottom, there is a conſtant ſupply of water poured in at the top, ſo as to keep the veſſel always full, but that the other having no ſuch ſupply runs till it is quite empty. The water will continue to run with the ſame or an uniform velocity out of the veſſel, that is kept full, from the beginning of the time to the end of it, by propoſition 144: becauſe as the veſſel is always full the perpendicular height of the water above the hole, and conſequently the ſquare root of the perpendicular height will be the ſame, during the whole time. But the water will run with an uniformly retarded velocity out of the veſſel that emptys itſelf, by propoſition 161. Now ſince both the veſſels are full at firſt, the velocity, which continues uni-

uniform in one of the veſſels, will be the ſame with the firſt velocity in that where the water is uniformly retarded, by propoſition 144. Therefore the quantities diſcharged out of the former veſſel will be to the quantity diſcharged in an equal time out of the latter, as the ſpace deſcribed by a body moving uniformly to the ſpace deſcribed in an equal time by a body, that ſets out with the ſame velocity and then is uniformly retarded. But the ſpace deſcribed by the former body will be double the ſpace deſcribed by the latter, by propoſition 110 of mechanics. Therefore the quantity diſcharged out of the former veſſel will be double the quantity diſcharged out of the latter.

CHAP. XIV.

Of ſpouting fluids.

168. *A fluid, when it ſpouts perpendicularly from any hole in the bottom or ſide of a veſſel, has ſuch a velocity as is ſufficient to carry it to the ſame height that the fluid ſtands at in the veſſel.*

IF the water ſtands at the level EFL in the veſſel ABCD, Plat. XX. fig. 8, and a tube or pipe DHK is ſoldered into a hole at D, the ſhape of this tube is ſuch that the water, as it comes out of the hole, muſt begin to move perpendicularly upwards in the direction HK. Now the water as it comes out at D has a velocity, that will be ſufficient to carry it to the perpendicular height HF or to the level EFL, at which the water ſtands in the veſſel.

The velocity that a heavy body would acquire in falling freely from the ſurface of the water to the place of the hole, or from F to D, would be ſufficient, if the direction of the body was changed, to carry it up to the ſame perpendicular height from whence it fell, by propoſition 115 of mechanics. But the water comes out of the hole D with the ſame velocity, that a heavy body would acquire in falling from the ſurface of the water to the place of the hole, or from F to D, by propoſition 154. Therefore the water, ſince it is a heavy body, has ſuch a velocity as is ſufficient to carry it up to the height DF, or to the ſame height that the water ſtands at in the veſſel.

The propoſition would be equally true, whatever was the length of DH, or however diſtant the hole, where the water ſpouts, is from the reſervoir or veſſel ABCD. Such artificial jets of ſpouting water are called fountains: and from hence we ſee the reaſon, why the jet will never riſe higher than the level of the water in the reſervoir. It would al-

ways riſe to the ſame height, if the water, as it riſes, was retarded by no other force but that of its own gravity, as has been already ſhewn in proving the propoſition. But in fact it never riſes quite ſo high, becauſe, beſides the force of gravity, there are three other cauſes, which retard it, and prevent it from aſcending to this height. Firſt, as the water riſes it is to move through the air, which is a reſiſting fluid, and conſequently will retard the water, by propoſition 117. Secondly, it cannot aſcend without rubbing againſt the ſides of the tube or pipe, and againſt the edges of the hole at K; and this friction will retard the water, as was obſerved in a ſimilar caſe under propoſition 98 of mechanics. Thirdly, the water, that has riſen to F its greateſt perpendicular height, will have loſt all its velocity, and would fall back again in the ſame line FK, in which it aſcended, if the water that follows it did not prevent its falling in this line by throwing it off ſide-ways: but as much motion as the water that follows it communicates to this, which is at the top of the column, in throwing it off ſide-ways, juſt ſo much it muſt loſe itſelf; and conſequently the aſcending column will be retarded by that water, which had aſcended before it.

169. *A ſtream of any fluid, which ſpouts obliquely, is a parabola.*

Every drop in a ſtream of water, that ſpouts obliquely out of a hole in the bottom or ſide of a veſſel, is a heavy body projected obliquely; it is a heavy body, by propoſition 8, and ſince the whole ſtream ſpouts obliquely, each drop is projected or made to move obliquely by the force or preſſure, which drives it out of the hole. Now heavy bodies deſcribe a parabola, when they are projected obliquely, by propoſition 144 of mechanics. Therefore every drop of the oblique ſtream will deſcribe a parabola, and conſequently the whole ſtream conſiſting of innumerable drops will be a parabola.

170. *When a fluid ſpouts obliquely, if the perpendicular depth of the hole below the ſurface of the fluid in the veſſel is made the diameter of the circle, the greateſt perpendicular height, to which the fluid will riſe, is equal to the verſed ſine of double the angle of elevation.*

If AB, Plat. V. fig. 8, is the perpendicular height of the water in a veſſel, out of which water ſpouts obliquely from a hole at B; then making AB the diameter of the circle, the greateſt perpendicular height, to which ſuch an oblique ſtream will riſe, is equal to the verſed ſine of double the angle of elevation.

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The reader has already ſeen this matter explaned, in propoſition 146 of mechanics, and from the ſame propoſition we may prove the truth of this, which is now before us. When a heavy body is thrown obliquely, if the height, from whence it muſt have fallen to acquire the velocity with which it is thrown, is made the diameter of the circle, the greateſt perpendicular height, to which the body will riſe in ſuch an oblique projection, is equal to the verſed ſine of double the angle of elevation, by propoſition 146 of mechanics. But every drop of water ſpouting out of a hole in the bottom or ſide of a veſſel, and conſequently the whole ſtream, is a heavy body projected obliquely, by propoſition 8, and the velocity of it is the ſame that any other heavy body would have acquired by falling freely from the ſurface of the water in the veſſel to the place of the hole. Therefore if the diſtance of the hole from the ſurface is made the diameter of the circle, the greateſt perpendicular height, to which the oblique ſtream will riſe, is equal to the verſed ſine of double the angle of elevation.

171. *When a fluid ſpouts obliquely, if the perpendicular depth of the hole below the ſurface of the fluid in the veſſel is made the diameter of the circle, the greateſt horizontal diſtance, to which the fluid will be thrown, is equal to four times the ſine of double the angle of elevation.*

If AB, Plat. V. fig. 8, is the perpendicular depth of the hole from the ſurface of the water, that is contained in a veſſel, out of which an oblique ſtream ſpouts at B, then making AB the diameter of a circle, the greateſt random of the ſtream, or greateſt horizontal diſtance, to which it will be thrown, is equal to four times the ſine of double the angle of elevation.

The reader has ſeen this matter farther explaned in propoſition 147 of mechanics, from which propoſition we may likewiſe deduce the truth of this. When a heavy body is thrown obliquely, if the height, from whence it muſt have fallen to acquire the velocity of projection, is made the diameter of a circle, the greateſt random is equal to four times the ſine of double the angle of elevation, by propoſition 147 of mechanics. But in an oblique ſtream every drop, and conſequently the whole ſtream, is a heavy body projected obliquely, by propoſition 8, and the velocity, with which it ſpouts or is projected, is the ſame that any heavy body would acquire by falling freely from the ſurface of the water in the veſſel to the place of the hole, by propoſition 154. Therefore if the diſtance of the hole from the ſurface is made the diameter of a circle, the greateſt random of the water will be equal to four times the ſine of double the angle of elevation.

172.

172. *When a fluid ſpouts obliquely, if the diſtance of the hole from the ſurface of the fluid in the veſſel is given, the ſtream will ſpout to the greateſt horizontal diſtance, when the elevation of it is half a right angle.*

When the perpendicular diſtance of the hole from the ſurface of the water in the veſſel is given, the velocity of the ſtream is likewiſe given, by propoſitions 144, 154. But the ſtream is a heavy body projected obliquely. Therefore the random will be the greateſt poſſible with that velocity, if the angle of elevation is half a right one, by propoſition 148 of mechanics.

173. *When a fluid ſpouts obliquely, if the diſtance of the hole from the ſurface of the fluid in the veſſel is given, the horizontal diſtances, to which the ſtream will ſpout, will be equal at two different elevations, provided thoſe elevations differ equally from half a right angle.*

When the perpendicular diſtance of the hole from the ſurface of the water in the veſſel is given, the velocity of the ſtream is likewiſe given. But the ſtream is a heavy body projected obliquely. Therefore the randoms will be equal at two different elevations, provided thoſe elevations differ equally from half a right angle, by propoſition 149 of mechanics.

174. *When a fluid ſpouts obliquely, if the diſtance of the hole from the ſurface of the fluid in the veſſel is given, the greateſt horizontal diſtance, to which the ſtream will ſpout, is double the greateſt height, to which it would have riſen, if it had ſpouted perpendicularly with the ſame velocity.*

When the perpendicular diſtance of the hole from the ſurface of the water in the veſſel is given, the velocity of the ſtream is likewiſe given, by propoſitions 144, 154. But the ſtream is a heavy body projected obliquely. Therefore the random, when the elevation is 45 degrees, which by propoſition 172, is the greateſt random, will be double the height, to which the ſame ſtream would riſe, if it was to ſpout perpendicularly out of the ſame hole, by propoſition 150 of mechanics.

Now if water ſpouts perpendicularly out of any hole in a veſſel, the greateſt height to which it will riſe is the perpendicular height of the water in the veſſel above the hole, by propoſition 168. Therefore if the ſtream ſpouts obliquely at an angle of 45 degrees, the horizontal diſtance, to which it will be thrown, or the random, will be equal to twice the height of the water in the veſſel above the hole.

175.

175. *The horizontal diſtances to which a ſtream of any fluid ſpouts, when the elevation is given, is as the ſquare of the fluids velocity, or as the perpendicular height of the fluid in the veſſel above the hole from which the ſtream ſpouts.*

A ſtream of water ſpouting obliquely is a heavy body projected obliquely. But in all oblique projections of heavy bodies and conſequently in an oblique ſtream of water, the randoms are as the ſquares of the velocities, when the elevations are given, by propoſition 151 of mechanics. Now the ſquare of the velocity, with which water ſpouts out of any hole in the bottom or ſide of a veſſel is as the perpendicular height of the water in the veſſel above the hole, by propoſition 144. Therefore, ſince the randoms are as the ſquares of the velocities, it follows that they will likewiſe be as the perpendicular height of the water above the hole.

The reader will obſerve that in the foregoing propoſitions we have conſidered the caſes of ſpouting fluids, only where the force, that drives them out, is the preſſure of the fluid contained in the veſſel from whence the ſtream ſpouts. The water is projected out of fire-engines by the force of the piſton, and not by the preſſure of the water in the tub. Therefore the heights and randoms of the ſtream in ſuch engines is not to be determined by theſe rules, but by thoſe explaned in chap. xi. of mechanics: ſo that the diameter of the circle determining the heights and randoms of ſuch ſtreams is not to be the perpendicular depth of the water in the engine, but is to be the height from whence a heavy body muſt fall to acquire the velocity of projection, or the velocity, with which the ſtream is driven out by the engine.

176. *When a fluid ſpouts horizontally out of any hole in the ſide of a veſſel that is full, if the ſide of the veſſel is made the diameter of a circle, and a line is drawn perpendicular to the ſide from the hole to the circumference, then the diſtance, to which the fluid will ſpout, will be double this perpendicular.*

If AB, Plat. XX. fig. 9, is the ſide of a veſſel, that is filled with water, and a hole is made any where in the ſide of it, as at *e*, then make AB the diameter of a circle, and draw *ed*, which is a line perpendicular to the ſide of the veſſel, from *e* the hole to the circumference at *d*, then as the water is driven out of the hole at *e*, ſince there is no pipe to make it riſe perpendicularly, it will ſpout horizontally, and BF, which is the horizontal diſtance, to which it ſpouts, will be equal to twice *ed*.

In

In the caſe of a fluid ſpouting horizontally and deſcending by its own weight to a horizontal plane BG, that is level with the bottom of the veſſel, there are two velocities to be diſtinctly conſidered; firſt, the velocity, with which the fluid ſpouts, which we will call the horizontal velocity; and ſecondly, the velocity, which it acquires in falling or deſcending, after it comes out of the hole, which we will call the deſcending velocity.

Now the fluid ſpouts out at *e* with the ſame velocity that a heavy body would acquire in falling from A to *e*, by propoſition 154. And this motion of the water, as it is in a horizontal direction, can neither be accelerated nor retarded by the force of gravity: for the force of gravity cannot accelerate the motion of a body, unleſs it acts in the ſame direction in which the body moves, nor can that force retard the motion, unleſs it acts in a contrary direction: but ſince the water by the force, which drives it out at *e*, is made to move horizontally and neither downwards nor upwards, the force of gravity acts neither in the ſame nor in a contrary direction: and conſequently this force will neither accelerate nor retard the waters motion. Therefore the horizontal velocity will continue uniform.

The deſcending velocity is uniformly accelerated, by propoſition 102 of mechanics. And this deſcending velocity, when the water ſtrikes upon the plane GB, is the ſame that any other heavy body would have acquired by falling through an equal ſpace or from *e* to B.

But ſince the horizontal velocity is the ſame that a heavy body would acquire by falling from A to *e*, and the deſcending velocity, when the water ſtrikes upon the horizontal plane GB, is the ſame that a heavy body would acquire by falling from *e* to B, it follows, by propoſition 105 of mechanics, that the ſquare of the laſt deſcending velocity is to the ſquare of the horizontal velocity, as *e*B to A*e*. Now AHB is a ſemicircle, and for that reaſon A*d*B is a right angled triangle. Euc. b. III. prop. 33. *de* is perpendicular to AB by the conſtruction, and conſequently the triangle B*ed* is ſimilar to the triangle *ed*A. Euc. b. VI. prop. 8. Therefore *e*B is to *ed*, as *ed* is to A*e*. Euc. b. VI. prop. 4. But ſince *e*B, *ed*, and A*e* are in continued geometrical progreſſion; the ſquare of *e*B is to the ſquare of *ed*, as *e*B is to A*e*. Euc. b. V. def. 10. But the ſquare of the laſt deſcending velocity, when the water ſtrikes the plane, is to the ſquare of the horizontal velocity as *e*B to A*e*. Therefore the ſquare of the laſt deſcending velocity is to the ſquare of the horizontal velocity as the ſquare of *e*B to the ſquare of *ed*; and conſequently the laſt deſcending velocity is to the horizontal velocity as *e*B to *ed*. Now the ſpaces deſcribed in the ſame time are as the velocities, with which a body moves,

by

by propoſition 10 of mechanics: and conſequently, if the water had begun to fall from *e* with the ſame velocity, that it acquires at laſt, and had fallen uniformly, in the time of deſcent the ſpaces deſcribed by the deſcending and horizontal velocities would have been reſpectively as thoſe velocities, or as *e*B to *ed*, that is, whilſt the water was deſcending, till it ſtruck upon the plane GB and was ſtopped there, the horizontal velocity would have carryed it forwards through a ſpace equal to *ed*; ſo that, upon this ſuppoſition, the water would fall upon ſome point of the plane GB, the diſtance of which from B would be equal to *ed*, or the horizontal diſtance, to which the water would be carryed, would be equal to *ed*. But this ſuppoſition is not true; for the deſcending velocity is not the ſame at firſt that it is at laſt; but it begins from nothing and is uniformly accelerated, and conſequently the time of deſcent is twice what it would have been upon this ſuppoſition, by propoſition 110 of mechanics. But the horizontal velocity is uniform, and therefore in twice the time, or in the true time of deſcent, the water will be carryed horizontally to twice the diſtance *ed*. Therefore, if BF is taken equal to twice *ed*, whilſt the water is deſcending from *e* to the plane GB, it will be carryed out horizontally to twice the diſtance *ed*, and will fall upon the plane at the point F.

In like manner we might ſhew that, if the water was to ſpout at C, the horizontal diſtance would be twice CH, and if BG is taken equal to twice CH, G would be the place where the water ſpouting out at C would fall, ſo that BG equal to twice CH would be the horizontal diſtance. Or if the water was to ſpout at E, the horizontal diſtance would be twice ED. Or univerſally, if the ſide of the veſſel is made the diameter of a circle, and a line is drawn perpendicular to the ſide from the hole to the circumference, the horizontal diſtance will be equal to twice the length of that line.

177. *If a fluid ſpouts horizontally out of a hole in the ſide of a veſſel, and the perpendicular height of the veſſel is given, the diſtance, to which it ſpouts, will be the greateſt, when the hole is made in the middle of the ſide.*

If AB, Plat. XX. fig. 9, is the perpendicular height of the water in the veſſel, and the veſſel is filled with water, then, making AB the diameter of a circle, C or the middle of the ſide AB will be the center of the circle, and conſequently a line CH drawn from C to the circumference perpendicular to AB will be the ſemidiameter of the circle. Now if the hole is made at C, the water will ſpout farther than if it had been made at E or at *e*, or any where elſe in the ſide AB. The horizontal di-

ſtance, to which the water ſpouts, when the hole is at C, will be BG equal to twice CH, and when the hole is at E, the horizontal diſtance will be BF equal to twice ED, by propoſition 176. But CH is longer than ED, becauſe the ſemidiameter is longer than any line that can be drawn in a ſemicircle parallel to the ſemidiameter. Euc. b. III. prop. 15. Therefore twice CH is longer than twice ED, or the horizontal diſtance, to which the water ſpouts from G, will be greater than the horizontal diſtance to which it ſpouts from E.

This demonſtration is applicable to any other part of the ſide as well as to the point E. For the ſemidiameter CH is longer than any other line that can be drawn parallel to CH in the ſemicircle AHB: and conſequently twice CH will be longer than twice any other line that is thus drawn parallel to CH. But the horizontal diſtance, when the hole is at C, or in the middle of the ſide, is twice CH, or twice the ſemidiameter; and when the hole is made any where elſe, the horizontal diſtance will be twice ſome other line in the ſemicircle parallel to the ſemidiameter. Therefore the horizontal diſtance, to which the water ſpouts, when the hole is in the middle of the ſide, will be greater than the horizontal diſtance, to which it will ſpout, when the hole is any where elſe.

178. *If two holes are made in the ſide of a veſſel at equal diſtances from the middle of the ſide, the fluid, with which the veſſel is filled, will ſpout horizontally to equal diſtances from each of thoſe holes.*

If the holes E and *e*, Plat. XX. fig. 9, are made in AB the ſide of a veſſel, at equal diſtances from the middle of the ſide, and the veſſel is filled with water; the water will ſpout horizontally to the ſame diſtance out of each hole.

The horizontal diſtance, to which the water will ſpout from the hole E, is equal to twice ED, and the diſtance, to which it will ſpout from the hole *e*, is equal to twice *ed*. But ſince E and *e* are at equal diſtances from the center C, by the ſuppoſition, ED will be equal to *ed*. Euc. b. III. prop. 14. Therefore twice ED is equal to twice *ed*, or the horizontal diſtance, to which the water ſpouts from each hole, is the ſame. Thus if BF is equal to twice ED, then F will be the point where the water will fall, that ſpouts from E. But ſince twice *ed* is equal to twice ED, BF, which is equal to twice ED, muſt be equal likewiſe to twice *ed*. Therefore F will likewiſe be the point, where the water will fall, that ſpouts from the hole *e*.

CHAP.

CHAP. XV.

Of rivers, and the force of ſtreams that run againſt any obſtacle.

179. *A river is a ſtream of water, that runs by its own weight down the inclined bottom of an open channel.*

180. *A ſection of a river is an imaginary plane, that cuts the ſtream, and is perpendicular to the bottom.*

181. *A river is ſaid to flow uniformly, when it runs in ſuch a manner that the depth of the water in any one part continues always the ſame.*

182. *If a river flows uniformly, the ſame quantity of water paſſes, in an equal time, through every ſection of it.*

IF AB, Plat XX. fig. 5, is the reſervoir, BC the bottom of the river, ZX a ſection of the ſtream in one part, and QR a ſection of it in another part; then, upon ſuppoſition that the river flows uniformly, the ſame quantity of water, that paſſes in any given time through the upper ſection ZX, will paſs in an equal time through the lower ſection QR. No more water can paſs through QR in an hour, than paſſes through ZX in an hour. For if there was more water to run off in an hour through the lower ſection QR, than comes down in an equal time through the upper ſection ZX, the quantity of water in the ſpace ZQXR between thoſe ſections would decreaſe, ſo that the water would ſink there, or the depth of it in that place would decreaſe, which is contrary to the ſuppoſition of the rivers flowing uniformly. No leſs water can paſs through QR in an hour than paſſes through ZX in an hour. For if there was leſs water to run off in an hour through the lower ſection QR than comes down in an equal time through the upper ſection ZX, the quantity of water in the ſpace ZQXR between thoſe ſections would encreaſe, ſo that the water would riſe there, or the depth of it would encreaſe; which is likewiſe contrary to the ſuppoſition of the rivers flowing uniformly. Therefore ſince neither more nor leſs water can paſs through the upper ſection ZX in an hours time, than what paſſes by the lower ſection QR in an equal time, the conſequence is that in an hour, or in any other equal time, equal quantities muſt paſs through each of thoſe ſections.

183. *If the breadth of the channel is given, the water in rivers is accelerated in the ſame manner with any heavy body that is moving down an inclined plane.*

The loweſt drops in the river, or thoſe which form the loweſt ſheet of water, are ſo many heavy bodies moving down upon the bottom, which is an inclined plane. The ſecond ſheet of water from the bottom moves upon the loweſt ſheet as upon an inclined plane. And in the ſame manner every ſheet or lamina of the water in the river moves down upon the ſheet or lamina, that is next below it. Therefore each drop of water is a heavy body moving down upon an inclined plane; and conſequently each drop muſt be accelerated in the ſame manner with any other heavy body in the ſame circumſtances.

184. *If the breadth of the channel is given, the velocity of each drop of water in a river is the ſame that a body would acquire in falling from the level of the ſurface of the water in the reſervoir to the place of the drop.*

If AB, Plat. XX. fig. 5, is the depth of the reſervoir, then AP is the level of the waters ſurface or is the plane of that ſurface continued. The velocity of any drop of water, as E, will be the ſame that a heavy body would acquire in falling from M, which is in the level of the waters ſurface, to E, which is the place of the drop. Let BC be the bottom of the channel. Now the drop E, after it came out of the reſervoir, rolled down an inclined plane, which is parallel to the bottom, by propoſition 183: conſequently, if the line EK is drawn parallel to CB and continued back to the reſervoir, KE is the plane down which the drop has rolled, and K is the place where it came out of the reſervoir. But a drop of water coming out of the veſſel AB has the ſame velocity that a heavy body would acquire in falling from A the ſurface of the water to K the place where the drop comes out, by propoſition 154. After the drop comes out at K with this velocity, it rolls down the inclined plane KE, and, in rolling down, it acquires the ſame velocity that any heavy body would acquire, in falling down GE, which, if KG is parallel to AM and conſequently to the horizon, is the perpendicular height of the plane, by propoſition 126 of mechanics. Now if EG is continued to M, ſo that MG may be parallel to AK, then MG and AK will be equal becauſe they are oppoſite ſides of a parallelogram. Euc. b. I. prop. 34. But the drop came out at K with the ſame velocity that a heavy body would acquire in falling down AK or the equal line MG. After it came out, it acquired the

ſame

ſame velocity that a heavy body would acquire in falling down GE. Therefore, when it comes to E, it muſt have the ſame velocity that a heavy body would acquire in falling down MG and GE together or down ME, which is the diſtance between the level of the waters ſurface at the reſervoir and the place of the drop.

185. *If the breadth of the channel is given, the velocity of each drop of water in a river is as the ſquare root of its diſtance from the level of the waters ſurface in the reſervoir.*

The velocities that heavy bodies acquire in falling from different heights, are as the ſquare roots of thoſe heights, by prop. 107 of mechanics. But the velocities of each drop of water in a river is the ſame that a heavy body would acquire in falling from the level of the waters ſurface at the reſervoir to the place of the drop, by propoſition 184. Therefore the velocity of each drop is as the ſquare root of its diſtance from that level.

If E and R, Plat. XX. fig. 5, are two drops in different parts of a river the velocity of the drop E is to the velocity of the drop R in a ſubduplicate ratio of ME to NR, that is, as the ſquare root of ME to the ſquare root of NR. For if AP is the level of the waters ſurface at the reſervoir, the velocity of the drop E is the ſame that a heavy body would acquire by falling down ME, and the velocity of the drop R is the ſame that a heavy body would acquire by falling down NR, by propoſition 184. Therefore, by propoſition 107 of mechanics, theſe velocities are to each other as the ſquare root of ME to the ſquare root of NR.

186. *If the breadth of the channel is given, the water at the bottom of the river will run faſter than the water at the ſurface.*

The diſtance of the water from the level of the ſurface in the reſervoir is greater at the bottom of the river than at the top; the diſtance of R from that level is NR, Plat. XX. fig. 5, and the diſtance of Q from the ſame level is PQ. But the velocities of the drops at R and Q are reſpectively as the ſquare roots of theſe diſtances NR and PQ, by propoſition 185. Therefore, ſince the ſquare root of NR muſt be greater than the ſquare root of PQ, the velocity of any drop of water at R the bottom of the river will be greater than the velocity of any drop at Q the ſurface of the river.

187. *If the breadth of the channel is given, the depth of the river continually decreaſes as it runs.*

The

The river will be ſhallower at any lower ſection QR, Plat. XX. fig. 5, than at any upper ſection ZX. The ſame quantity of water paſſes through each of theſe ſections in an equal time, by propoſition 182. But the water runs faſter at the lower ſection than at the upper ſection, by propoſition 183. Now it is impoſſible that an equal quantity of water ſhould paſs through each of theſe ſections in an equal time, if it runs faſter through one of them than it does through the other, unleſs the area of that ſection, where the velocity is the greateſt, ſhould be leſs than the area of the other ſection, where the velocity is leſs; for where the velocities are unequal, the ſections muſt be inverſely as the velocities, if equal quantities paſs through thoſe ſections in equal times, as appears from what has been ſaid in a ſimilar caſe under propoſition 10. But the breadths of theſe ſections are equal, by the ſuppoſition. Therefore their areas are as their heights. Euc. b. VI. prop. 1: and conſequently the heights of theſe ſections, or the depths of the water, which are the heights of them, will be inverſely as the velocities; ſo that the depth of the water at the lower ſection will be as much leſs as the velocity is greater than at the higher.

In fact we find indeed that rivers grow deeper as they run, but this is owing to the additional water, which is received from other ſtreams that in the progreſs of the river run into the ſame channel with it. Whereas in the propoſition we were conſidering what would happen to a river, if only the ſame water, that comes from the reſervoir, was to run on in the ſame channel without having any other additional water joyned to it.

188. *As the river runs on, the velocities of the water at the bottom and at the ſurface come nearer to an equality.*

As the river runs on, the water grows ſhallower, or the ſurface is nearer to the bottom, by propoſition 187. If the ſurface and bottom were cloſe together, the water at the ſurface of the river and the water at the bottom of it would be at the ſame diſtance from the level of the ſurface in the reſervoir, and conſequently, their velocities would be equal, by propoſition 184. Therefore, ſince the water grows ſhallower as it runs on, the velocities of it at the ſurface and at the bottom become nearer equal.

The depth of each ſection decreaſes continually, as the water runs on, by propoſition 187. But as QR, Plat. XX. fig. 5, the depth of a ſection decreaſes, PQ and NR will be more nearly of the ſame length. Now the velocities of the water at Q and R are reſpectively as the ſquare roots of PQ and NR. But whilſt PQ and NR approach to a ratio of equality their

their ſquare roots will approach ſtill faſter to a ratio of equality. Therefore as the river runs on, the velocity of the water at the bottom and at the ſurface come nearer to an equality.

Indeed beſides this reaſon why the water at the bottom and at the ſurface ſhould come nearer to an equality, as the river runs on, there are other accidental reaſons, ſuch as cannot be reduced to any certain rule. In the progreſs of the river, the water at the bottom and the water at the ſurface communicate with one another, therefore the lower water will accelerate the higher, and the higher water will retard the lower, till their velocities become equal. The lower water, as it runs upon the bottom of the channel, will be retarded by any roughneſs on the bottom: whereas the upper water glides down without any ſuch obſtruction upon that water which is below it, as upon a perfectly ſmooth inclined plane, and conſequently is not ſubject to the ſame retardation. Therefore, though the lower water would of itſelf move faſter than the higher water, yet as the former is retarded in its progreſs more than the latter, their velocities will conſtantly approach to a ratio of equality.

189. *At a given diſtance from the reſervoir, the velocity of the water will be inverſely as the breadth of the channel, if the river flows uniformly.*

If the river flows uniformly, the depth of it at any given part is the ſame, and the ſame quantity of water flows in a given time through every ſection of it. But if a given quantity of water flows in a given time through any ſection, then the velocity muſt be inverſely as the area of the ſection, as appears from what has been ſaid in a ſimilar caſe under propoſition 10. Now the area of the ſections when the depth of it is given, will be as its breadth. Therefore the velocity of the water, which is inverſely as the area of the ſection, muſt be inverſely as its breadth.

190. *If the depth of the river is given, the preſſure upon any part of the bank will be the ſame, whatever is the breadth of the river.*

The preſſure upon any given part in the bank of a river, like the preſſure of water upon any given part in the ſide of a veſſel that contains it, will be as the diſtance of that part from the ſurface of the water, by propoſition 24. But the diſtance of any part in the bank from the ſurface of the water continues the ſame, whilſt the depth of the river is the ſame, however the breadth of it may be altered. Therefore the preſſure upon any given part in the bank will not be altered by the breadth of the river, if the depth of it is not altered.

From

From hence it follows that the water of the ſea, or of any other broad lake, does not require ſtronger banks to hold it, than the water in a narrow canal of the ſame depth. For, where the depth of the water is given, the preſſure againſt the banks is the ſame, whatever is the breadth of the body of water that preſſes againſt them.

191. *If the breadth of a river is given, the preſſure againſt any part of the bank will encreaſe in proportion to the depth of the river.*

The preſſure againſt any given part in the bank of a river, like the preſſure of water againſt the ſide of a veſſel that contains it, is as the diſtance of that part from the ſurface, by propoſition 24. But the diſtance between the ſurface and any given part of the bank will be greater in proportion as the depth of the river is greater or in proportion as the river ſwells and the ſurface of it is raiſed higher. Therefore the preſſure againſt any given part of the bank will be greater in proportion as the depth of the river is encreaſed.

From hence it follows that the banks of a river may break in a flood, though they were ſtrong enough not to be broken by the preſſure of the water at another time. For as the flood ſwells the river, or encreaſes the depth of it, the preſſure upon each part of the bank encreaſes, and conſequently ſuch banks, as were ſtrong enough to bear the preſſure, when the river was ſhallower, may be two weak to bear the preſſure, when a flood has encreaſed it.

192. *The preſſure againſt any given ſurface in the bank of a river, if that ſurface reaches from the bottom to the top of the ſtream, is equal to the weight of a column of water, whoſe baſe is the ſurface, and whoſe height is half the depth of the ſtream.*

ZQXR, Plat. XX. fig. 5, is a given ſurface in the bank and reaches from BC the bottom to AD the top of the ſtream. Now the preſſure againſt this ſurface in the bank is half the preſſure upon an equal ſurface at the bottom, as appears from what has been ſaid under propoſition 24. But the preſſure upon an equal ſurface at the bottom is equivalent to the weight of a column of water whoſe baſe is the ſurface and whoſe height is equal to the depth of the ſtream, by prop. 24. Therefore the preſſure againſt this ſurface in the bank is equal to the weight of half ſuch a column, or to the weight of a column whoſe baſe is the ſurface and whoſe height is half the depth of the ſtream.

193.

193. *When a ſtream, that moves with the ſame velocity in every part, ſtrikes perpendicularly upon any obſtacle; the force, with which it ſtrikes, is equal to the weight of a column of the ſame fluid, whoſe baſe is the obſtacle, and whoſe height is the ſpace through which a heavy body muſt fall to acquire the velocity of the ſtream.*

If a ſtream of water flows horizontally out of a hole *e*, Plat. XX. fig. 9, in the direction *ed*, the force of gravity will indeed make it deſcribe the curve *e*F: but, neglecting this force of gravity, we may conſider the force of the ſtream in the horizontal direction *ed*.

Now if this ſtream was to ſtrike upon an obſtacle juſt as broad every way as the ſtream itſelf is, then it is plane that the ſtream muſt ſtrike that obſtacle with its whole force. But this whole force is equal to the force, which drives the water out at *e*: and the force, which drives the water out at *e*, is the weight of a column of water whoſe baſe is *e* and whoſe height is A*e*. Now A*e*, by propoſition 154, is the height from whence a heavy body muſt fall to acquire the velocity, with which the ſtream ſpouts from *e*. Therefore the force, with which this ſtream would ſtrike ſuch an obſtacle, is equal to the weight of a column of the ſame fluid, whoſe baſe is *e*, and whoſe height is that from whence a heavy body muſt fall to acquire the velocity of the ſtream.

We have here ſuppoſed that the obſtacle is juſt as broad every way as the ſtream. But if the ſame obſtacle was to be ſtruck by a ſtream flowing with the ſame velocity, the ſtroke could neither be greater nor leſs, though the ſtream was ever ſo much broader: for let the ſtream be ever ſo broad, no part of it can ſtrike the obſtacle, but juſt ſo much of it as is contained within a ſection equal to the obſtacle. Thus ſuppoſe the diameter of the hole at *e*, and conſequently the breadth of the ſtream *ed*, fig. 9, to be equal to BC, Plat. XX. fig. 4, then, by what has been ſaid, an obſtacle, whoſe diameter is BC, being ſtruck by the whole ſtream HBCI will ſuſtain a force equal to the weight of a column of water, whoſe baſe is the obſtacle and whoſe height is the ſpace through which a heavy body muſt fall to acquire the velocity of the ſtream. But if the obſtacle was to continue the ſame, whilſt the diameter of the ſtream by making the hole wider is encreaſed to AD, the force, with which this greater ſtream would ſtrike the ſame obſtacle BC, would be ſtill the ſame: becauſe the part of this greater ſtream, which ſtrikes the obſtacle, is juſt equal to the obſtacle, and conſequently is juſt equal to the leſſer ſtream HBIC, which we ſuppoſed to ſtrike it before. So that, if the obſtacle

BC is given, the force of the ſtream, let the breadth of it be ever ſo great, will be equal to the weight of a column of water, whoſe baſe is given, and whoſe height will be equal to the ſpace through which a heavy body muſt fall to acquire the velocity of the ſtream. But then if in a broad ſtream running with a given velocity the obſtacle is greater than BC, the force of the ſtream will be greater in the ſame proportion: thus if the obſtacle is AD, the ſtream, that ſtrikes it will be FAGD; and the force, with which this ſtream ſtrikes the obſtacle AD, is as much greater than the force, with which the leſſer ſtream ſtruck the obſtacle BC, as the former ſtream is greater than the latter, or as the obſtacle AD is greater than the obſtacle BC.

We have here proved the truth of the propoſition by ſuppoſing that the ſtream is made to flow by being forced out of a hole in the ſide of a veſſel. But, if it is made to flow horizontally with the ſame velocity by any other means, it will ſtrike an obſtacle, that is oppoſed to it, in the ſame manner and with the ſame force. Conſequently, if a river has run to ſome diſtance from the fountain-head, till, by propoſition 188, the water at the bottom and at the ſurface runs with the ſame velocity, the force, with which the ſtream ſtrikes upon any obſtacle, that is oppoſed to it, ſuch as a ſluice or a dam, will be equal to the weight of a column of water, whoſe baſe is the ſluice or dam, and whoſe height is equal to the ſpace, through which a heavy body muſt fall perpendicularly to acquire the velocity of the river.

Let AD be the ſail of a ſhip, which is here repreſented by a right line, becauſe the edge of the ſail is ſuppoſed to be towards the eye, ſo that the breadth of it cannot be ſeen. Then if the wind blows, that is, if a ſtream of air flows againſt it, the ſail will be ſtruck with the ſame force, as if ſuch a ſtream of water was to ſtrike upon it, only allowance muſt be made for the difference of denſity, weight, or quantity of matter contained, in equal columns of air and water. Therefore as we eſtimate the force of a ſtream of water by the weight of a column of water, ſo we muſt eſtimate the force of a ſtream of air by the weight of a column of air: and conſequently the force, with which the wind ſtrikes upon the ſail, will be equal to the weight of a column of air, whoſe baſe is the ſail, and whoſe height is equal to the ſpace, through which a heavy body muſt fall perpendicularly to acquire the velocity of the wind.

194. *When the obſtacle is given, the force, with which a ſtream of any given fluid ſtrikes upon it, will be as the ſquare of the fluids velocity.*

If

If a ſtream of water ſtrikes upon a given obſtacle, the force, with which it ſtrikes, will be equal to the weight of a column of water, whoſe baſe, being equal to the obſtacle, is given, by the ſuppoſition, and whoſe height is equal to the ſpace, through which a heavy body muſt fall perpendicularly to acquire the velocity of the ſtream. Now ſince the baſe of this column is given, the weight of it will be as its height. But the ſpaces, through which heavy bodies fall to acquire different velocities, are as the ſquares of thoſe velocities, by propoſition 106 of mechanics. Therefore the height of this column, and conſequently its weight, and conſequently the force of the ſtream, which is equivalent to this weight, will be as the ſquare of the velocity with which the ſtream moves.

Or otherwiſe. The force with which an immoveable obſtacle is ſtruck by a ſtream of any given fluid, that runs againſt it, will be as the quantity of matter that ſtrikes upon the obſtacle in a given time, and as the velocity conjoyntly, by propoſition 13 of mechanics. But when the obſtacle AD, Plat. XX. fig. 4, is given, the quantity of any given fluid, that ſtrikes upon it in a given time, whilſt the ſtream is running againſt it, will be as the velocity of the ſtream; the number of particles, that are driven againſt the obſtacle, will be greater in proportion as the ſtream runs faſter and leſs in proportion as it runs ſlower. Now if the quantity of matter is as the velocity, the quantity of matter multiplyed into the velocity will be as the velocity multiplyed into itſelf, or as the ſquare of the velocity. But the force, with which the obſtacle is ſtruck, is as the quantity of matter and velocity conjoyntly, or as the quantity of matter multiplyed into the velocity. Therefore that force will likewiſe be as the ſquare of the velocity, with which the ſtream flows.

It is more difficult to ſwim againſt a ſtream that runs faſt, than againſt one that runs ſlow. For our body is a given obſtacle, and conſequently the force, with which the ſtream ſtrikes againſt this given obſtacle, will be as the ſquare of its velocity. So that, if the velocities of different ſtreams are to each other as 1 to 2, the ſquares of thoſe velocities are as 1 to 4; and conſequently the latter ſtream would ſtrike upon us, as we are ſwimming, with 4 times the force of the former; or it would require 4 times the ſtrength to ſwim againſt the latter ſtream, that would be ſufficient to carry us up againſt the former.

From this propoſition we may likewiſe conclude, that, when the obliquity of the ſail of a ſhip is given, the force, with which the wind ſtrikes upon it, is as the ſquare of the velocity with which the air moves. Thus if BE is the ſail, and the wind blows obliquely upon it in the direction HBIC, the baſe of the column of air, that blows againſt it, will

be BC; for all the air, that ſtrikes upon the ſail, will be included within the ſpace HBIE. Whilſt the obliquity is given, the baſe or obſtacle is given, and conſequently the force of the ſtream will be as the ſquare of the velocity, with which the ſtream moves.

195. *The force, with which a river ſtrikes againſt a ſluice, is greater than the force, with which it preſſes upon an equal ſurface of the bank.*

If QR, Plat. XX. fig. 5, is a ſluice ſtanding acroſs a river, and ZQRX is a part of the ſurface of the bank equal to the ſurface of the ſluice; then the ſluice will be more preſſed than this equal part of the bank. The force, with which the bank is preſſed, is equal to the weight of a column of water, whoſe baſe is ZQRX the ſurface preſſed, and whoſe height is half the depth of the river, or half QR, by propoſition 192. But if the river has run far enough for the upper and lower water to have the ſame velocity, by propoſition 188, then the force, with which the ſluice is ſtruck will be equal to the weight of a column of water, whoſe baſe is the ſluice, and whoſe height is equal to the ſpace, through which a heavy body muſt fall to acquire the velocity of the ſtream, by prop. 183. When all the parts of the ſtream move with one and the ſame velocity, if this velocity was the ſame with that, which the water at R would regularly have, then the height of the column, whoſe weight is equivalent to the force of the ſtream would be NR, by propoſition 193: or if the velocity was the ſame with that which the water at Q would regularly have, then the height of this column would be PQ, by propoſition 193. But ſince whilſt the velocity of the ſtream becomes the ſame in all its parts, the lower waters at R are retarded and the higher at Q are accelerated, it follows that the velocity, when it is become the ſame in all parts of the ſtream, will be the middle velocity between that which the water would regularly have at R, and that which it would regularly have at Q. Now the regular velocity at R is what a heavy body would acquire in falling from the level of the ſurface in the reſervoir to R the bottom of the river; and the regular velocity at Q is what a heavy body would acquire in falling from the ſame level to the ſurface of the river, by prop. 184. Therefore the middle velocity is what a heavy body would acquire in falling from the ſame level to the middle between Q and R, or through $PQ + \frac{1}{2}QR$. Thus a column of water, whoſe weight is equivalent to the preſſure againſt the bank ZQRX, and a column, whoſe weight is equivalent to the force with which a ſluice QR is ſtruck, have equal baſes. But the height of the former column is equal to $\frac{1}{2}QR$ or to the diſtance of the ſurface of the river from the middle of the ſtream: whereas the height of the lat-

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ter column is equal to PQ + $\frac{1}{2}$ QR or to the diſtance of the level of the ſurface in the reſervoir from the middle of the ſtream. Therefore, as much as this latter diſtance is greater than the former, ſo much greater will be the weight of the latter column than the weight of the former. But the force, with which the ſtream ſtrikes againſt the ſluice, and the force, with which it preſſes the bank, are reſpectively equal to the weights of thoſe columns, and conſequently the force, with which the ſluice is ſtruck, will be greater than that, with which the bank is preſſed.

196. *The force, with which the wind ſtrikes upon the ſail of a ſhip, when the velocity of the air and the ſize of the ſail are given, will be proportional to the ſquare of the coſine of incidence.*

Let AD, Plat. XX. fig. 4, repreſent the ſail of a ſhip with its edge towards the eye, and let a circle be drawn upon the center K, which is the middle of the ſail, ſo that AD the length of the ſail may be the diameter of the circle. If the wind blows perpendicularly againſt the ſail, then all the air included within the ſpace FAGD will ſtrike upon it. But if the ſail is inclined into the poſition BE, ſo that the wind may blow obliquely upon it, then all the air, that ſtrikes upon it, is included within the ſpace HBIE.

The force, with which the oblique ſail BE is ſtruck, is to the force, with which the perpendicular one AD is ſtruck, as the ſquare of BL to the ſquare of AK, when the ſails are equal and the air ſtrikes againſt them with a given velocity, or as the ſquare of the radius to the ſquare of the coſine of incidence. That AK is the radius or ſemidiameter of the circle ABDE is evident: and that BL is the coſine of the angle of incidence may eaſily be ſhewn; for let LK be drawn parallel to HB, which is the direction in which the wind is ſuppoſed to blow, and let BC be drawn perpendicular to LK, then, ſince LK is the direction in which the wind ſtrikes the ſail, BKL is the angle contained between the direction of the ſtriking body and the ſurface that is ſtruck, and conſequently BKL is the complement of incidence, by propoſition 89 of mechanics. But BL is the ſine of the angle BKL. Therefore BL is the ſine of the complement of incidence, or the coſine of incidence.

Firſt, if it was poſſible that the ſail ſhould be ſtruck with the ſame quantity of air, whether it is in the poſition AD and the wind blows directly againſt it, or in the poſition BE and the wind blows obliquely againſt it; yet, as the ſame quantity of air would ſtrike the ſail obliquely in one caſe and directly in the other caſe, the quantity of the oblique ſtroke would be to the quantity of the direct ſtroke, as the coſine of incidence to the radius, or as BL to AK, by prop. 97 of mechanics.

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Secondly, if it was poſſible for the quantity of an oblique ſtroke upon the ſail BE to be equal to the quantity of a direct ſtroke upon the ſail AD, yet if the quantities of air that ſtrike the ſails are different, this would make the ſtrokes unequal. Now, when the ſail is in the poſition AD and is ſtruck directly, the column of air FAGD, that ſtrikes it, has AD for its baſe; and, when it is in the poſition BE and is ſtruck obliquely, the column HBIE, that ſtrikes it, has BC for its baſe. Therefore the quantity of matter in the latter caſe, when the ſail is oblique, will be to the quantity of matter in the former caſe, when the ſail is perpendicular to the wind, as BC to AD, or as $\frac{1}{2}$ BC to $\frac{1}{2}$ AD, or as BL to AK. But, ſince the velocities of the wind in either caſe are ſuppoſed to be the ſame, the moments or forces, with which the ſails are ſtruck, will be as the quantities of matter, or as BL to AK, or as the coſine of incidence to the radius.

Thus we have ſeen that the force, with which the wind ſtrikes the ſail BE obliquely, is to the force, with which it ſtrikes an equal ſail AD directly, as BL to AK, or as the coſine of incidence to the radius, upon two accounts: firſt, becauſe an oblique ſtroke is to a direct one in this proportion; and ſecondly, becauſe the quantity of air, which ſtrikes the oblique ſail, is to the quantity, which ſtrikes a direct one, in the ſame proportion. Conſequently upon both accounts together the oblique force is to the direct one as BL to AK, and as BL to AK conjoyntly, or as BL×BL to AK×AK, or as the ſquare of BL the coſine of incidence to the ſquare of AK the radius.

This is equally true in all other oblique poſitions of the ſail, as well as in the poſition BE: for the ſame reaſoning may be applyed in all other oblique poſitions. Therefore univerſally, the force of the wind, when it blows obliquely upon a ſail, is to the force of it, when it blows directly with the ſame velocity upon an equal ſail, as the ſquare of the coſine of incidence to the ſquare of the radius.

This is the proportion, which the oblique force of the wind always bears to its direct force. But then the oblique force will be different according to the different obliquity of the ſail. And the propoſition affirms that this force will always vary as the ſquare of the coſine of incidence varys.

Now the length of the ſail AD is given, by the ſuppoſition, and conſequently half its length, or AK, or the radius, is given; ſo that in all obliquities of the ſame ſail the ſquare of the radius muſt likewiſe be a given quantity. Therefore the proportion, which the ſquare of the coſine of incidence bears to this given quantity, muſt vary as the ſquare of the co-

cosine of incidence varys. But the force of the wind in different obliquities of the same sail varys in the proportion of the square of the cosine of incidence to the square of the radius, as has been shewn already: and consequently the force of the wind in different obliquities of the same sail varys as the square of the cosine of incidence varys, or is proportional to the square of the cosine of incidence.

CHAP. XV.
Of capillary tubes.

197. *The ascent of water into capillary tubes, that are made of glass, is not owing to the pressure of the air.*

LET *hl*, Plat. XX. fig. 11, be a very small tube, that is made of glass, and is open at both ends. Then if the end *l* is dipped into a vessel of water, the water will rise into the tube as high as *f*. This ascent of the water cannot be owing to the pressure of the air. For, since the tube is open at the top *h*, the air will press as much through the tube upon that part of the surface of the water in the vessel, which is immediately under the tube, as it presses any where else: and consequently the surface, when it is so pressed equally in every part, will not be prevented by these equal pressures from continuing level, by proposition 14. Therefore the water under the tube cannot be raised above the level of the rest, or cannot be made to rise into the tube by these pressures. This reasoning will be farther confirmed by tryal. For the water ascends into such tubes not only in the open air, but in an exhausted receiver: and we are sure that the ascent of the water, when the tube and vessel are in an exhausted receiver, cannot be owing to the pressure of the air.

198. *The quantities of water, that ascend into different tubes, are as the diameters of the tubes.*

If tubes have unequal diameters, as the tubes *hl* and *kn* have, Plat. XX. fig. 11, 12, the water will rise into these tubes to different heights, and the heights are found to be inversely as the diameters of the tubes. Thus *lf*, the height to which the water rises in the smaller tube *hl*, is found to be as much greater than *nm*, the height to which it will rise in the greater tube *kn*, as the diameter of the tube *hl* is less than the diameter of the tube *nm*: so that if the diameters of those tubes are to each other as 1 to 5, the heights to which the water will respectively rise in them will be as the reciprocals of these diameters or as

$\frac{1}{1} = 1$ to $\frac{1}{5}$, or the water will riſe 5 times as high in the leſſer tube as it does in the greater. Univerſally, if the diameter of the tube is called D, the height, to which the water will riſe, is as the reciprocal of D, or as $\frac{1}{D}$.

The columns of water that aſcend into theſe capillary tubes are cylinders, and the baſes of them, if the tubes are perpendicular to the ſurface of the water in the veſſel, will be circles. But the contents of all cylinders are to each other as their baſes and perpendicular heights conjoyntly. Euc. b. XII. prop. 11. And the baſes, which are circles, are as the ſquares of the diameters of the tubes, Euc. b. XII. prop. 2. Therefore the contents or quantities of water in the tubes, will be as the ſquares of the diameters and as the heights conjoyntly. Now we have already obſerved that the heights, to which the water aſcends, will be inverſely as the diameters: and conſequently the quantities of water, that riſe into different tubes, will be as the ſquares of the diameters directly and as the diameters inverſely, or as the ſquares of the diameters divided by the diameters. But if the ſquares of the diameters are divided by the diameters the quotients will be the diameters. And the quantities of water, that riſe into different tubes, are as theſe quotients. Therefore the quantities of water are as the diameters of the tubes.

Thus if the diameters of the two tubes *hl* and *kn* are to each other as 1 to 5; the heights of the water will be as $\frac{1}{1}$ to $\frac{1}{5}$ and the baſes will be as 1 to 25. But the contents or quantities of water, when the heights are as $\frac{1}{1}$ to $\frac{1}{5}$ and the baſes as 1 to 25, will be as the baſes and heights conjoyntly, or as $1 \times \frac{1}{1} = 1$ to $25 \times \frac{1}{5} = \frac{25}{5} = 5$. Therefore when the diameters are as 1 to 5, the quantities will likewiſe be as 1 to 5, or as the diameters.

Univerſally; if the diameter of the tube is called D, the baſe will be as DD and the height of the water as $\frac{1}{D}$. But the quantities or contents of cylinders are as the baſes and perpendicular heights conjoyntly. Therefore the quantities of water will be as $DD \times \frac{1}{D} = \frac{DD}{D} = D$, that is, the quantities of water will be as D or as the diameter of the tube.

199. *In all capillary tubes the inward ſurface, which contains the water, is equal.*

Though the water riſes higher in the tube *hl* than in the tube *kn*, Plat. XX. fig. 11, 12, yet the inward ſurfaces of that part of each tube, which

which is filled with water, are equal to each other. One tube is filled with water to the height *fl* and one tube to the height *mn*. Both *fl* and *mn* are hollow cylinders, in which the water is contained; and the inside surfaces of these hollow cylinders are the surfaces which we are now speaking of, and which we affirm to be equal. The area of the base of each tube is the base of the whole space that is filled with water in the tube: but the circumference of the base of each tube is the base of that hollow cylindrical surface, in which the water is contained. Now the circumferences of circles are as their diameters: and consequently the bases of these hollow surfaces are respectively as the diameters of the tubes. But the heights of the containing surfaces, being equal to the heights to which the water ascends, are inversely as the diameters. Therefore the heights of these surfaces are inversely as their bases. But surfaces, whose heights are reciprocally as their bases, are equal to each other. Euc. b. VI. prop. 14. Therefore the containing surfaces in these tubes are equal.

Let the diameter of the tube *hl* be to the diameter of the tube *kn* as 1 to 5, then the circumferences of their bases, being to each other as the diameters, will be as 1 to 5. But these circumferences are the bases of the containing surfaces. Therefore the bases of the containing surfaces are to each other as 1 to 5. But the heights of the water and consequently of the containing surfaces are inversely as the diameters or as $\frac{1}{1}$ to $\frac{1}{5}$. And the surfaces themselves are as their heights multiplyed into their bases respectively. Euc. b. VI. prop. 23. Therefore the containing surface in one tube is to the containing surface in the other tube as $\frac{1}{1} \times 1 = 1$ to $\frac{1}{5} \times 5 = \frac{5}{5} = 1$.

Call the diameter of any tube D, then the circumference of it, which is the base of the hollow cylindrical surface that contains the water, is as the diameter or as D, and the height to which the water rises, or the height of that surface, in which it is contained after it has risen, will be as the diameter inversely or as $\frac{1}{D}$. The containing surface is as its base and its perpendicular height conjoyntly, Euc. b. VI. prop. 23, or as $D \times \frac{1}{D} = \frac{D}{D}$. But $\frac{D}{D}$ is a given or invariable quantity being always equal to 1. Therefore the containing surface is the same or a given quantity in all capillary tubes whatever.

200. *The water in capillary tubes is not supported by any attraction of the whole surface of glass that contains it.*

The force with which water endeavours to deſcend in any cylindrical veſſel, ſuch as a capillary tube, is proportional to the quantity of water contained in the veſſel, by propoſition 17. Now the quantities contained in different capillary tubes are as the diameters of the tubes, by propoſition 198, and conſequently the force with which the water, that is ſupported, endeavours to deſcend is likewiſe as the diameters of the tubes. The cauſe muſt always be proportional to the effect, or as the moment of the water, that is ſupported in one tube, is to the moment in any other tube, ſo the cauſe, that ſupports the water, in the former tube muſt be to the cauſe that ſupports it in the latter. But the containing ſurfaces are equal in all tubes, by propoſition 199. Therefore, ſince the moments of water are different in tubes of different diameters, but the containing ſurfaces and conſequently the attractions of theſe ſurfaces are equal in all tubes, and ſince equal cauſes cannot produce unequal effects, it follows, that the water in capillary tubes cannot be ſupported by the attractions of the whole ſurfaces of glaſs, in which that water is contained.

201. *The water is made to riſe into capillary tubes, and is ſupported in them, when it is raiſed, by the attraction of a ring of glaſs that is contiguous to the top of the column of water, and the breadth of this ring is equal to the diameter of the ſphere of attraction of any ſingle particle of the glaſs.*

Water will adhere to glaſs, and the force, which makes it thus adhere, we call attraction. Now if a capillary or ſmall tube made of glaſs, ſuch as ABCD, Plat. XX. fig. 10, is dipped into a baſon of water; the water, which is under the tube at CD, is attracted upwards by the glaſs that is above it, whilſt the other parts of the water are not attracted upwards at all. Conſequently that part of the ſurface, where the water is thus attracted, will riſe above the level of the reſt and will enter into the tube. The ſame cauſe, that makes the water riſe at firſt, will continue to raiſe it, till the weight of the water becomes equal to this attraction.

Now it is not the attraction of the whole inward ſurface of the tube that raiſes the water and ſupports it: this has been ſhewn already under propoſition 200, and will be made farther evident by ſhewing that it is the attraction of only a ring of this ſurface contiguous to the upper part of the water, which ſupports the weight of the column that riſes into the tube. Let *l*C be the height of that column, then *yz wx* or any other part of the glaſs that is far below the top of the column, attracts the water,

ter, that is above it, as much downwards, as it attracts the water, that is below it, upwards: and conſequently any ſuch part of the tube can contribute nothing towards the ſupport of the column *l*C. The water can be ſupported by no other part of the tube but a ring that is contiguous to *lm* the top of the column: for ſuch a ring will attract upwards, but does not attract any part downwards, becauſe there is no part of the column above it. The breadth of this ring may be thus determined. If *l* is a particle of glaſs, which would attract water at the diſtance *le* from it one way and at the diſtance *lr* the other way, ſo that *le* or *lr* is the ſemidiameter, and *er* is the diameter of the ſphere, through which the attraction of this particle extends all round it; then the breadth of the ring will be equal to *er*. From *e* to *y* let circles be conceived to be drawn round the tube at equal diſtances from each other: as the edges of theſe circles are towards the eye, they may be repreſented by the lines *ef*, *gh*, *ik*, *lm*, *no*, *pq*, *rs*, *tu*, *wx*, *yz*. Theſe imaginary circles from *lm* to *yz* divide the water into laminas or thin plates, and the thickneſs of each plate is *ln*, *np*, *pr*, *rt*, *tw*, *wy*. Let us ſuppoſe that the attraction of any ſingle particle of glaſs reaches both ways through a ſpace equal to the thickneſs of three of theſe plates; thus let us ſuppoſe that the attraction of the particle *r* reaches from *r* to *y* one way, and from *r* to *l* the other way; that the attraction of the particle *p* reaches from *p* to *w* one way, and from *p* to *i* the other way; and ſo on.

Now it is evident that neither the particle *r* nor any other particle in the circle *rs* can contribute at all towards the ſupport of the column *l*C: for as the circle attracts as much of the column upwards, as lies between *rs* and *yz*, ſo it attracts an equal part of it downwards or as much of it as lies between *rs* and *lm*. The ſame may be ſaid of any other particle or of any other circle of particles below *rs*: they attract as much of the column downwards as they attract upwards, and conſequently the column cannot be ſupported by their attraction. But if the diſtance of any particle *p* or any circle of particles *pq* from the top of the column is leſs than *lr*, which is the ſemidiameter of a ſingle particles ſphere of attraction, then theſe particles contribute ſomething towards the ſupport of the water. For ſuch particles attract three laminas upwards from *p* to *w* and only two downwards from *p* to *l*, which is the top of the column: indeed the attraction of any particle in the circle *pq* reaches from *p* to *i*, which is higher than *l*; but as there is no part of the column above *l*, any attraction above *l* will not affect the water. In like manner the particles in the circle *no* attract three laminas of water upwards from *n* to *t*, and only one lamina downwards; for there is only one lamina above this

circle. And for the ſame reaſon the particles in the circle *lm* attract three laminas upwards, from *l* to *r*, and none downwards. Thus we have ſeen that all the particles of glaſs from *rs* to *lm* contribute to the ſupport of the water, for they attract it more upwards than downwards. But *lr* is only half the breadth of the ring, by which the water is ſupported. For if *ip* is equal to *lr*, then all the particles contained in the circle *ik* will attract the water, that lies between *ik* and *pq*, ſo that this circle will attract two laminas of water upwards, becauſe there are two laminas within the ſphere of its attraction; but it will attract no water downwards, becauſe there is none above it. The particles contained in the circle *gh* will likewiſe attract all the water upwards, that lies within the ſphere of their attraction: but if *gn* is the ſemidiameter of this ſphere, there is only one lamina between *lm* and *no* that is within the attraction of theſe particles. But then this circle *gh* has none of the water above it, and conſequently does not attract any of it downwards. The ſame may be ſaid of any other circle that is below *ef*. But if *el* is the ſemidiameter of the ſphere of attraction, this circle of particles *ef* and all circles above it will not contribute to the ſupport of the water.

Thus we have ſeen that all the glaſs from *ef* to *rs* aſſiſts by its attraction to the ſupport of the water. But *er* is the diameter of the ſphere of attraction of any ſingle particle of glaſs. Therefore the breadth of the ring which ſupports the column is equal to this diameter: and half the breadth of it *el* is above the water, and the other half *lr* is below it.

Since then the breadth of this ring is always given, or is the ſame in all tubes, becauſe it is always equal to the diameter of the ſphere of attraction of a ſingle particle of glaſs, the number of particles contained in it will be as its circumference. But the circumference of the ring is the ſame as the circumference of the tube: and the circumference of the tube is proportional to the diameter of the tube. Therefore the number of particles and conſequently their attraction will be as the diameter of the tube. Now we have already ſhewn, in propoſition 198, that the weight of water, which is ſupported in any tube, is proportional to the diameter of the tube. Therefore ſince, as we have ſeen, the glaſs does attract the water, ſince no part of it can contribute to the ſupport of the water but ſuch a ring as is contiguous to the top of the column, and ſince the attraction of ſuch a ring is proportional to the weight of water that is ſupported; we may conclude that this attraction is what raiſes the water into the tube at firſt, and ſupports it afterwards.

The aſcent of water into the capillary pores of a ſpunge, and the aſcent of oyl or melted tallow into the wick of a lamp or candle, are owing

ing to the ſame cauſe with the aſcent of water into capillary tubes made of glaſs. From hence likewiſe we may underſtand, why, if any part of a piece of cap-paper or of ſugar is wetted, the moiſture will be propagated along the pores of it to the other parts.

The reader muſt obſerve, that *er*, which we have here ſuppoſed equal to the diameter of the ſphere of attraction of any ſingle particle of glaſs, is in this figure made much two large, in order to keep the parts of the figure ſufficiently diſtinct.

202. *If a capillary tube is firſt filled with water, and the lower end is not dipped into a baſon of water, the column, that will be ſupported, is ſomething longer, than would have riſen ſpontaneouſly into the ſame tube.*

Let *Cl*, Plat. XX, fig. 10, be the height to which the water would riſe into the tube AC, when the end CD is dipped in a baſon of water. Then, if inſtead of dipping the end CD in water, water is poured into AB the upper end of the tube, and the end CD is not dipped into water at all, the column, which will be ſupported, will be longer than *Cl*.

As the glaſs attracts the water, ſo the particles of water attract one another, and conſequently the water in the baſon attracts the water in the tube. Now if the attraction between the water in the tube and that in the baſon was equal to the attraction between the glaſs and the water, theſe equal forces would deſtroy each other, and no water would riſe; or if the tube had been filled with water, upon dipping the end of it into the baſon all the water would fall out by its own weight. But if the water in the tube is more attracted by the glaſs than it is by the water in the baſon, the weight of water that riſes and is ſupported, when the end of the tube is dipped into the baſon, will be proportional to the difference between theſe two attractions. Whereas if the tube has water poured into AB the upper end of it, and the lower end CD is not dipped into the water in the baſon, the weight of the water, that is ſupported, will be proportional to the attraction between the water and the glaſs. Now as this attraction muſt be greater than the difference between this attraction and the attraction of the water in the baſon upon that in the tube; it follows that a greater weight of water, and conſequently a longer column will be ſupported in the latter caſe than in the former.

203. *When the water in a capillary tube has aſcended to its uſual height, if the tube is laid in a horizontal poſition, the water will move a little way towards the middle of the tube, ſo as to leave a ſmall part empty at that end which was dipped in the baſon.*

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circle. And for the ſame reaſon the particles in the circle *lm* attract three laminas upwards, from *l* to *r*, and none downwards. Thus we have ſeen that all the particles of glaſs from *rs* to *lm* contribute to the ſupport of the water, for they attract it more upwards than downwards. But *lr* is only half the breadth of the ring, by which the water is ſupported. For if *ip* is equal to *lr*, then all the particles contained in the circle *ik* will attract the water, that lies between *ik* and *pq*, ſo that this circle will attract two laminas of water upwards, becauſe there are two laminas within the ſphere of its attraction; but it will attract no water downwards, becauſe there is none above it. The particles contained in the circle *gh* will likewiſe attract all the water upwards, that lies within the ſphere of their attraction: but if *gn* is the ſemidiameter of this ſphere, there is only one lamina between *lm* and *no* that is within the attraction of theſe particles. But then this circle *gh* has none of the water above it, and conſequently does not attract any of it downwards. The ſame may be ſaid of any other circle that is below *ef*. But if *el* is the ſemidiameter of the ſphere of attraction, this circle of particles *ef* and all circles above it will not contribute to the ſupport of the water.

Thus we have ſeen that all the glaſs from *ef* to *rs* aſſiſts by its attraction to the ſupport of the water. But *er* is the diameter of the ſphere of attraction of any ſingle particle of glaſs. Therefore the breadth of the ring which ſupports the column is equal to this diameter: and half the breadth of it *el* is above the water, and the other half *lr* is below it.

Since then the breadth of this ring is always given, or is the ſame in all tubes, becauſe it is always equal to the diameter of the ſphere of attraction of a ſingle particle of glaſs, the number of particles contained in it will be as its circumference. But the circumference of the ring is the ſame as the circumference of the tube: and the circumference of the tube is proportional to the diameter of the tube. Therefore the number of particles and conſequently their attraction will be as the diameter of the tube. Now we have already ſhewn, in propoſition 198, that the weight of water, which is ſupported in any tube, is proportional to the diameter of the tube. Therefore ſince, as we have ſeen, the glaſs does attract the water, ſince no part of it can contribute to the ſupport of the water but ſuch a ring as is contiguous to the top of the column, and ſince the attraction of ſuch a ring is proportional to the weight of water that is ſupported; we may conclude that this attraction is what raiſes the water into the tube at firſt, and ſupports it afterwards.

The aſcent of water into the capillary pores of a ſpunge, and the aſcent of oyl or melted tallow into the wick of a lamp or candle, are owing

ing to the ſame cauſe with the aſcent of water into capillary tubes made of glaſs. From hence likewiſe we may underſtand, why, if any part of a piece of cap-paper or of ſugar is wetted, the moiſture will be propagated along the pores of it to the other parts.

The reader muſt obſerve, that *er*, which we have here ſuppoſed equal to the diameter of the ſphere of attraction of any ſingle particle of glaſs, is in this figure made much two large, in order to keep the parts of the figure ſufficiently diſtinct.

202. *If a capillary tube is firſt filled with water, and the lower end is not dipped into a baſon of water, the column, that will be ſupported, is ſomething longer, than would have riſen ſpontaneouſly into the ſame tube.*

Let C*l*, Plat. XX, fig. 10, be the height to which the water would riſe into the tube AC, when the end CD is dipped in a baſon of water. Then, if inſtead of dipping the end CD in water, water is poured into AB the upper end of the tube, and the end CD is not dipped into water at all, the column, which will be ſupported, will be longer than C*l*.

As the glaſs attracts the water, ſo the particles of water attract one another, and conſequently the water in the baſon attracts the water in the tube. Now if the attraction between the water in the tube and that in the baſon was equal to the attraction between the glaſs and the water, theſe equal forces would deſtroy each other, and no water would riſe; or if the tube had been filled with water, upon dipping the end of it into the baſon all the water would fall out by its own weight. But if the water in the tube is more attracted by the glaſs than it is by the water in the baſon, the weight of water that riſes and is ſupported, when the end of the tube is dipped into the baſon, will be proportional to the difference between theſe two attractions. Whereas if the tube has water poured into AB the upper end of it, and the lower end CD is not dipped into the water in the baſon, the weight of the water, that is ſupported, will be proportional to the attraction between the water and the glaſs. Now as this attraction muſt be greater than the difference between this attraction and the attraction of the water in the baſon upon that in the tube; it follows that a greater weight of water, and conſequently a longer column will be ſupported in the latter caſe than in the former.

203. *When the water in a capillary tube has aſcended to its uſual height, if the tube is laid in a horizontal poſition, the water will move a little way towards the middle of the tube, ſo as to leave a ſmall part empty at that end which was dipped in the baſon.*

After

After the water has riſen into the tube ABCD, Plat. XX. fig. 10, to the height C*l*, if this tube is laid horizontally, the water will move towards the middle and will leave a ſmall part of the tube empty at CD.

When the tube is laid horizontally the ring at *lm* attracts the water one way, and the ring at CD attracts it the other way. But then the ring at *lm* is twice as broad as that at CD. For, by propoſition 201, half the breadth of the ring at *lm* is within the waters ſurface, and the other half is beyond it; whereas the water, when the tube is firſt laid down, is cloſe to CD; and conſequently, as there can be no part of the ring beyond the water, this ring will have but half the breadth of the other. From hence it follows that the water is more attracted at *lm* than at CD; and therefore it will move from CD, till ſo much of the tube is left empty as will make the ring at this end equal to that at the other end; that is, the water will move from CD, till the ſpace that it leaves empty is equal in length to *le*; for then, the rings being equal at both ends, the water will be attracted equally both ways and conſequently will reſt.

204. *The water will not run out at the top of a capillary tube, though the tube is ſhorter than the height, to which water would riſe in a longer tube of an equal diameter.*

Though water would riſe in the tube AB CD, Plat. XX. fig. 10, to the height C*l*, yet if an equal tube was no longer than C*r*, the water would riſe in this ſhort tube only to *rs* and would not run out at the top. For as the water is raiſed only by the attraction of the glaſs that is above it; when it has riſen to the top *rs*, there is no more glaſs above it, and conſequently it can riſe no higher, but will ſtop there without running out at the top of the tube.

205. *Water riſes to the ſame perpendicular height in a capillary tube, whether the tube is erect or inclined.*

If DN, Plat. XIX. fig. 8, is a capillary tube, that is open at both ends, and the water will riſe in it to the perpendicular height NL, or to the level AB, when it is erect; then ſuppoſing the ſame or an equal tube to be inclined and to ſtand in the poſition EN, the water will riſe to M or to the ſame level AB, ſo that the perpendicular diſtance of the top of the column from the ſurface of the water in the baſon FG will be the ſame as when the tube was erect.

We have already ſhewn in a ſimilar caſe, under propoſition 130, that there will be more water in the inclined tube than in the erect one, and conſequently as there ſeems to be a greater weight to be ſupported, the

ring

ring, that ſupports the water, ought to be greater in one tube than it is in the other: whereas this ring and conſequently the attraction of it is equal in both.

If we ſay that the water preſſes in proportion to its perpendicular height, and conſequently that the preſſure of the water which is to be ſupported by the attraction of the ring, is equal in both tubes; the anſwer is obvious: for water preſſes in proportion to its perpendicular height, only when the baſe of the containing veſſel is given, by propoſition 17. But the baſe of the column in the inclined tube is greater than the baſe of the column in the erect one, as appears from what has been ſaid under propoſition 130; aud conſequently, by propoſition 17, the preſſure of the water ought to be greater in the inclined tube than in the erect one. Indeed if the tube DN is inclined into the poſition EN, the ſhape and ſize of the bottom of the tube is not altered by this change of poſition: but the bottom of the inclined tube is not the baſe of the column of water contained in it: for the baſe of the column is the loweſt horizontal ſection; which ſection in the inclined tube is an ellipſis and in the erect one is a circle leſs than that ellipſis.

Should it be ſaid that the ſurface of the water in the inclined tube is level or parallel to the horizon, and conſequently that the ring of glaſs at the top of the inclined column, is an oblique ſection of a cylinder, which being elliptical is bigger than the circular ring of glaſs, which ſupports the water in an equal erect tube; this account of the matter will be as little ſatisfactory as the former. For in the firſt place we may deny the fact; when a capillary tube is inclined, the ſurface of the water is not level, but is inclined to the horizon ſo as to be circular as well in this poſition of the tube, as when the tube is erect. Or if we grant the fact, we may deny the concluſion that is to be drawn from it. For ſuppoſing the ſurface of the inclined column of water to be an horizontal level, then indeed the ring of glaſs, which is at the top of it, will be elliptical and will be bigger than the circular ring, which is at the top of the erect column. But if this was the reaſon why more water is ſupported in the inclined tube than in the erect one, the elliptical ring ought to be as much greater than the circular one, as the quantity of water in the inclined tube is greater than the quantity of water in the erect one. Now the quantities of water in theſe tubes are as the areas of their baſes, the quantity of water in the inclined tube is to the quantity of water in an equal erect tube, as the area of the elliptical baſe of the former is to the area of the circular baſe of the latter. Thus if, when any tube is erect, the baſe of the column contained in it is the area of

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the circle HEID, Plat. XX. fig. 7, then by inclining this tube the baſe of the column may be made equal to the area of the ellipſis AEBD. But the rings, which are ſuppoſed to ſupport the water in the different poſitions of this tube, are to one another only as the circumference of the circle HEID to the circumference of the ellipſis AEBD. And ſince the areas encreaſe in a greater proportion than the circumferences, it follows, that the quantity of water in the inclined tube bears a greater proportion to the quantity contained in the erect one, than the elliptical ring at the top of the former bears to the circular one at the top of the latter. Therefore though we grant that it is an elliptical ring, which ſupports the water in the inclined tube, and a circular ring, which ſupports it in the erect one; yet this will not prove what is intended: ſince if this was the true cauſe why more water ſhould be ſupported in one tube than in the other, the cauſe ought to encreaſe in the ſame proportion with the effect, the attracting rings ought to be proportional to the quantities of water ſupported, or to the baſes of the columns.

That the area of the baſe of any tube encreaſes faſter by inclining the tube than the circumference of that baſe, or than the circumference at the top of the column, may be eaſily ſhewn in one inſtance. If the area of the circle EHCID is the baſe of an erect tube, and this tube is inclined till the baſe of the column contained in it becomes equal to the area of the ellipſis AECBD, ſo that AB the longer axis of this ellipſis may be double ED the diameter of the inſcribed circle, then, the area of the elliptical baſe will be double the area of the circular one, becauſe the area of the ellipſis is to the area of the inſcribed circle as AB the longer axis of the ellipſis to ED the diameter of the circle. But the circumference of the ellipſis is not double the circumference of the inſcribed circle. For ſince the circumferences of circles are to one another as their diameters, and ſince AB is double ED, it follows that the circumference of the circumſcribed circle AFBG is only double the circumference of the inſcribed one HEID. But the circumference of the ellipſis is leſs than that of the circumſcribed circle. Therefore the circumference of the ellipſis is not double that of the inſcribed circle.

Let us now try whether no other reaſon can be given, why the ſame attracting ring ſhould ſupport more water in an inclined capillary tube, than in an equal erect one. We obſerved, in propoſitions 130, 21, that water in an inclined cylindrical veſſel cannot be conſidered as a heavy body upon an inclined plane: becauſe, though the lower ſide of the veſſel ſupports a part of the waters weight juſt as an inclined plane ſupports a part of a bodys weight that is deſcending upon it, yet the water preſſes upwards,

wards, and the reaction of the upper ſide adds as much preſſure upon the baſe of the veſſel as the action of the lower ſide takes off. Now in theſe circumſtances the water preſſes upon the bottom of the veſſel, and there is nothing to keep it from running out but that bottom: ſo that the cauſe, which ſupports the weight of the water, does not act at the top of the column, but the upper water preſſes every where upon the water below it: and it is from this preſſure of the upper water upon the lower that the preſſure againſt the upper inclined ſide, and conſequently the reaction againſt the bottom, ariſes. From hence it is, that though the water in ſuch a veſſel reſts upon an inclined plane, and conſequently the relative weight of the water is leſs than its whole weight, by propoſition 120 of mechanics, yet the preſſure upon the bottom becomes greater than its relative weight and is equal to its whole weight, by propoſition 21. But if the water could be ſupported and be kept from falling out of the veſſel by any force applyed to the top of the column; ſuch a force would prevent the upper parts from preſſing upon the lower: ſo that each part of the water would endeavour to deſcend only by its own weight; and, as it would bear no preſſure from above, it would not preſs either ſideways or upwards, by propoſition 16. This is the caſe of water in an inclined capillary tube: the ring of glaſs, that ſupports it, is at the top of the column; ſo that the upper parts of it being ſuſtained by the attraction of this ring are prevented from preſſing upon the lower parts: and ſince by this means the water in ſuch a tube does not preſs againſt the upper inclined ſide of the tube, the lower ſide will ſupport a part of the waters weight, juſt as an inclined plane ſupports a part of the weight of any other heavy body. From hence it follows that though there is more water ſupported in a capillary tube that is inclined, than in the ſame tube when it is erect, yet the ſame ring of glaſs may be ſufficient to ſupport the water in either caſe; becauſe the lower ſide of the inclined tube is an inclined plane, which ſuſtains the difference between the weight of the water in the inclined tube and the erect one.

304. *If the diameter of a capillary tube is given, at that part which is contiguous to the top of the column of water ſupported in it; the perpendicular height, at which the water will be ſupported, is always the ſame whatever is the diameter of the lower part of the tube.*

The tube *hl*, Plat. XX. fig. 11, which has the ſame dimenſions from one end to the other is called a ſimple one, and the tube *ec*, which is ſmaller at the top than at the bottom, is called a compound one. Suppoſe that the ſimple tube would raiſe water to the perpendicular height *lf*,

then if water is poured into the compound tube, and the lower end of the tube is dipped in ſtagnant water at *c*, the column *dc*, which this tube will ſupport, is equal in height to *lf*, provided the diameter of the compound tube at *d* the top of the column is equal to the diameter of the ſimple tube. The water indeed riſes ſpontaneouſly into the tube *bl* to the height *lf*: but though a column of an equal height will be ſupported in the compound one, yet ſuch a column would not aſcend out of the baſon; the water muſt firſt be put into the tube, and when it is there, the tube will ſupport it.

It is plane that the quantity of water, which is ſupported in this caſe by the compound tube, is greater than what is ſupported by the ſimple one: for the tubes by the ſuppoſition are equal at the top; and the compound tube at the bottom is bigger than the ſimple one. The moment of the water, or force, with which it endeavours to deſcend in the compound tube, is greater than the moment in the ſimple one; as is obvious from one of the firſt principles in hydroſtatics. For the moment, that is ſupported in either tube, is the force with which the water would deſcend, if it was not ſupported, or is the force, with which the water would preſs againſt the reſpective baſes *l* and *c*. But the perpendicular heights are equal, and conſequently, by propoſition 22, theſe preſſures will be as the baſes *l* and *c*. Therefore as much as the baſe of the compound tube is greater than that of the ſimple one, ſo much the moment of the water, that is ſupported in the former, will be greater than the moment, that is ſupported in the latter.

Indeed, in propoſition 10, we have ſhewn that, if a tube is wider in one part than in another, the velocity of water, when it is forced through ſuch a tube will be inverſely as the area of the ſection, through which it is paſſing: and if it would follow from hence that as the quantity of matter at *c* the baſe of the compound tube is greater than at *l* the baſe of the ſimple one; ſo the velocity at *c* will be leſs than at *l* in the ſame proportion, the conſequence would be that the moment in one tube muſt be equal to the moment in the other. For, ſince the quantity of matter in the lower part of the compound tube is to that in an equal part of the ſimple tube as their reſpective baſes, if the velocities at *c* and *l* were inverſely as thoſe baſes, the velocities would then be inverſely as the quantities of matter, and conſequently, the moments would be equal, by propoſition 14 of mechanics. But though we have ſhewn, in propoſition 10, that, when water is paſſing through different parts of one and the ſame tube, the velocity will be inverſely as the area of the ſection, through which it is paſſing; yet we can by no means conclude from hence,

hence, that, when water is paſſing through different tubes, the velocity in any ſection of one tube will be to the velocity in any ſection of the other inverſely as the areas of the ſections. Particularly in the inſtance now before us; if the cauſe, which ſupports the water in theſe tubes, whatever that cauſe is, was to ceaſe acting, and the water was to run out at their reſpective bottoms *c* and *l*, it would begin to run out of each tube with an equal velocity. For, by propoſition 144, the velocity, with which water runs out at the bottom of a veſſel, is as the ſquare root of its perpendicular height above the hole. But the perpendicular height of the water is the ſame in both theſe tubes. Therefore the ſquare roots of that height, and conſequently the velocity of the water, is the ſame in both.

Since therefore the velocity, with which the water would run out of either tube at their reſpective baſes, is the ſame, the moments at the baſes will be as the quantities of matter diſcharged in a given time, by propoſition 13 of mechanics. But the quantities of matter are as the baſes, by propoſition 148. Therefore, if the cauſe, which ſupports the water, was to ceaſe acting, the moment, with which the water would begin to run out of the compound tube, is to the moment, with which it would begin to run out of the ſimple tube, as the baſe of the compound tube to the baſe of the ſimple tube.

Now the attracting ring of glaſs in the compound tube at *d* is equal to the attracting ring at *f* in the ſimple one, by the ſuppoſition. And the moment, with which the water would run out, if this ring was to ceaſe acting, is greater in the compound tube than in the ſimple one, as has been already ſhewn. Therefore, if the ring of glaſs at the top of the column is what prevents the water from running out, the attraction of equal rings muſt produce unequal effects. The attraction of the ring at *f* ſupports a leſſer moment of water, and the attraction of an equal ring at *d* ſupports a greater. This is a very conſiderable objection againſt the opinion, that we have been endeavouring to eſtabliſh, and may ſeem to prove, that the attracting ring at the top of the column is not the cauſe by which the water is ſupported. For the cauſe ought always to be proportional to the effect: equal attractions ought to ſupport only equal moments of water. Whereas if we compare the ſimple tube *hl* with the compound one *ec*, and ſuppoſe that the ring contiguous to the top of the column of water in each tube is the cauſe that ſuſpends the water, we ſhall find, as above, that it is not as cauſe to cauſe ſo effect to effect, for the ſuppoſed cauſes are equal, and the effects are different; the attracting ring in one tube is equal to the attracting ring in the other, but the moment of water ſuſpended is leſs in the ſimple tube than in the compound one.

In the caſe, that we have been explaning the compound tube contains more water than the ſimple one. But there is another caſe, in which the compound tube contains leſs water than the ſimple one. If the ſimple tube *kn*, Plat. XX. fig. 12, would raiſe water to the height *nm*, then, ſuppoſe that at *t* the diameter of the compound tube *ig* is equal to the diameter of the ſimple tube, and if water is put into the compound tube, and it is held in ſuch a manner that *bt* the height of the column of water in it may be equal to *nm*, this water and no more will be ſupported in the tube; ſo that if the tube was drawn farther out of the water in the baſon, the top of the column of water would keep ſinking, till it came into the lower part of the tube, where the diameter is leſs.

The ring of glaſs at *t* the top of the column in the compound tube is equal to the ring at *m* the top of the column in the ſimple one; becauſe the diameters are equal at *t* and *m* by the ſuppoſition. But there is evidently a leſs quantity of water in the compound tube than there is in the ſimple one. And it may be proved from the firſt principles of hydroſtatics that the moment of the water is leſs in the compound tube than in the ſimple one. For the perpendicular height is the ſame in both, and conſequently the water in each will preſs downwards in proportion to the baſe, by propoſition 22. Therefore the moment in the compound tube is as much leſs than in the ſimple tube as *b* the baſe of the column in the compound one is leſs than *n* the baſe of the column in the ſimple one. And though, in propoſition 10, we have proved, that, if a tube is wider in one part than it is in another, the velocity of water, as it moves through thoſe different parts, will be inverſely as the area of the ſection, through which it is paſſing, yet we cannot conclude from hence that the velocity, with which the water would run out of the compound tube, is as much greater than the velocity, with which it would run out of the ſimple tube, as the baſe of the compound one is leſs than the baſe of the ſimple one. If this concluſion could be drawn from propoſition 10, then indeed we might prove that the moment of the water at the baſe of each tube is the ſame. For the quantity of water at the baſe is directly as the area of the baſe; and if the velocity was inverſely as the baſe, the velocity would be inverſely as the quantity of matter, and conſequently the moments would be equal, by propoſition 14 of mechanics. But ſince all that has been proved, in propoſition 10, relates to different parts of one and the ſame tube, and not to different parts of different tubes; we cannot conclude from thence that the velocity of water running out through the ſections *b* and *n*, in different capillary tubes, will be inverſely as the areas of thoſe ſections. Nay, by propoſition,

poſition 144, it appears that, if the water was to run out of theſe tubes, the velocity of it at the baſes *b* and *n* would be equal. For the velocity is as the ſquare root of the perpendicular heights: and ſince the heights *bt* and *nm* are equal, the ſquare roots of thoſe heights, and conſequently the velocities of the water at *b* and *n*, will be equal. But ſince the velocities at *b* and *n* are equal, the moments will be as the areas of the orifices at *b* and *n*, by propoſition 147. Therefore the moment of the water in the compound tube is to the moment of it in the ſimple tube, as the baſe of the compound tube to the baſe of the ſimple one. Now, if the water in a capillary tube is ſuſpended by the attraction of a ring of glaſs contiguous to the top of the column, this attraction is what keeps the water from running out of the tube, and conſequently muſt be equivalent to the moment, with which it would begin to run, ſuppoſing this attraction was to ceaſe acting. But the ring, which is contiguous to the top of the column in the compound tube at *t*, is equal to the ring, which is contiguous to the top of the column in the ſimple tube at *m*: and conſequently theſe attractions are equivalent to equal moments. Why therefore ſhould an equal attraction ſupport a leſs moment of water in the compound tube and a greater moment in the ſimple one?

The water is ſuſpended by compound capillary tubes in the manner already deſcribed as well in an exhauſted receiver as in the open air. From whence we may conclude that the water is not ſupported by the preſſure of the atmoſphere. Indeed a receiver can never be ſo perfectly exhauſted, but ſome air will be left behind, by propoſition 90. But the ſmall remainder of air, that is left after frequent pumping, will not be ſufficient to ſupport the water to ſuch a height as it has been found to ſtand at in compound capillary tubes: for Dr. Jurin found that in a tube, which was large at the bottom and drawn out into a fine capillary at the top, water might be ſuſpended to the height of 35 inches. This is $\frac{1}{12}$ of the height to which water could be ſupported by the preſſure of the whole atmoſphere, by propoſition 46: and, as a receiver may be exhauſted till much leſs than $\frac{1}{12}$ of the air is left in it, the preſſure of this remaining air will not be ſufficient to ſupport the water to the height of 35 inches.

If it is the preſſure of the air remaining in the receiver, which ſupports a greater quantity of water in the tube *dc* than the attraction of the ring at *d* could ſupport alone; it muſt be becauſe the ring at *d* is ſufficient to ſupport the drop of water that is contiguous to it, ſo as to prevent the air from preſſing down through the tube upon the water in the lower part; and then the water in the lower part of the tube having no preſſure from above will be ſupported in the ſame manner as mercury

or water would be in the Torricellian experiment. But if this was true, we ſhould find that if the tube *dc*, inſtead of having a capillary at *d*, was ſealed up at that end, as the tube of a common barometer is, the water might ſtill be ſupported at the ſame height in an exhauſted receiver, as if there was a capillary there. For the remaining air in the receiver could no more preſs downwards upon the water in the tube, when it is ſealed at the top, than it can, when there is an open capillary at the top: and conſequently, if the air, which remains in the receiver, would be ſufficient to ſupport the water in one caſe, it would likewiſe be ſufficient to ſupport it in the other.

In the compound tube *th* a leſs quantity of water is ſupported than in the ſimple one *mn*, but the rings of glaſs at *t* and *m* are equal. Here the difficulty is to ſhew from whence it ſhould happen that the ring *t* in the compound tube ſhould not ſupport as great a weight as the ring *m* in the ſimple one. Now if the preſſure of the ſmall remainder of air in an exhauſted receiver is the cauſe why more water ſhould be ſupported in the compound tube *dc* than in the ſimple one *fl*; this preſſure muſt likewiſe be the cauſe why leſs ſhould be ſupported in the tube *th* than in the tube *mn*. But this preſſure cannot make the weight ſupported in the compound tube equal to the weight ſupported in the ſimple one. The tube *th* is indeed wider at the top than it is at the bottom: and conſequently we may imagine that the air in the receiver preſſes more downwards upon the ſurface at *t*, than it preſſes upwards againſt the ſurface at *h*. Whereas the tube *mn* is of the ſame dimenſions both at the top and at the bottom, ſo that in this tube the air preſſes equally both downwards and upwards. It is evident, upon this ſuppoſition, that theſe equal preſſures in contrary directions upon the tube *mn* will deſtroy one another, and the ring of glaſs at *m* will have nothing to ſupport but the weight of the water, that is contained in the tube. But we cannot ſay that, ſince the preſſure downwards upon the ſurface *t* in the compound tube is greater than the preſſure upwards againſt *g* the baſe of this tube, therefore the difference of theſe preſſures will make the weight, which the ring *t* is to ſupport, equal to the weight, which the ring *m* ſupports. For the queſtion is, how it comes to paſs, that the weight in *rb* the lower part of the compound tube *th* ſhould be equal to the weight in the lower part of the ſimple one *mn*. And though we grant, that the ſurface *t* is more preſſed downwards than the baſe *g* or *h* is preſſed upwards, yet this does not account for the difficulty. Becauſe this difference of preſſure will be ſupported by the bottom of the upper part of the tube, where the two parts joyn, and the upper ſurface at *r* will be no more preſſed downwards than

the

the lower part at *b* is preſſed upwards. Therefore upon that part of the tube, where we want to account for the encreaſe of weight, there is no difference of preſſure at all.

That this is an unſatisfactory account of the matter will appear farther from conſidering that the difference of theſe preſſures, ſuppoſing there was any ſuch difference, cannot be the ſame in an exhauſted receiver that it is in the open air. Let us ſuppoſe that the ſurface at *c* is to the ſurface at *b* as 10 to 1: then the preſſure downwards will be 10 times as great as the preſſure upwards; and conſequently the difference of theſe preſſures will be $\frac{9}{10}$ of the weight of the atmoſphere, when the tube is in the open air. But when the ſame tube is in an exhauſted receiver, this difference will be only $\frac{9}{10}$ of the weight of the ſmall quantity of air that is left in the receiver after exhauſting. It is plane therefore that the difference in the former caſe will be greater than in the latter. And conſequently, if we were to grant that the difference of preſſures, when the tube is in the open air, would make the weight to be ſupported in the compound tube as great as that in the ſimple one, yet in the receiver, where this difference is leſs, it could not make the weight equal.

Let us now endeavour to find out ſome other way of removing this difficulty. If the force, whatever it is, that ſupports the water, in the compound tube *ec* and in the ſimple one *hl*, was to ceaſe acting, the water would then run out of them: and the quantities diſcharged from each of them in an equal time would be as their reſpective orifices, that is, as their reſpective baſes, by propoſition 147. But as the water is diſcharged at the bottom of the tube, the top of the column keeps ſinking; and the ſpaces deſcribed by the top of the column, whilſt it ſinks, will be proportional to the quantities diſcharged, as has been obſerved in propoſition 162. Therefore the ſpaces deſcribed in equal times by the top of the column in each tube will be as the reſpective baſes of the tubes. Now, the velocities of bodies are as the ſpaces deſcribed by them in equal times, by propoſition 9 of mechanics: and conſequently the velocity of the water at the top of the column of each tube will be as the reſpective baſes; the velocity, with which the water will deſcend at *d* the top of the column in the compound tube *ec*, will be as much greater, than the velocity, with which the water will deſcend at *f* the top of the column in the ſimple tube, as the baſe of the compound tube is greater than the baſe of the ſimple one. From hence we may ſhew that though the attracting rings at the top of each column are equal to one another, yet the moments, with which theſe columns are attracted, will be proportional to the baſes of the tubes, or to the moments, with which the wa-

ter

ter endeavours to deſcend. A very ingenious friend of mine is of opinion that this may be proved in the following manner. The moment of that attraction, by which a column of water is ſupported in a capillary tube, is proportional to the force of attraction, that would act upon it in a given time, if it was falling out of the tube. Now, where the ſize of an attracting ring is given, the force of attraction, that would act in a given time upon the column of water, if it was falling out of the tube, would be proportional to the number of theſe rings that it muſt paſs by in a given time as it falls: for the greater the number of attracting rings is, that act in a given time, the greater will be the force of attraction exerted upon the water, and the leſs the number of rings, the leſs the force of attraction. But the number of rings, that the top of the column of water would paſs by in a given time, if it was to fall out of any tube, would be as the velocity, with which the top of the column moves; the greater the velocity, the more attracting rings it would paſs by in a given time, as it falls, and the leſs the velocity, the fewer ſuch rings it will paſs by in an equal time. Since then the moment, with which the water would be attracted, if it was falling out of the tube, is as the number of rings that it would paſs by in a given time, and ſince the number of rings is as the velocity, with which the top of the column would begin to move; it follows, that the moment, with which the water would be attracted, if it was falling, would be proportional to the velocity of the top of the column. But the velocity of the top of the column, is as the baſe of the tube, as we have proved already. Therefore the moment with which the water would be attracted, if it was to begin to fall, would be as the baſe of the tube. Now the moment, with which the water endeavours to deſcend out of a capillary tube, where the height is given, has been ſhewn to be as the baſe of the tube: but it cannot deſcend without being attracted by a force that is likewiſe proportional to the baſe of the tube: and conſequently, if this force of attraction is ſufficient to ſupport the water in the ſimple tube *hl* it will be ſufficient to ſupport it to the ſame height in the compound tube *ec*; becauſe though the moment of water is greater in the compound tube than in the ſimple one, yet the moment of the attraction that would act upon it, if it was to begin to fall, is greater in the ſame proportion.

This method of reaſoning is applicable to a compound tube *ig* that is leſſer at the bottom than at the top. The water, if it was to fall out of the compound tube *th*, or out of the ſimple one *mn*, would begin to move with the ſame velocity at the bottom of each tube, by propoſition 144, becauſe the perpendicular height is the ſame in both. But then

then the quantities diſcharged in equal times at *n* and *b* will be as the baſes, by propoſition 148: and the ſurfaces *m* and *t* ſink proportionally to the quantities diſcharged. Therefore the velocities, with which theſe ſurfaces ſink, or the ſpaces, which the tops of the columns deſcribe, are as the baſes. But the number of attracting rings, which thoſe ſurfaces would paſs by, in a given time, as they ſink, are as their reſpective velocities: and the moments, with which they would be attracted in falling, are as the number of attracting rings, that they paſs by in a given time. Therefore the moments with which the columns would be attracted, if they were to begin to move, would be as their velocities, that is, as the baſes of the tubes. From hence it follows that, as the force, with which the water is attracted in the ſimple tube *mn* prevents it from falling; becauſe the attraction is equal to the moment with which it would begin to fall; ſo the force, with which the water is attracted in the compound tube *th*, will be only juſt ſufficient to keep it from falling; becauſe, though the moment, with which the water would begin to fall, is leſs in the compound tube than in the ſimple one in proportion as the baſe is leſs, yet the attraction at the top of the column is leſs in the ſame proportion.

305. *When water has aſcended into the wider part of a conical tube, if the tube is laid in a horizontal poſition, the water will run to the leſſer end of it.*

If a capillary tube that is cylindrical is laid in a horizontal poſition, the water runs only a little way from the end towards the middle of the tube by propoſition 203. But if the bore of the tube is conical, the water, when the tube is laid along horizontally, will run quite to the ſmaller end. In order to ſhew the reaſon of this, we muſt prove that the water is attracted more forcibly towards the narrower end of the tube than towards the wider end.

When the water has aſcended into the tube, and the tube is firſt laid horizontally, there is a larger ring of glaſs at the wider end of the column of water, than there is at the narrower end, as is plane from the figure of the tube. And upon this account the water will be more attracted towards the wider end of the tube than towards the narrower. Call the diameter of the tube at the wider end D and at the narrower end d. Then theſe rings, and conſequently their attractions, if all other circumſtances are equal, will be as D to d reſpectively, by what has been ſaid under propoſition 201. But the velocity of the water at each end of the column will be inverſely as the ſections of the tube, by propoſi-

tion 10. And the ſections of the tube at each end of the column are as the ſquares of the diameters, or as DD to dd. Euc. b. XII. prop. 2. Therefore the velocities, with which the water would begin to move at each end, are as the ſquares of the diameters inverſely, or as $\frac{1}{DD}$ $\frac{1}{dd}$. Now we have ſhewn, under propoſition 304, that the moment, with which water is attracted in a capillary tube, where all other circumſtances are equal, is as the velocity, with which it would begin to move if it was to fall out: and conſequently the moments of the attraction at each end of the column in a conical tube are inverſely as the ſquares of the diameters, or as $\frac{1}{DD}$ to $\frac{1}{dd}$.

Thus we have ſeen, firſt that the attractions at each end are as D to d, becauſe the attracting rings are in this proportion; and ſecondly the attractions at each end are as $\frac{1}{DD}$ to $\frac{1}{dd}$, becauſe the velocities are in this proportion. Therefore the attractions at each end, upon both accounts together, will be in a ratio compounded of D to d and $\frac{1}{DD}$ to $\frac{1}{dd}$, or as $D \times \frac{1}{DD}$ to $d \times \frac{1}{dd}$, or as $\frac{D}{DD}$ to $\frac{d}{dd}$, or as $\frac{1}{D}$ to $\frac{1}{d}$, that is, the attractions will be inverſely as the diameters. Conſequently, ſince the attraction is the greateſt at that end where the diameter is leaſt, the water will begin to move towards the leſſer end of the tube, and will, for the ſame reaſon, continue to move that way till it comes cloſe to the leſſer end.

The end of Hydroſtatics.

PLATE XX.

Fig. 1.

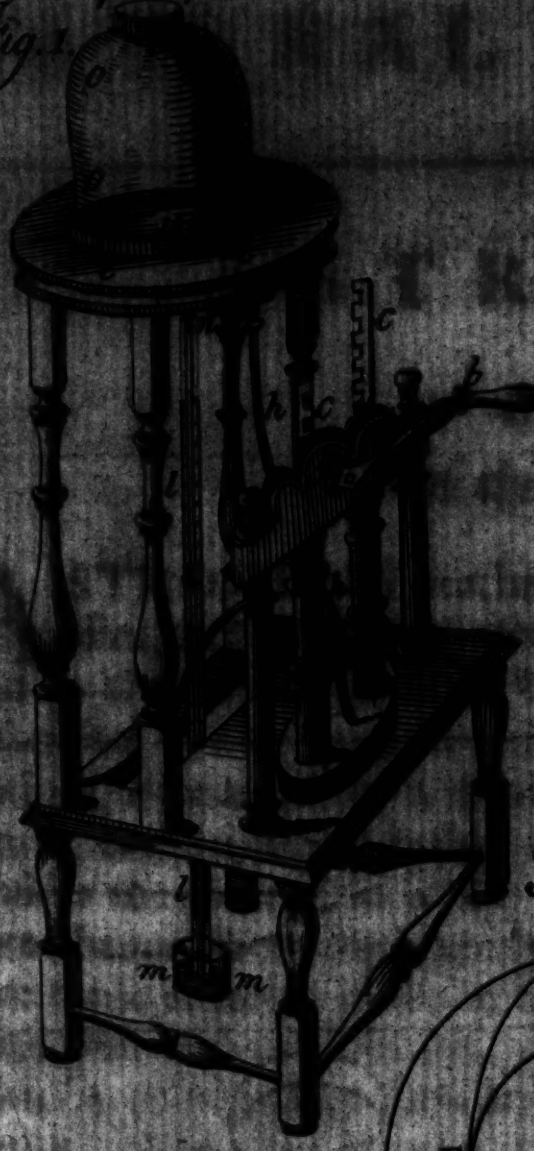


Fig. 2.

Fig. 3.

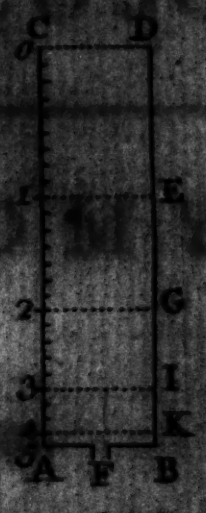


Fig. 4.

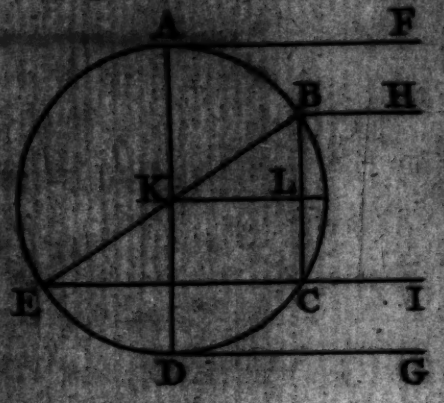


Fig. 5.

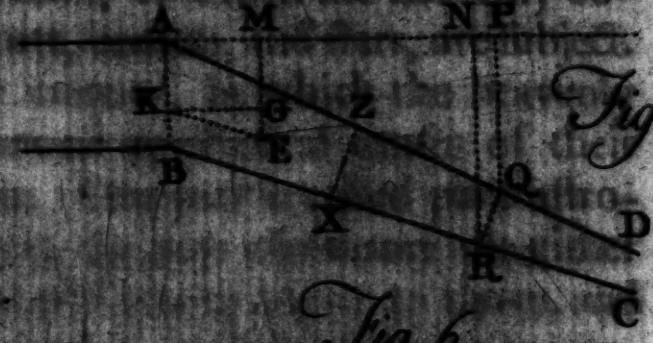


Fig. 6.

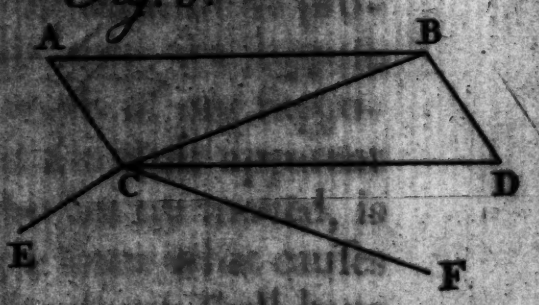


Fig. 7.

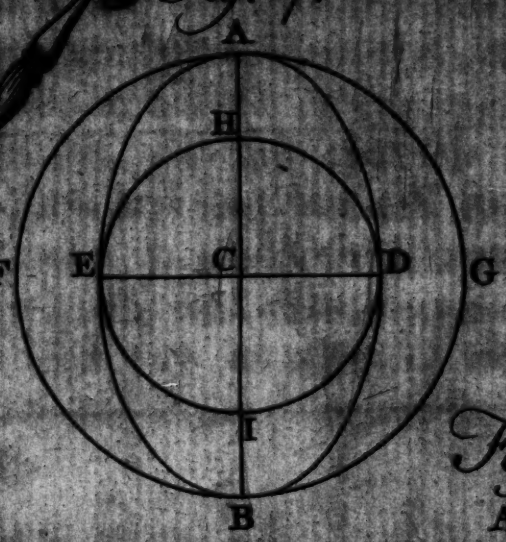


Fig. 8.

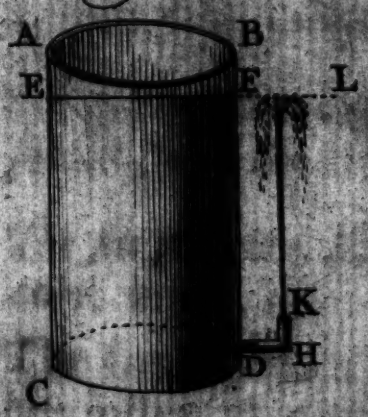


Fig. 9.

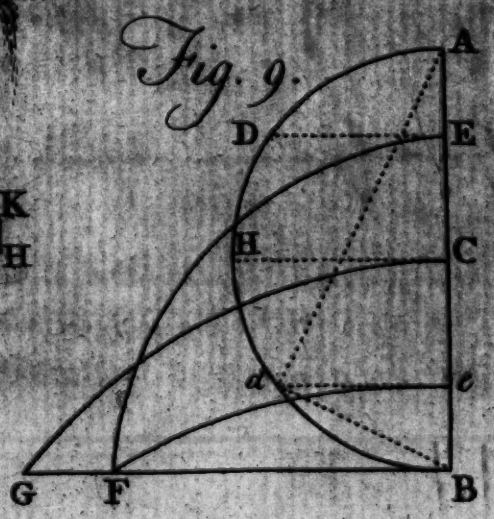
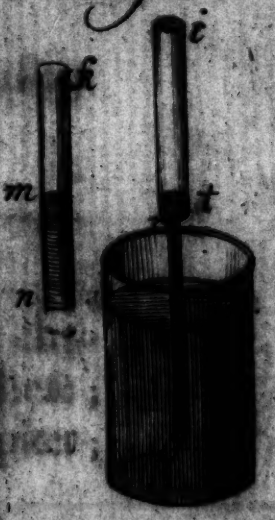
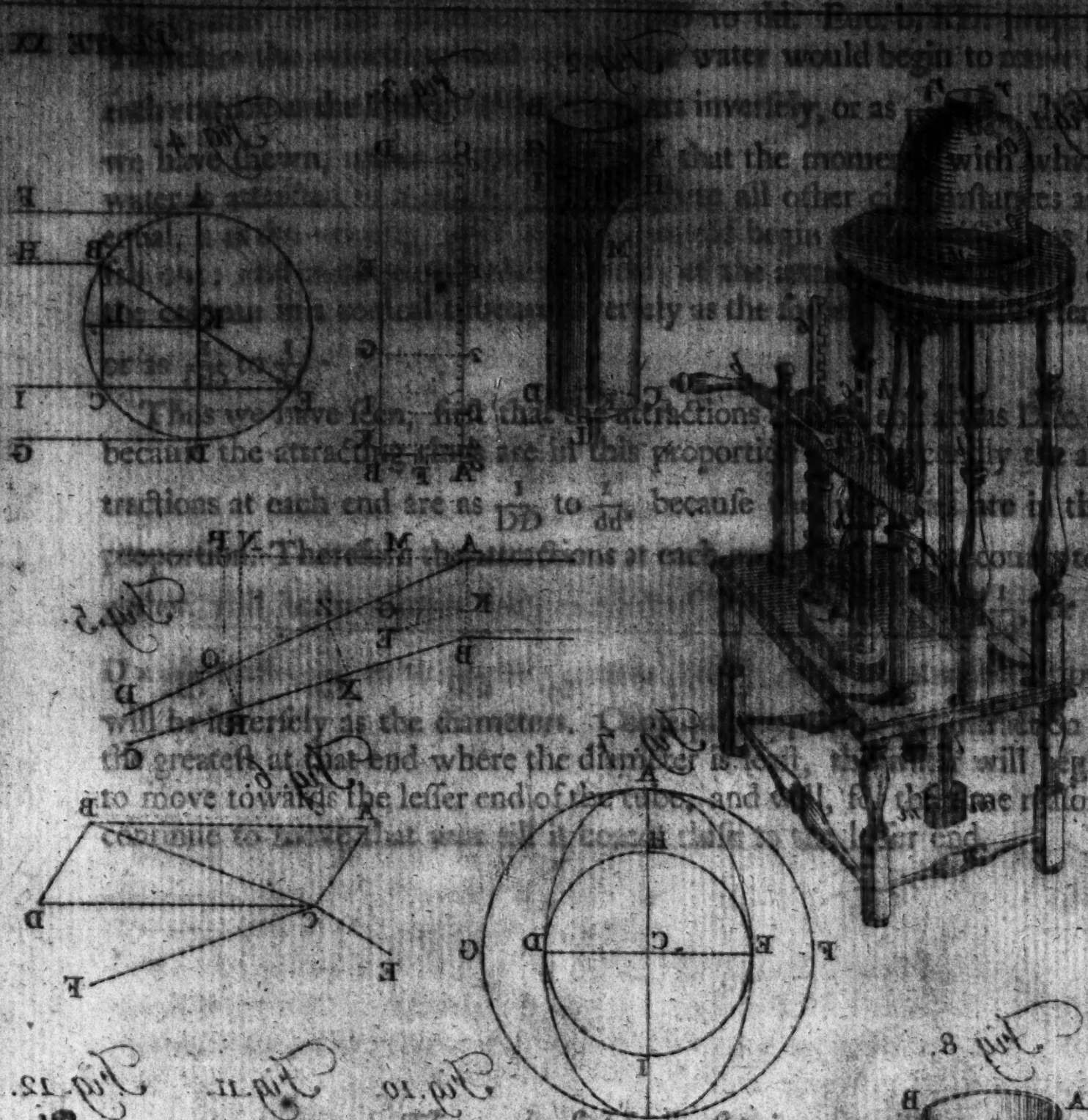


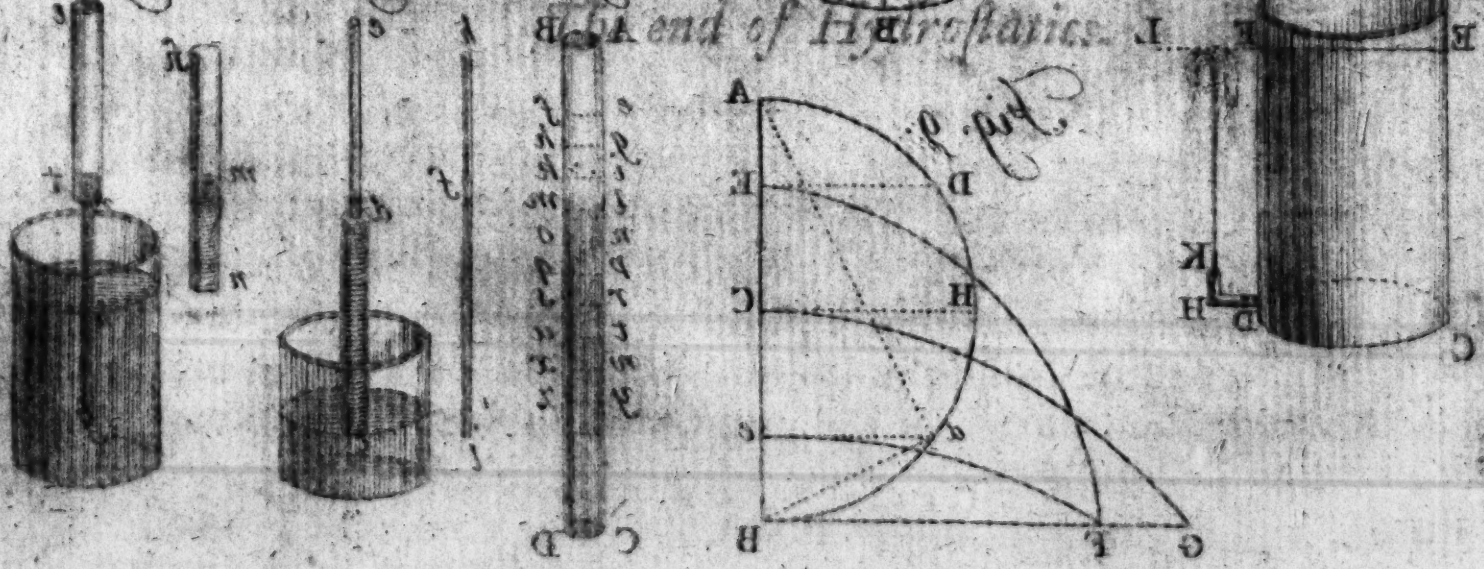
Fig. 10.

Fig. 11.

Fig. 12.







The end of Hydrostatics.

A SYSTEM OF NATURAL PHILOSOPHY.

ASTRONOMY.

CHAP. I.

Of the ſolar ſyſtem.

ASTRONOMY teaches us the laws, to which the heavenly bodies, ſuch as the ſun, moon, planets, comets, and ſtars, are ſubject. And as this ſcience explanes both the manner, in which the heavenly bodies are moved, and likewiſe the natural or phyſical cauſes of their motions, we may divide it into two parts. One part is called pure aſtronomy; and the other phyſical aſtronomy. In pure aſtronomy we either conſider the manner, in which the heavenly bodies appear to move, and from thence determine what their real motions muſt be, in order to produce thoſe apparent ones; or elſe we reaſon the other way, we ſuppoſe what their real motions are, and then eſtabliſh the truth of our ſuppoſition by proving that ſuch real motions will account for their apparent motions. When the manner, in which the heavenly bodies are moved, is thus ſettled, we proceed in phyſical aſtronomy to ſhew from what cauſes theſe motions ariſe. Under one or other of theſe heads we ſhall have occaſion to conſider the light, ſhape, magnitude, and quantities of matter of the heavenly bodies.

1. *The ſolar ſyſtem conſiſts of the ſun, ſix primary planets, ten ſecondary planets, and the comets.*

2. *The ſix primary planets, according to the Copernican ſyſtem, are mercury, venus, the earth, mars, jupiter, and ſaturn.*

The ſun and theſe planets are ſometimes expreſſed by marks or characters inſtead of writing their names at length. The characters are theſe; ☉ the ſun; ☿ mercury; ♀ venus; ♁ the earth; ♂ mars; ♃ jupiter; ♄ ſaturn.

3. *The planets are deſtinguiſhed from the fixed ſtars by their motion and by the ſteadineſs of their light.*

The fixed ſtars have no motion in reſpect of one another, but always keep in the ſame places. But the planets or wandering ſtars change place; they are ſometimes ſeen in one part of the heavens and ſometimes in another. The fixed ſtars are ſo remote, and in conſequence of their remoteneſs their apparent diameters are ſo ſmall, that every ſmall particle of duſt, as it floats in the atmoſphere, intercepts their light: and as their light is ſo frequently ſeen and intercepted by turns, they appear to twinkle. But as the planets are nearer to us, their apparent magnitude is ſuch that thoſe ſmall particles of duſt will not intercept their light: and conſequently their light not being ſubject to the ſame interruptions with that of the fixed ſtars will be more conſtant and ſteady.

4. *The ten ſecondary planets are the moon, which is the earths ſatellite; four moons or ſatellites, that move round jupiter; and five, that move round ſaturn.*

This character ☽ is ſometimes uſed for the moon or ſatellite of the earth. The other nine ſecondary planets are not viſible to the eye without the aſſiſtance of a teleſcope: and there are no aſtronomical marks to expreſs them.

5. *The number of comets belonging to the ſolar ſyſtem is uncertain.*

The comets move round the ſun in orbits of ſuch a ſhape as carryes them to great diſtances from us. When they are at their greateſt diſtances, we cannot ſee them even with the help of teleſcopes; and their returns into our neighbourhood are not frequent enough to determine their number.

6. *The ſyſtem of the world conſiſts of the ſolar ſyſtem and all the other heavenly bodies in the univerſe.*

It is conjectured, not without ſome appearance of probability, that, as the ſun has a number of planets and comets moving round it, ſo every fixed ſtar may be a ſun to a ſyſtem of its own and may be attended in the ſame manner with its planets and comets. When we ſpeak of the ſyſtem of the world we include all theſe, we mean by it all the bodies in the univerſe, the ſun and all its planets and comets, and the fixed ſtars with all their planets and comets, if they have any.

7. *The*

7. *The Copernican ſyſtem ſuppoſes that the ſun is fixed in the center, that the ſix primary planets move round it from weſt to eaſt, and that the ſecondary planets, as they move from weſt to eaſt round their reſpective primaries, are carryed with their primaries round the ſun.*

By the ſyſtem of Copernicus we mean the opinion of Copernicus concerning the order, in which the bodies that compoſe the ſolar ſyſtem, are placed in reſpect of one another, and the manner, in which they perform their motions. The opinion of this aſtronomer is, that if S, Plat. XXI. fig. 1, repreſents the ſun, this is to be conſidered as the center of the ſyſtem, and the ſix primary planets perform their revolutions round this center at different diſtances and in different periodical times. And whilſt thoſe primary planets, which have ſatellites, are thus performing their revolutions in their reſpective orbits, the ſatellites continue to revolve round their primaries and attend them in their motion round the ſun.

The primary planets, if we reckon from the ſun, are placed in this order, mercury, venus, the earth, mars, jupiter, ſaturn. Mercury is repreſented in its orbit at *a*, venus at *b*, the earth at *t* with the moons orbit *d*, mars at *e*, jupiter with the orbits of its four ſatellites at *h*, and ſaturn with the orbits of its five ſatellites at *i*. Whilſt the earth is moving in its orbit from *t* to *g*, ſo as to get half round the ſun, the moon keeps pace with it and will have gone half round the ſun too. In reſpect of this part of the moons motion the ſun may be conſidered as the center. But the earth and not the ſun is the center of the moons motion in its own orbit: for whilſt it has thus attended the earth, it has kept conſtantly revolving in its orbit *d* round the earth. In the ſame manner the ſatellites of jupiter revolve round jupiter, and the ſatellites of ſaturn revolve round ſaturn; but at the ſame time theſe ſatellites revolve round the ſun along with their reſpective primaries.

We could not repreſent the proportion, which the diſtances of the primary planets from the ſun bear to one another, within the compaſs of this ſmall figure. The reader will have a better notion of this proportion from the following numbers. If the diſtance of the earth from the ſun is divided into 10 equal parts, the diſtance of mercury from the ſun is 4 of thoſe parts, the diſtance of venus is 7, the diſtance of mars 15, the diſtance of jupiter 52, the diſtance of ſaturn 95; that is, the diſtances of the ſix planets from the ſun are to one another as theſe numbers 4, 7, 10, 15, 52, 95. Theſe numbers are not indeed very exact; but they are not far from the truth: and ſuch ſmall numbers will be more eaſily remem-

membered, and therefore to the generality of my readers will be more uſeful, than larger numbers that are more exact. The reader muſt obſerve that theſe numbers are not deſigned to determine the diſtances of the planets from the ſun in any real meaſures, as miles, leagues, or ſemidiameters of the earth; they only expreſs the proportion that theſe diſtances bear to one another.

The periodical times of the primary planets, or the times that they take up to complete a revolution round the ſun, are very nearly theſe; the periodical time of mercury is 3 months, of venus 7 months, of the earth 12 months, of mars 2 years, of jupiter 12 years, of ſaturn 30 years.

The comparative magnitude of the planets is better expreſſed in figure 2, than it could be within the compaſs of figure 1. If we call the diameter of the earth 1, then the diameters of the planets are nearly as the following numbers; the diameter of mercury $\frac{1}{3}$; of venus 1; of the earth 1; of mars $\frac{1}{2}$; of jupiter $9\frac{1}{3}$; of ſaturn $8\frac{1}{2}$. The diameter of the ſun is much greater than any of theſe, it is 100 times greater than the diameter of the earth.

There are ſome particulars to be mentioned hereafter, which are not taken notice of in this general account of the Copernican ſyſtem. Firſt, we have ſuppoſed that the ſun is abſolutely at reſt in the middle of the ſyſtem: whereas we ſhall find hereafter that it has a ſmall motion. But as this motion does not hinder its being taken for the center of the ſyſtem without any remarkable error, we have here neglected it. Secondly, we have not taken notice of the figure of thoſe orbits, which the planets deſcribe round the ſun, but have repreſented them as circles having the ſun in their common center: whereas thoſe orbits are ellipſes, and the ſun is not in the center but in one of the focuſes. But as the orbits of the planets, though they are elliptical, do not differ much from circles, it was not neceſſary in this general account to take any notice of the difference. Though theſe niceties were unknown to Copernicus, yet, as they do not much affect the general acount that he gave of the order, in which the ſun and planets are diſpoſed, and the manner, in which the planets revolve round the ſun, the ſyſtem with theſe corrections is ſtill called by his name.

The periodical times of the ſecondary planets and their diſtances from their reſpective primaries ſhall be taken notice of hereafter.

8. *The orbits of the primary planets are inclined to one another.*

In Plat. XXI. fig. 1, the orbits of all the planets are repreſented as if they were in one and the ſame plane: and in repreſenting the whole ſyſtem

ſyſtem it could not be otherwiſe without great perplexity. The reader may underſtand, from a view of fig. 3, what we mean by circles, that are inclined to one another. The view of the two circles in this figure is a perſpective one, and as the planes of the circles are ſuppoſed to be oblique to the eye they are drawn as if they were ellipſes: for a circle in ſuch a view of it appears elliptical.

The orbits of the planets having the ſun for their common center are inclined to one another in this manner but at different angles. The plane of the earths orbit is the ſtandard, to which we refer the reſt, and is called the plane of the ecliptic. The angles, which the orbits of the other five primary planets make with the plane of the ecliptic, are as follows. The orbit of mercury makes an angle of 6°. 52′. with the plane of the ecliptic: the orbit of venus makes an angle of 3°. 23′. with the ſame plane; the orbit of mars makes an angle 1°. 52′; the orbit of jupiter makes an angle of 1°. 20′; the orbit of ſaturn makes an angle of 2°. 30′.

9. *The Ptolemaic ſyſtem ſuppoſes that the earth is fixed in the center, and that ſeven planets move round it.*

Ptolemy does not reckon the earth a planet but conſiders it as the center of the ſyſtem. The planets, which were known in his time were the moon, mercury, venus, the ſun, mars, jupiter, and ſaturn; for the ſatellites of jupiter and ſaturn were not then diſcovered. He ſuppoſed them all to move round the earth in the order, in which we have reckoned them up. The ſyſtem is repreſented Plat. XXI. fig. 4. Where *s* is the earth, *r*, *i*, *o*, *p*, *t*, *l*, *q*, are the orbits of the ſeven planets, that were known, when this ſyſtem was invented.

10. *The Tychonic ſyſtem ſuppoſes that the earth is fixed, that the moon and the ſun revolve round the earth as the center of their orbits, but that the other planets revolve round the ſun and are carryed with it in its revolution round the earth.*

This ſyſtem is repreſented Plat. XXI. fig. 5, *t* is the earth, *b* the orbit of the moon, *c* the orbit of the ſun, *l* the orbit in which mercury revolves round the ſun, *ed* the orbit of venus, *n* the orbit of mars, *o* the orbit of jupiter, and *p* the orbit of ſaturn. There is one thing, which we ſhould obſerve in this contrivance of Tycho. *as* the ſemidiameter of the orbit, in which venus is ſuppoſed to move round the ſun, is ſhorter than *at* the ſemidiameter of the orbit in which the ſun is ſuppoſed to move round the earth. By this means it would be impoſſible for venus ever to be on one ſide of the earth and the ſun on the oppoſite ſide: for in this caſe the earth

muſt

muſt be within the orbit of venus: whereas upon Tycho's ſuppoſition, it will always be on the outſide of this orbit. But *am* the ſemidiameter of the orbit, which mars is ſuppoſed to deſcribe round the ſun, is longer than *at* the ſemidiameter of the orbit, which the ſun is ſuppoſed to deſcribe round the earth. By this means though the earth at *t* is not in the center of mars's orbit, yet it is within that orbit: and conſequently, as mars may be at *m* in its own orbit, whilſt the ſun is at *a*, mars and the ſun may be on oppoſite ſides of the earth.

What we have ſaid in this and the laſt propoſition concerning the Ptolemaic and Tychonic ſyſtems will be ſufficient for our future purpoſes: ſince from this general account of them we may ſhew, that they are not true; for the order and real motions of the heavenly bodies, as ſuppoſed in theſe ſyſtems, will not account for the manner, in which they appear to move to us, who live upon the earth.

CHAP. II.

Of the apparent diurnal and annual motions of the ſun, and its real motion upon its own axis.

11. *The earth is a ſpherical body.*

BY the earth we do not here mean the land only, but the land and ſea together. We ſhall find hereafter that the earth, like every other opake body, caſts a ſhadow oppoſite to the ſun, and that this ſhadow is what occaſions an eclipſe of the moon, But in every poſition of the earth the ſhadow of it is found to be circular. And none but a ſpherical body can caſt ſuch a ſhadow in all poſitions of it. Therefore the figure of the earth is ſpherical.

The convexity of the ſurface of the ſea, which is owing to its being a part of the earths ſpherical ſurface, is evident to ſenſe. For ſailors from the top of a maſt can ſee ſhips at a diſtance, which would be hid from them by the convexity of the water, if they ſtood upon the deck. When a ſhip is going out to ſea, if we ſtand upon the ſhore, the hull of it will be hidden by the convexity of the water, whilſt the top of the maſt, that is above this convexity, will continue to appear. Nor can we now doubt but that the ſurface of the ſea is round, ſince ſeveral have been known to ſail round the globe.

12. *The axis of the earth, or any other planet, is an imaginary line, which is conceived to be drawn through the center of the earth or other planet, upon which line its rotations are performed.*

13. *The*

13. *The poles of the earth, or of any other planet, are the extremities of its axis.*

14. *The equator is an imaginary circle, which is ſuppoſed to be drawn round the earths ſurface in the middle between the two poles.*

It will be of great ſervice to the reader in many of the propoſitions that follow to make uſe of artificial globes. In this definition he will find that from the manner, in which the equator is deſcribed, it follows that the plane of the equator paſſes through the earths center, and that the axis of the earth is perpendicular to the plane of the equator.

15. *All the heavenly bodies appear to us, as if they were in one and the ſame concave ſurface. This ſurface is called the concave ſphere of the heavens.*

The reaſon for the appearance mentioned in the firſt part of this propoſition has already been aſſigned in propoſition 135 of optics.

16. *If we conceive the axis of the earth to be continued both ways, as far as the concave ſphere of the heavens, we then call it the axis of the world.*

17. *The poles of the world are the extremities of the axis of the world.*

18. *If we conceive the plane of the earths equator to be continued all ways as far as the concave ſphere of the heavens, the circle marked out in the ſphere by that plane ſo extended is called the equator in the heavens.*

Since the equator upon the earths ſurface is in the middle between the poles of the earth, and ſince the poles of the world are the extremities of the earths axis produced to the concave ſphere of the heavens; it is evident from this definition, that the equator in the heavens is a circle conceived to be drawn upon that concave ſphere in the middle between the poles of the world.

19. *If innumerable circles are conceived to be drawn through the poles of the world, ſo as to be perpendicular to the plane of the equator; theſe are called ſecondaries of the equator and circles of declination.*

20. *Two imaginary circles, one on each ſide of the equator, that are at about 23°. 30′ minutes from the equator are called tropics.*

21. *The horizon is an imaginary circle, that ſeparates the viſible part of the heavens from the inviſible.*

When objects are upon the earths ſurface the plane of the horizon touches the place where the ſpectator ſtands. If there are no mountains, or hills to hinder his view, he will be able to ſee all ſuch objects as are above this plane; and all ſuch as are below it will be hid by the convex ſurface of the earth. This is called the ſenſible horizon.

But when the objects are as remote as the heavenly bodies, the plane of the horizon paſſes through the earths center, and conſequently divides the concave ſphere, in the middle of which the earth appears to be placed, into two equal parts or hemiſpheres. Such objects as are in the upper hemiſphere will be viſible, and ſuch as are in the lower hemiſphere will be inviſible. This is called the rational horizon.

From what has been ſaid it appears that there is no other diſtinction between the ſenſible and rational horizon, but in reference to the diſtance of the objects that we are looking at; the ſenſible horizon is an imaginary circle, which terminates our view, when the objects, that we are looking at, are upon the earths ſurface, and the rational horizon is an imaginary circle, which terminates our view, when the objects, that we are looking at, are as remote as the heavenly bodies. The plane of the ſenſible horizon HO Plat. XXI. fig. 6, touches the earths ſurface at S, where the ſpectator ſtands; the plane of the rational horizon RI is parallel to the former, and paſſes through C the earths center. Conſequently as one touches the ſurface and the other paſſes through the center, the diſtance between them is SC the ſemidiameter of the earth. And this diſtance at *ab*, *cd*, OI, or any where elſe is the ſame, becauſe the two planes HO, and RI are parallel to one another. But at very great diſtances from S the apparent diſtance of one from the other is nothing, ſo that the two planes as to ſenſe coincide; or the plane of the ſenſible horizon, when objects are very remote, coincides with the plane of the rational horizon. If any object was in the circle Z*a*L, the apparent diſtance between the ſenſible and rational horizon in reſpect of that object would be the angle *a*S*b*, or the angle, that *ab* equal to SC the earths ſemidiameter would ſubtend, if it was viewed at the diſtance S*a*. If any object was in the circle M*c*N, the apparent diſtance between the ſenſible and rational horizon, in reſpect of that object, would be *c*S*d*, or the angle, that *cd* equal to SC the earths ſemidiameter would ſubtend, if it was viewed at the diſtance S*c*. In like manner if the object is at a ſtill greater diſtance, in the circle POQ the apparent diſtance between the ſenſible and rational horizon will be OSI, or the angle which OI=SC would ſubtend ſeen from the earth. And univerſally, the apparent diſtance between the ſenſible and rational horizons in reference to any object is the angle, which the ſemidiameter

of

of the earth would ſubtend if it was a viſible line placed as far from the ſpectators eye as the object is. Now the apparent length of ſuch a line would be inverſely as its diſtance from the eye, or inverſely as the diſtance of the object, by propoſition 126 of optics. Therefore this apparent length, and conſequently the diſtance between the ſenſible and rational horizon, will be little or nothing, where the viſible object is as remote as the heavenly bodies are from the earth.

HO and RI, though they are deſigned to repreſent circles, are here only right lines: becauſe, if the edge of a circle is towards the eye, the circle would appear like a right line; and in the figure we could not eaſily have drawn theſe circles any otherwiſe, than with their edge towards our eye as we are looking upon the paper where the figure is drawn.

22. *That point in the heavens, which is directly over the head of a ſpectator is called the zenith; and the oppoſite point is called the nadir.*

If the ſpectator is at S, Plat. XXI. fig. 6, P is the zenith and Q is the nadir.

23. *If the earth revolves upon its axis from weſt to eaſt in 23 hours and 56 minutes, thoſe heavenly bodies, which have no motion of their own, will appear in the mean time to revolve round the earth from eaſt to weſt.*

Let S, Plat. XXI. fig. 6, be the place of the ſpectator, RI the horizon, and *e* a ſtar cloſe to the eaſtern edge or eaſtern ſide of the horizon. Let SFTG be the earth, C one end of its axis or one of its poles upon which it turns round from weſt to eaſt, in the order of the letters S, F, T, G. Then, as the earth moves in this direction, any perſon at S will move the ſame way. But, as the ſpectator moves, the plane of his horizon RI will move with him. For HO touches the earths ſurface at the place where the ſpectator ſtands, and conſequently as the place where he ſtands moves, the imaginary plane HO, which touches this place muſt move too. But RI is always parallel to HO. Therefore RI will always be carryed in the ſame direction with HO, that is, in the ſame direction with S, or from weſt to eaſt. By ſuch a motion as this R the eaſtern edge of the horizon will ſink down below the ſtar *e*, and I the weſtern edge of the horizon will approach towards it. In the mean time, as the ſpectator does not attend to, or is not able to obſerve, the motion of himſelf and the place where he ſtands, and yet the ſtar is more and more remote from R the eaſtern edge of his horizon; this will appear to him to be owing to a motion of the ſtar from that edge, though it is really owing to a motion of that edge from the ſtar. And thus as the horizon, upon account of the

earths motion, moves from the ſtar, the appearance will be the ſame as if the ſtar had moved away from the horizon; or as the earth, the ſpectator, and the horizon with him, turn round from weſt to eaſt, the ſtar will appear to move in the contrary direction or from eaſt to weſt.

When the earth has performed one quarter of its rotation, the ſpectator will be at F, and his horizon RI will be in the ſituation QP. At this time the ſtar *e* will appear in the zenith, and conſequently it will ſeem to have moved round one quarter of the earth. When the earth has performed half its rotation the ſpectator will be at T, R the eaſtern edge of the horizon will be got to I, and I the weſtern edge of it will be got to the ſtar *e*. Soon after this the ſtar will diſappear, as the weſtern horizon will riſe above it. But as the ſpectator ſuppoſes the place, where he is, to be at reſt, this will ſeem to him to have been owing to the ſtars motion; or the ſtar will ſeem to deſcend below the weſtern horizon, or to ſet. When the earth has performed three quarters of a rotation upon its axis, the ſpectator will be at G, the horizon RI, as it revolves with him, will be in the ſituation PQ, and the ſtar at *e* will be in his nadir. When the whole rotation is completed, the ſpectator will be at S, his horizon in the poſition RI, and the ſtar at *e* will be riſing upon the eaſtern edge of it, as at firſt.

Thus whilſt the earth has turned once round upon its axis from weſt to eaſt, the ſtar *e* will appear to have revolved once round the earth in the contrary direction. And the ſame might be ſhewn of other ſtars in other parts of the heavens. Conſequently, whilſt the earth is performing a rotation upon its axis, the whole concave ſphere of the heavens will appear to have turned round from eaſt to weſt. Now ſince this motion of the earth is upon its axis, and ſince the axis of the world or concave ſphere of the heavens is only the earths axis produced; the apparent motion of the heavens, that is owing to the earths rotation, will be performed upon the axis of the world. But if a ſphere is turned round upon its axis, the axis itſelf and extreme points of it or the poles of the ſphere, will not partake of this motion. Therefore, when we ſay that the whole concave ſphere of the heavens appears to turn round upon the axis of the world, whilſt the earth is performing one rotation round its own axis, we muſt be underſtood to except the two poles of the world, for theſe do not partake of this apparent motion.

In the Ptolemaic and Tychonic ſyſtems, the earth is ſuppoſed to have no motion round its axis, and this conſtant rotation of all the heavenly bodies from eaſt to weſt is looked upon to be real: ſo that beſides the motion of the planets round the earth from weſt to eaſt in their reſpective or-

bits, the whole ſphere of the heavens *fg*, Plat. XXI. fig. 4, 5, and all the planets along with it, muſt revolve every day from eaſt to weſt upon the axis of the world.

24. *The alternate changes of day and night are owing to the rotation of the earth round its axis.*

The rotation of the earth round its axis makes all the heavenly bodies appear to move from eaſt to weſt, by propoſition 23. By this motion the ſun, which is one of thoſe bodies, will appear to riſe above the horizon in the eaſt, and after having deſcribed a portion of a circle it will appear to ſink below the horizon in the weſt. It is day whilſt the ſun continues above the horizon, and night after it has ſunk below it.

25. *The time, that the ſun takes up to complete one apparent revolution is longer than the time that the earth takes to revolve once upon its axis.*

We have already ſeen in the foregoing propoſitions that the diurnal or daily motion of the ſun from eaſt to weſt is not a real but an apparent one, and is owing to the rotation of the earth round its axis. Now if the ſun had no other motion but this apparent one, it would ſeem to go once round the earth in the time of one complete rotation, or in 23 hours and 56 minutes. This is the caſe of any fixed ſtar. If the earth turns upon P as upon an axis in the direction STVW, Plat. XXII. fig. 1, then in the time of one complete rotation any ſtar R, which is fixed in its place or has no other motion of its own, either real or apparent, but what ariſes from the earths rotation, will appear to have gone once round the earth: ſo that, if it was exactly ſouth of any place S at the beginning of the rotation, it will be returned to the ſouth again at the end of it, or after 23 hours 56 minutes. But the ſun is found to take up a longer time to complete its apparent revolution: for if it is in the ſouth of any particular place at 12 o'clock at noon to day, it will not complete an apparent revolution, ſo as to return to the ſouth of that place again, till 12 o'clock at noon to morrow, and conſequently the time of ſuch an apparent revolution is 24 hours. This is owing to another motion of the ſun, which is called its annual or yearly motion. For the ſun does not continue always in the ſame place in the heavens, as the fixed ſtars do: but, if it is ſeen at M one day, and is near the fixed ſtar R, it will have ſhifted its place the next day, and will be near ſome other fixed ſtar L. This motion of the ſun is from weſt to eaſt, and one entire revolution is completed in a year.

Suppoſe therefore that the ſun, when it is at M near to the fixed ſtar R, appears in the ſouth of any particular place S, and then imagine the earth

earth to turn once round upon its axis from weſt to eaſt, or in the direction STVW, ſo that the place S may be returned to the ſame ſituation: after this rotation is completed, the ſtar R will be in the ſouth of the place as before: but the ſun having in the mean time moved eaſtwards and being near to the ſtar L or to the eaſt of R, will not be in the ſouth of the place S, but to the eaſtward of it. Upon this account the place S muſt move on a little farther and muſt come to T, before it will be even with the ſun again, or before the ſun will appear exactly in the ſouth.

We may perhaps illuſtrate this matter by an inſtance. The two hands of a clock are cloſe together or even with one another at 12. They both turn round the ſame way and upon the ſame axis, but the minute-hand turns round in a ſhorter time than the hour-hand. When the minute-hand has completed one rotation and is come round to 12, the hour-hand will then be before it, or will be at 1, ſo that the minute-hand muſt move more than once round, in order to overtake the hour-hand ſo as to be even with it again. In like manner any place S upon the earths ſurface revolves from weſt to eaſt round the earths axis, and the ſun by its annual motion appears to revolve round the earths center in the ſame direction; but the place upon the ſurface of the earth turns round in a ſhorter time than the ſun. Therefore, if the ſun is in the ſouth of the place, or is even with it, when any rotation of the earth is ſuppoſed to begin, then, after the rotation is completed, and the place S is returned to the ſame ſituation, the ſun having in the mean time moved forwards will be before the place S, and conſequently the place muſt move more than once round, in order to overtake the ſun, ſo as to be even with it again, or ſo as to make it appear again exactly in the ſouth.

This annual motion of the ſun is ſuppoſed to be real both in the Ptolemaic and in the Tychonic ſyſtem, but it is only apparent if the Copernican ſyſtem is true: which of theſe ſyſtems is the true one ſhall be ſhewn hereafter: for the preſent it will be ſufficient to prove, that the appearance will be the ſame, whether the ſun moves round the earth, as is ſuppoſed in the Ptolemaic and Tychonic ſyſtems, or the earth moves round the ſun, as in the Copernican ſyſtem.

26. *The ſun, when ſeen from the earth, will appear to move in the ſame manner, whether the ſun itſelf revolves round the earth, or the earth revolves round the ſun.*

The ſun, the planets and the fixed ſtars appear to be placed in the ſurface of the ſame concave ſphere, by propoſition 135 of optics: and the ſuns annual motion is performed in a great circle of this ſphere. Let EFGH, Plat. XXII. fig. 2, be this circle. The apparent motion of the

ſun

ſun will be the ſame, whether the earth is at reſt at S and the ſun revolves round it in the orbit ABCD, or the ſun is at reſt at S and the earth revolves in the ſame orbit ABCD.

Firſt, let the earth be ſuppoſed at S, without any motion of its own, and let the ſun be ſuppoſed to revolve round it in the orbit ABCD, and let EFGH be a circle in the concave ſphere of the heavens. As the ſun moves in the order of the letters A, B, C, D, in its orbit, it will appear to a ſpectator on the earth to have deſcribed the circle EFGH. When the ſun is at A it will appear as if it was amongſt the fixed ſtars that are at E; when it is at B, it will appear amongſt the fixed ſtars at F; when it is at C, it will appear amongſt the fixed ſtars at G; and when it is at D, it will appear amongſt the fixed ſtars at H. Indeed the fixed ſtars and the ſun are not ſeen at the ſame time; for the fainter light of the ſtars is not ſufficient to make them viſible, when the ſun ſhines: but yet we may tell in what part of the heavens the ſun is, or what fixed ſtars it is near to, by knowing what fixed ſtars are oppoſite to it. Thoſe ſtars to which the ſun is near, come to the ſouth at noon, or at the ſame time with the ſun, and thoſe, that are oppoſite, come to the ſouth at midnight. Therefore if we find that any ſet of ſtars, as thoſe at G for inſtance, come to the ſouth at midnight, we may be ſure that they are oppoſite to the ſun, and conſequently that, if we could ſee the ſtars in that part of the heavens where the ſun is, we ſhould find them to be thoſe at E. We ſhould obſerve farther, that the ſun in the orbit ABCD is nearer than the fixed ſtars, but as theſe diſtances are either of them too great for the eye to judge of them, they appear to be equal, by propoſition 135 of optics.

Secondly, let us ſuppoſe that S is the ſun, that the ſun has no motion of its own, but that it reſts within the orbit ABCD, in which we will now ſuppoſe the earth to move in the order of the letters A, B, C, D. Upon this ſuppoſition, when the earth is at A, the ſun will appear in that part of the heavens where the ſtars G are; when the earth is at B, the ſun will appear in that part of the heavens where the ſtars H are; when the earth is at C, the ſun will appear in that part of the heavens where the ſtars E are: and when the earth is at D, the ſun will appear in that part of the heavens where the ſtars F are. Thus as the earth revolves round the ſun in the orbit ABCD, the ſun will appear to a ſpectator on the earth to deſcribe the circle GHEF.

Thus whether the earth is at reſt and the ſun revolves in the orbit ABCD, or the ſun is at reſt and the earth revolves in this ſame orbit, a ſpectator on the earth in one year, or in the time of one revolution, will ſee the ſun deſcribe the ſame circle EFGH in the concave ſphere of the heavens.

27. The

27. *The circle, which the ſun appears to deſcribe every year in the concave ſphere of the heavens, is called the ecliptic.*

This circle is called the ecliptic, becauſe in all eclipſes the moon is either in or near the plane of it.

28. *If the earth moves round the ſun, the plane of the ecliptic is likewiſe the plane of the earths orbit.*

ABCD, Plat. XXII. fig. 2, is the earths orbit, any line, as AS, drawn from the earth to the ſun is in the plane of this orbit, and when the earth is at A, the ſun will appear ſomewhere in this line AS continued to G the concave ſphere of the heavens. As the end of this line next to the earth deſcribes the orbit ABCD along with the earth, the line may be conſidered as turning round upon the ſun S, and then the other end of it deſcribes the ecliptic GHEF. One part of this line, which is between the earth and the ſun, as AS, BS, CS, DS, runs over the plane of the earths orbit, and the other part of it, which is between the ſun and the ecliptic, as SG, SH, SE, SF, runs over the plane of the ecliptic. But all the parts of the ſame right line are in the ſame plane. Euc. b. XI. prop. 1. Therefore ſince AS is in the ſame plane with SG, BS with SH, CS with SE, and DS with SF, and ſince AS, BS, CS, DS, are in the plane of the earths orbit, and SG, SH, SE, SF in the plane of the ecliptic, it follows that the earths orbit is in the ſame plane with the ecliptic, or with that circle, which the ſun by its annual motion appears to deſcribe in the concave ſphere of the heavens.

29. *If we conceive a zone or belt about ſixteen degrees broad in the concave ſphere of the heavens with the ecliptic paſſing through the middle of it, this zone is called the zodiac.*

The orbits of all the other planets are inclined to the orbit of the earth, that is, by propoſition 28, the orbits of all the planets are inclined to the plane of the ecliptic. But, as the angle made by the orbit of any planet with the plane of the ecliptic is leſs than 8°, none of the planets will ever be ſeen out of the zodiac. The orbit of mercury, for inſtance, makes an angle of 6°. 52′ with the plane of the ecliptic, which is a greater angle than the orbit of any other planet makes with the ſame plane. Therefore mercury may be 6°. 52′ from the ecliptic: but ſtill it will be within the zodiac; becauſe the zodiac reaches 8° on each ſide of the ecliptic.

30. *The*

30. *The ſtars in the zodiac are divided into twelve ſigns, aries, taurus, gemini, cancer, leo, virgo, libra, ſcorpio, ſagittarius, capricornus, aquarius, piſces.*

The fixed ſtars in every part of the heavens are reduced to conſtellations. By a conſtellation we mean a number of ſtars, that are near to one another, and may be drawn, at their proper diſtances from one another upon an artificial globe, within the figure of ſome animal or ſome other thing, which either our imagination has invented, or our ſenſes have obſerved. Thus a number of ſtars near the north pole are repreſented upon the globes within the figure of a bear: theſe ſtars make a conſtellation, which is called the bear. Another conſtellation is called the archer; becauſe the ſtars, which compoſe it, are drawn at ſuch diſtances from one another, that they may be repreſented upon the globe within the figure of a centaur ſhooting with a bow and arrow. Another conſtellation is called the ſhip, becauſe that collection of ſtars, which compoſe it, when they are repreſented upon the globe, are drawn upon ſome part of the figure of a ſhip.

The ſtars, which are ſeen in that part of the heavens, that we call the zodiac, are reduced by aſtronomers to twelve conſtellations. The twelve ſigns in the zodiac are theſe twelve conſtellations. The latin names of them, as in the propoſition, are uſed by the aſtronomers of our own country, and we ſeldom hear them called by engliſh names. Sometimes theſe twelve ſigns are each of them expreſſed in writing by a mark or character inſtead of the name at length. Their characters are theſe, ♈ aries, ♉ taurus, ♊ gemini, ♋ cancer, ♌ leo, ♍ virgo, ♎ libra, ♏ ſcorpio, ♐ ſagittarius, ♑ capricornus, ♒ aquarius, ♓ piſces. Theſe ſigns, when reckoned in this order, lie from weſt to eaſt, ſo that each ſign is to the eaſt of the ſign that is before it.

31. *The axis of the earth is inclined to the plane of the ecliptic at an angle of about* 66 *degrees* ½; *and the different ſeaſons of the year are owing to this inclination.*

EC, Plat. XXII. fig. 3, repreſents the plane of the earths orbit or of the ecliptic: it is here drawn upon the paper as a right line, becauſe the edge of it, as we look at the paper, is ſuppoſed to be towards our eye. S is the ſun in the middle of the earths orbit. If the earth is at C the axis of it is not perpendicular to the plane of the ecliptic, as AB is, but is inclined to it, as PR is; ſo that the angle P*i*E, which the axis of the earth makes with this plane, is about 66°. 30′. If the earth, by moving half round the ſun, is arrived at E the oppoſite point in its orbit, the axis PR

is ſtill inclined to the plane of the ecliptic at the ſame angle as before, that is, the angle PnE is 66°. 30′.

Let us firſt conſider what would be the conſequence, if the axis of the earth was perpendicular to the plane of its orbit. If AB perpendicular to EC was the earths axis, then the plane of the equator would be in the plane of the ecliptic. For the axis is perpendicular to the ecliptic by the ſuppoſition, and it is perpendicular to the plane of the equator, by propoſition 18. Now as the ſun is always in the plane of the ecliptic, by propoſition 27, it would always be in the plane of the equator, if the equator and ecliptic coincided, as they would do, if the earths axis was perpendicular to the plane of its orbit. But the equator is juſt in the middle between the two poles. Therefore, if the earths axis was perpendicular to the plane of its orbit, the ſun would at all times of the year be juſt in the middle between the two poles: and conſequently we ſhould have no change of the ſeaſons from ſummer to winter; becauſe our ſummer is made by the ſuns approach to one of the poles, and our winter by its departure from thence towards the other pole.

But if the earths axis is inclined to the plane of the ecliptic, or, which is the ſame thing, to the plane of its orbit, upon this ſuppoſition, when the earth is at C, the pole P will be towards the ſun, and the pole R will be turned from it; and juſt the contrary will happen, when the earth, by going half round the ſun, has arrived at E the oppoſite point in its orbit. By this means the ſun will not always be in the equator, but at one time of the year it will appear nearer to one of the poles, and at the oppoſite time it will appear nearer to the other. The change of ſeaſons is owing to this; for, as we have already obſerved, when the ſun leaves the equator and approaches to one of the poles, it will be ſummer on that ſide of the equator; and when the ſun departs from thence and approaches to the other pole, then it will be winter. From this poſition of the earths axis each part of the earth enjoys the benefit of ſummer in its turn: for it is evident, from what has been ſaid already, that when it is winter towards one of the poles, or on one ſide of the equator, it will be ſummer towards the other pole, or on the other ſide of the equator. This change of the ſeaſons will be further explaned in ſome of the following propoſitions.

32. *The equator makes an angle of about* 23 *degrees* ½ *with the plane of the ecliptic.*

The angle, which the equator makes with the plane of the ecliptic, is the complement of that which the earths axis makes with the ſame plane. PR,

Plat.

Plat. XXII. fig. 3, is the earths axis: this axis is perpendicular to the plane of the equator, by propoſition 18: therefore, if QT is drawn at right angles to PR, QT will be in the plane of the equator. Now ſince P*n*Q is a right angle, it follows that Q*n*E is the compleiment of P*n*E. But P*n*E is the angle, which the axis makes with the plane of the ecliptic, and Q*n*E is the angle, which the equator makes with the ſame plane. The angle P*n*E is 66°. 30′. by propoſition 31. Therefore its complement Q*n*E is an angle of 23°. 30′.

If PR coincided with AB, that is, if the earths axis was perpendicular to EC the plane of the ecliptic, then QT would coincide with GC or HE. But the ſun is always in the plane of the ecliptic, by propoſition 27; and QT is the plane of the equator. Therefore, if the earths axis was perpendicular to the plane of the ecliptic, the ſun would always be in the plane of the equator, or at equal diſtances from each pole. But the axis PR, inſtead of being perpendicular to EC the plane of the ecliptic, makes an angle of 66°. 30′. with this plane; and conſequently the equator QT makes an angle of 23°. 30′. with the ſame plane; that is G*i*Q or T*n*H are either of them an angle of 23°. 30′. Now if the earth is at C, the ſun appears in the plane SGC. But G is 23°. 30′. from Q. Therefore when the earth is at C, the ſun does not appear in the plane of the equator, but is 23°. 30′. nearer to the pole P. In like manner if the earth is in the oppoſite part of its orbit at E, the ſun does not appear in the plane of the equator QT, but in the plane SHE, with which QT makes an angle of 23°. 30′. Therefore, ſince H is 23°. 30′. from T, the ſun does not appear in the equator, but is 23°. 30′. nearer to the pole R.

33. *If we conceive the ecliptic to be divided into twelve equal parts, each of theſe parts is called a ſign: and the names of the ſigns in the ecliptic are the ſame with thoſe in the zodiac.*

A ſign in the zodiac is a conſtellation or number of ſtars; a ſign in the ecliptic is only the twelfth part of that circle, which the ſun by its annual motion appears to deſcribe in the concave ſphere of the heavens. Now as every circle contains 360°, the twelfth part of a circle, is 30°; ſo that a ſign in the ecliptic means no more than an arc of 30°. in the ecliptic.

But as theſe ſigns are not viſible to the eye, as thoſe in the zodiac are, we might call an arc of 30°. in any part of the ecliptic by the name of aries or taurus, or gemini, &c, and one aſtronomer might call the ſame arc aries, which another would call cancer, unleſs there was ſomething to determine the point in the ecliptic, from whence we are to begin to rec-

kon the ſigns, ſo that the firſt 30°. from that point is to be called aries, the next 30°. taurus and ſo on. Now we obſerved, in the laſt propoſition, that the equator is inclined to the plane of the ecliptic, and conſequently the equator cuts the plane of the ecliptic in two points: and from one of theſe points it is that we begin to reckon the ſigns of the ecliptic. We ſhall ſee hereafter which point of interſection it is that this reckoning begins from.

In the mean time we ſhould obſerve, that, though the ſigns or conſtellations in the zodiac, are called by the ſame names with thoſe in the ecliptic, and though the ecliptic paſſes through the zodiac, yet the ſigns in one do not anſwer to thoſe in the other; the ſign aries, for inſtance, in the ecliptic is not that part of the ecliptic, which paſſes through the ſign or conſtellation aries in the zodiac; the ſign cancer in the ecliptic is not that part of the ecliptic, which paſſes through the ſign or conſtellation cancer in the zodiac; and the ſame is equally true of all the other ſigns in each. This the reader may ſee repreſented upon the celeſtial globe: where he will find that this mark ♈ and the 30°. reckoned from it, which make the ſign aries in the ecliptic, are not within the picture of the ram where the ſtars are drawn that make the conſtellation or ſign aries in the zodiac: the ſame he will find to be true of all the other ſigns.

The reaſon why the ſigns in the ecliptic and the zodiac, that have the ſame names, are not in the ſame places ſhall be ſhewn hereafter.

34. *The ſun is once in the year at each of the tropics, and twice in the year at the equator.*

The ſun appears to deſcribe the ecliptic once every year, by propoſition 28. And the equator is inclined to the ecliptic, by propoſition 32, ſo as to make with it an angle of 23°. 30′. Let ABCD, Plat. XXI. fig. 3, be the ecliptic and EFGH the equator, which makes with the ecliptic the angle GDC of 23°. 30′. Both theſe circles, as they are drawn upon the paper, are ſuppoſed to be ſeen obliquely, and for that reaſon in this perſpective view of them they are repreſented as ellipſes. The equator interſects the ecliptic at B and D. Therefore, whilſt the ſun is deſcribing the ecliptic ABCD when it comes to either of theſe interſections B or D, it will be at the equator. G and E are each of them 23°. 30′. from the ecliptic. Therefore when the ſun, whilſt it is deſcribing the ecliptic, is either at C or at A, it will be 23°. 30′. from G or E, that is, it will be 23°. 30′. from the equator. But the two tropics are at this diſtance from the equator, one on one ſide of it and the other on the other ſide: and conſequently, when the ſun is at A, it will be at one of the tropics, and when at C, it will be at the other tropic.

35. *The*

35. *The two points, where the equator cuts the ecliptic, are called the equinoctial points.*

Theſe are called the equinoctial points, becauſe the ſun is in one of thoſe points at the time of the equinoxes, or when the days and nights are of an equal length in all parts of the earth.

36. *That point, where the ſun is at the vernal equinox, is called the firſt of aries; and that, where it is at the autumnal equinox, is called the firſt of libra.*

The ſigns of the ecliptic begin to be reckoned from the vernal equinoctial point. The ſign aries in the ecliptic is an arc of 30°. beginning from this point. And the ſign libra in the ecliptic is an arc of 30° degrees beginning from the autumnal equinoctial point. The former is called the firſt of aries, becauſe the ſign aries begins there: and the other for a like reaſon is called the firſt of libra.

37. *The points, where the ecliptic touches the tropics, are called the ſolſtitial points.*

One of the ſolſtitial points is a quarter of a circle or 90°. from the firſt of aries; ſo that three ſigns in the ecliptic lie between the firſt of aries and this point; and conſequently this ſolſtitial point is the firſt of cancer. The other ſolſtitial point is at the ſame diſtance from the firſt of libra, and conſequently is the firſt of capricorn.

38. *A ſecondary of the equator, that paſſes through the equinoctial points, is called the colure of the equinoxes; and another, that cuts this at right angles, and paſſes through the ſolſtitial points, is called the colure of the ſolſtices.*

39. *That tropic, which touches the ecliptic at the firſt of cancer, is called the tropic of cancer: and the other, which touches it at the firſt of capricorn, is called the tropic of capricorn.*

40. *That pole, which is neareſt to the tropic of cancer, is called the north pole; and the other, which is neareſt to the tropic of capricorn, is called the ſouth pole.*

41. *An imaginary line, that paſſes through the center of the ecliptic and is perpendicular to the plane of it, is called the axis of the ecliptic.*

The axis of the earth, or of the world, paſſes through the center of the equator, and is perpendicular to the plane of it: this is ſometimes called the

the axis of the equator. And from hence a line, that paſſes through the center of any other great circle of the ſphere, and is perpendicular to the plane of it, is called the axis of that circle.

42. *The extremities of the axis of the ecliptic are called the poles of the ecliptic.*

43. *If innumerable circles are ſuppoſed to paſs through the poles of the ecliptic and to be perpendicular to the plane of it, theſe are called ſecondaries of the ecliptic.*

44. *The axis of the ecliptic makes an angle of* 23 *degrees* $\frac{1}{2}$ *with the axis of the earth or of the world.*

The plane of the equator makes an angle of 23°. 30′ with the plane of the ecliptic, by propoſition 32. Therefore the axiſes of theſe circles, or lines, which are reſpectively perpendicular to the planes of them, will make the ſame angle with each other.

If PR, Plat. XXII. fig. 3, is the axis of the earth, then the ſame line produced both ways to the concave ſphere of the heavens is the axis of the world, by propoſition 16. If AB is perpendicular to EC the plane of the ecliptic, then AB produced both ways will be the axis of the ecliptic. Now PR is perpendicular to QT, and AB is perpendicular to EH: and whatever angle is contained between QT and EH, an equal one will be contained between PR and AB, or the angle A*n*P is equal to the angle E*n*Q. For A*n*E is equal to P*n*Q, becauſe they are both of them right angles, by the conſtruction. Therefore, if from each of theſe equal angles the ſame angle P*n*E is taken, the remainder A*n*P will be equal to the remainder E*n*Q. But E*n*Q or the inclination of the equator to the ecliptic is 23°. 30′. and conſequently A*n*P or the inclination of the axis of the ecliptic to the axis of the world is likewiſe an angle of 23°. 30′.

45. *The polar circles are two circles deſcribed by the poles of the ecliptic round the poles of the world in the daily rotation of the heavens.*

If the earth QPTR, Plat. XXII. fig. 3, revolves upon its axis PR, the two extremities A and B of the line AB would in ſuch a rotation reſpectively deſcribe circles round the points P and R. In like manner as the heavens apparently revolve every day upon the line PR produced, by propoſition 23, the extremities of the line AB produced, or the poles of the ecliptic, will in this daily rotation reſpectively deſcribe circles in the concave ſphere of the heavens. Theſe are called the polar circles. And

ſince

ſince P*n* and A*n*, by propoſition 44, make with each other an angle of 23°. 30′, the circumference of one of theſe circles will be 23°. 30′. from the north pole, and the circumference of the other will be at an equal diſtance from the ſouth pole.

46. *The arctic circle is the polar circle next to the north pole: and the antarctic circle is the polar circle next to the ſouth pole.*

47. *The declination of any ſtar or planet is its diſtance from the equator: it is called north declination, when the ſtar or planet is on the north ſide of the equator, and ſouth declination, when it is on the ſouth ſide.*

Thus the ſun has no declination, when it is in thoſe points of the ecliptic where it croſſes the equator; for then its diſtance from the equator is nothing. When it is in that point of the ecliptic, which touches the tropic of cancer, it has 23°. 30′. north declination: and when it is in that point, which touches the tropic of capricorn, it has 23°. 30′. ſouth declination.

The degrees or minutes of any ſtars or planets declination are reckoned upon a ſecondary of the equator, that paſſes through that ſtar or planet.

48. *The right aſcenſion of any ſtar or planet is its diſtance from the firſt of aries reckoned upon the equator.*

In order to determine the right aſcenſion of a ſtar, or of the ſun, or moon, or any planet, we muſt conceive a ſecondary of the equator to paſs through the ſtar, or the ſun, or moon, or planet, and the arc of the equator, that is intercepted between the firſt of aries and the place where this ſecondary cuts the equator, is the right aſcenſion.

49. *The latitude of any ſtar or planet is its diſtance from the ecliptic: it is called north latitude, when the ſtar or planet is on the north ſide of the ecliptic, and ſouth latitude, when it is on the ſouth ſide.*

From this definition it appears, that the ſun has never any latitude, becauſe it always is in the ecliptic. But the planet venus, for inſtance, moves in an orbit that makes an angle of 3°. 23′. with the ecliptic, and conſequently this planet may have 3°. 23′. north or ſouth latitude, according as it happens to be on the north or ſouth ſide of the ecliptic.

The degrees or minutes of any ſtars or planets latitude are reckoned upon a ſecondary of the ecliptic, that paſſes through that ſtar or planet.

50. *The longitude of any ſtar or planet is its diſtance from the firſt of aries reckoned upon the ecliptic.*

In order to determine the longitude of a ſtar or of the ſun, or moon, or any planet, we muſt conceive a ſecondary of the ecliptic to paſs through the ſtar, or the ſun, or moon, or planet; and the arc of the ecliptic, that is intercepted between the firſt of aries and the place where this ſecondary cuts the ecliptic, is the longitude.

When the ſun is in the firſt of aries, at the vernal equinox, its longitude is nothing: as it moves on from thence, its longitude encreaſes; when it is in the firſt of cancer, its longitude is 90°; when in the firſt of libra, its longitude is half a circle or 180°; when it is in the firſt of capricorn, its longitude is 270°; and when it returns to the firſt of aries its longitude may then be ſaid to be 360°. or rather, as it has gone round the ecliptic and is ſetting out again, its longitude is nothing.

51. *Two of the heavenly bodies are ſaid to be in conjunction with each other, when they have the ſame longitude.*

The ſun for inſtance is ſaid to be in conjunction with venus, when the ſun and venus have the ſame longitude, that is, when they are at the ſame diſtance from the firſt of aries, or are on the ſame ſide of the heavens.

The reader ſhould be careful to obſerve, that thoſe heavenly bodies, which are in conjunction, though they have the ſame longitude, have not always the ſame latitude, ſo that, though they appear on the ſame ſide of the heavens, yet they do not always appear in the ſame point of the heavens. The ſun is always in the plane of the ecliptic, and conſequently has no latitude at any time: venus deſcribes an orbit, which is inclined to the ecliptic, and conſequently may have latitude. Now venus and the ſun may both of them be at the ſame diſtance from the firſt of aries, and ſo may be in conjunction with each other, when venus has ſome latitude, and for that reaſon is not in the plane of the ecliptic, where the ſun is. If the ſun appears at C, Plat. XXI. fig. 3, in the plane of the ecliptic ABCD, venus may at the ſame time appear at G. In this caſe they would both appear on the ſame ſide of the heavens or in conjunction; but they would not appear in the ſame point of the heavens, becauſe venus has ſome latitude and the ſun has none.

In like manner any two of the heavenly bodies, may have the ſame longitude, or may be at equal diſtances from the firſt of aries, ſo as to appear on the ſame ſide of the ecliptic; and yet thoſe two bodies will not ap-

appear in exactly the ſame point of the heavens, if their latitudes are different, that is, if they are at different diſtances from the plane of the ecliptic.

52. *Two of the heavenly bodies are ſaid to be in oppoſition to each other, when their longitudes differ half a circle or* 180 *degrees.*

Any two of the heavenly bodies may be in oppoſition, though they are not exactly in oppoſite points of the heavens: they may have different latitudes, when their longitudes differ 180°; and if they have, they will only be on oppoſite ſides of the heavens and not in oppoſite points.

The ſun may be at C, for inſtance, Plat. XXI, fig. 3, when venus is at E, they are then in oppoſition, becauſe they are at the diſtance of half a circle from one another: but they are not in oppoſite points of the heavens, becauſe the ſun is in the ecliptic or has no latitude, whereas venus in that ſituation has latitude or is not in the ecliptic.

53. *The apparent motion of the ſun in the ecliptic is not uniform.*

The apparent motion of the ſun in the ecliptic is owing to the motion of the earth in its orbit, by propoſition 26: and conſequently, unleſs the earth moves uniformly in its orbit, the ſun will not appear to move uniformly in the ecliptic.

Now the orbit of the earth ABCD, Plat. XXII. fig. 2, is an ellipſis, the ſun is placed at S in one of the focuſes, and the earth deſcribes equal areas round it, in equal times, in ſuch a manner as is explaned in propoſition 159 of mechanics. For this reaſon the earth, as it paſſes from its greateſt diſtance C, to its leaſt diſtance A, is accelerated; and then, as it paſſes from its leaſt diſtance A to its greateſt diſtance C, it is retarded, by propoſition 161 of mechanics. Therefore, as the earth moves ſometimes quicker and ſometimes ſlower, the apparent motion of the ſun, ariſing from this motion of the earth, will be ſometimes quicker and ſometimes ſlower, or will not be uniform.

54. *The greateſt diſtance of the earth, or any other planet, from the ſun is called its aphelion, and the leaſt diſtance is called its perihelion.*

The former of theſe is called the higher apſis, and the latter the lower apſis. See propoſition 173 of mechanics.

55. *The eccentricity of the earth, or any other planet, is the diſtance between the ſun and the center of the elliptical orbit, in which the earth or planet moves.*

If *anpm*, Plat. XXII. fig. 4, is the orbit of a planet, and S the focus in which the ſun is placed, *c* the center of this elliptical orbit; then *c*S or the diſtance between the ſun and the center of the orbit is the planets eccentricity.

56. *If the middle diſtance of a planet from the ſun, and the eccentricity of the planet are known, the greateſt and leaſt diſtances may be found.*

The greateſt diſtance of a planet from the ſun, when it revolves in the orbit *pnam*, Plat. XXII. fig. 4, is *a*S, its leaſt diſtance is *p*S, its middle diſtance is *m*S, its eccentricity *c*S. But if *m*S and *c*S are known, then either *a*S or *p*S may be found. For *m*S the middle diſtance is equal to *cp* or to *ac* half the longer axis of the elliptical orbit, as has been ſhewn under propoſition 152 of mechanics. Now $ac + Sc = aS$, and $cp - Sc = pS$. Therefore if to the middle diſtance or half the longer axis we add the eccentricity, the ſum is the greateſt diſtance; and if from the middle diſtance or half the longer axis we ſubtract the eccentricity, the remainder is the leaſt diſtance.

57. *Our ſummer is eight days longer than our winter.*

By ſummer is here meant the time that paſſes between the vernal and autumnal equinoxes, and by winter the time that paſſes between the autumnal and vernal. The ecliptic is divided into twelve ſigns: ſix of theſe ſigns, aries, taurus, gemini, cancer, leo, virgo, are on the north ſide of the equator, and the other ſix, libra, ſcorpio, ſagittarius, capricornus, aquarius, piſces, are on the ſouth ſide of it. The equator croſſes the ecliptic at the firſt of aries and at the firſt of libra: ſo that from the vernal equinox to the autumnal the ſun paſſes through the ſix northern ſigns, and from the autumnal equinox to the vernal it paſſes through the ſix ſouthern ſigns. The ſun is eight days longer in paſſing through the ſix northern ſigns than it is in paſſing through the ſix ſouthern ones.

Let BCDA, Plat. XXII. fig. 2, be the earths orbit, and S the ſun, which is placed in one of the focuſes of this orbit. Whilſt the earth is at B, the ſun appears at H in the firſt of aries, and whilſt the earth moves from B through C to D, the ſun appears to run through the ſix northern ſigns, aries, taurus, gemini, cancer, leo, virgo. Whilſt the earth is at D, the ſun appears at F in the firſt of libra, and whilſt the earth moves from D through A to B, the ſun appears to run through the ſix ſouthern ſigns, libra, ſcorpio, ſagittarius, capricornus, aquarius, piſces. Now the line HF drawn from the firſt of aries to the firſt of libra through the ſun, divides the ecliptic, which is a circle, into two equal parts: but the ſame line divides

vides the earths orbit into two unequal parts, as the ſun is not in the center but in one of the focuſes of this orbit. The greater part is BCD, which the earth deſcribes, whilſt the ſun appears in the northern ſigns: the leſſer part is DAB, which the earth deſcribes, whilſt the ſun appears in the ſouthern ſigns. The aphelion of the earth, where it moves the ſloweſt, is in the greater part BCD, and its perihelion, where it moves the faſteſt, is in the leſſer part DAB. There are therefore two reaſons, why our ſummer is longer than our winter, or why the ſun is longer in the northern ſigns than in the ſouthern. Firſt, becauſe the ſun continues in the northern ſigns, whilſt the earth is deſcribing the greater part of its orbit. And ſecondly, becauſe the earths real motion, and conſequently the ſuns apparent motion, is ſlower, whilſt the ſun appears in the northern ſigns, than whilſt it appears in the ſouthern ones.

58. *The apparent diameter of the ſun is greater in winter than in ſummer.*

We have ſeen, in propoſition 53, that, whilſt the ſun is in the ſouthern ſigns, or in our winter, the earth is in its perihelion; and that, whilſt the ſun is in the northern ſigns, or in our ſummer, the earth is in its aphelion: but when the earth is in its perihelion or at its leaſt diſtance from the ſun, the ſuns apparent diameter will be greater, than when the earth is in its aphelion or at its greateſt diſtance, by propoſition 126 of optics.

The difference between the ſuns apparent diameter in ſummer and winter is not much; it is 32 minutes 47 ſeconds in winter, when it is the greateſt, and 31 minutes 40 ſeconds in ſummer, when it is the leaſt. For the orbit of the earth, though it is an ellipſis, does not differ much from a circle, having the ſun in its center; and conſequently the earths greateſt and leaſt diſtances do not differ much from one another. If the earths middle diſtance from the ſun is called 1000, its eccentricity will be 17. Therefore, by propoſition 56, its greateſt diſtance from the ſun is $1000+17=1017$, and its leaſt diſtance is $1000-17=983$.

But if the earth is farther from the ſun in ſummer than in winter, it may be aſked why the weather ſhould be the hotteſt in ſummer. In anſwer to this queſtion we muſt obſerve, that the difference between the diſtances of the ſun in theſe two different ſeaſons of the year is inconſiderable; and that the heat or coldneſs of the weather depends upon other cauſes, and will be but little affected by this. The principal reaſons, why our ſummer is hotter than our winter, are, firſt the greater height to which the ſun riſes above our horizon in the ſummer, and ſecondly the greater length of the days. The ſun is much higher at noon in ſummer than in winter, and conſequently, as its rays, in ſummer are leſs oblique

than in winter, more of them will fall upon the earths ſurface. In ſummer the days are very long and the nights very ſhort; therefore the earth and air are heated by the ſun in the day-time, more than they are cooled in the night-time; and upon this account, the heat will keep encreaſing. Whereas in winter the nights are very long and the days very ſhort: therefore during the abſence of the ſun in the night-time, the earth and air will cool, more than they are heated by the ſun in the day-time; and upon this account the heat will keep decreaſing.

59. *A direct motion, or a motion in conſequentia, is from weſt to eaſt amongſt the ſtars, or according to the order of the ſigns.*

If any of the heavenly bodies, or any point in the heavens, moves from weſt to eaſt, from aries to taurus, gemini, cancer, &c. this motion is direct. It is called a motion in conſequentia, becauſe, when a body or a point in the heavens moves in this manner, it moves out of the ſign, where it is, in conſequentia ſigna or into the following ſigns.

60. *A retrograde motion, or a motion in antecedentia, is from eaſt to weſt amongſt the ſtars, or contrary to the order of the ſigns.*

If any of the heavenly bodies, or any point in the heavens, appears to move from eaſt to weſt, from aries to piſces, aquarius, capricorn, &c. this motion is retrograde. It is called a motion in antecedentia, becauſe, when a body or a point in the heavens moves in this manner, it moves out of the ſign, where it is, in antecedentia ſigna, or into the preceding ſigns.

The apparent daily motion of the ſun is indeed from eaſt to weſt; but yet this is not a motion in antecedentia. For a motion in antecedentia is a motion from eaſt to weſt in reference to the fixed ſtars, ſince it carryes the body ſo moving from eaſt to weſt according to the order of the ſigns in the zodiac. Whereas the ſuns apparent daily motion is in reference to the earths ſurface; and the ſun, though it appears to move in this manner round the earth, is not carryed by this diurnal revolution either from weſt to eaſt or from eaſt to weſt amongſt the fixed ſtars, its diurnal motion does not make it appear to move out of one ſign into another any way.

61. *A periodical year is the time that the ſun takes up to deſcribe the whole ecliptic.*

If the ſun ſets out from any particular fixed ſtar it will be 365 days 6 hours 9 minutes and 11 ſeconds before it returns to the ſame ſtar again.

Let

Let S, Plat. XXII. fig. 2, be the ſun; then if the earth is at A in its orbit, the ſun will appear in that part of the heavens where the ſtar G is, and when the earth has finiſhed one period in its orbit ABCD, and is returned to the ſame place A again, the periodical year is completed; for in that time, by propoſition 27, the ſun will appear to have deſcribed the whole ecliptic.

62. *The ſolſtitial and equinoctial points are called cardinal points.*

63. *A tropical year is the time that the ſun, after it has left any one of the cardinal points, takes up to return to the ſame cardinal point again.*

If the tropical year begins, when the ſun is in the vernal equinoctial point, or in the firſt of aries, the tropical year will be completed, when the vernal equinox returns, or when the ſun is come round to the firſt of aries again. The length of the tropical year is 365 days, 5 hours, and 49 minutes.

64. *The equinoctial points move in antecedentia or go backwards.*

If ABCD, Plat. XXI. fig. 3, is the plane of the ecliptic, in which the ſun appears to move, and GHEF the plane of the equator; then B and D will be the equinoctial points. Now the equator does not always interſect the plane of the ecliptic in the ſame two points B and D; but the equator in a courſe of time changes its ſituation, ſo that Z and X ſhall be the two points of interſection or the two equinoctial points. In like manner theſe points of interſection or equinoctial points will keep ſhifting their place by the plane of the equator changing its ſituation, till they come to P and Q. And thus in time they will go quite round the eliptic. This motion of the equinoctial points is in antecedentia; or if the ſun moves in the direction ABCD, the equinoctial points are found to move in the contrary direction DCBA.

It is not neceſſary that the inclination of the equator to the plane of the ecliptic ſhould be altered by this motion of the equinoctial points: for the plane of the circle EFGH may turn round in the circle ABCD; ſo as to change the points of interſection B and D, and yet the angle, which theſe two circles make with one another, may continue the ſame, whether they interſect one another at B and D, or at Z and X, or at P and Q, or at any other points.

The reader may eaſily repreſent ſuch a motion as this upon the celeſtial globe. The broad wooden horizon of the globe muſt repreſent the ecliptic, and the axis of the globe muſt be placed perpendicular to that wooden ho-

horizon. In this poſition the equator upon the globe will coincide with the wooden horizon, and the ecliptic upon the globe, will make an angle of 23°. 30′. with the wooden horizon. Now ſince in ſhewing this motion the horizon of the globe repreſents the ecliptic, it is plane that the equator muſt be repreſented by a circle, which makes an angle of 23°. 30′. with the horizon, for ſuch an angle the equator makes with the ecliptic, by propoſition 32. But the ecliptic upon the globe, when the globe is in the poſition juſt deſcribed, makes an angle of 23°. 30′. with the horizon. Therefore in ſhewing this motion the ecliptic upon the globe is to repreſent the equator. The ecliptic upon the globe, in this poſition, cuts the horizon in two points and is inclined to it, as was ſaid, at an angle of 23°. 30′. Theſe two points of interſection repreſent the two equinoctial points. If the globe, whilſt it is in this poſition, is turned ſlowly round upon its axis from eaſt to weſt, theſe points of interſection will move round the ſame way, and the inclination of the circle, which in ſhewing this motion repreſents the equator, to the wooden horizon, which repreſents the ecliptic, will not be altered by ſuch a revolution of the points of interſection or equinoctial points.

This motion is called the preceſſion of the equinoxes, becauſe it carryes the equinoctial points in præcedentia ſigna, that is, it carryes them backwards amongſt the fixed ſtars, or from any one ſign in the zodiac into thoſe ſigns which are before it. The motion is a very ſlow one: for the equinoctial points take up 72 years to move one degree, and therefore they are 25920 years in deſcribing 360 degrees or in completing a revolution.

65. *The preceſſion of the equinoxes makes the tropical year ſhorter than the periodical year.*

The ſun moves in the ecliptic from weſt to eaſt or in conſequentia, by propoſitions 26, 2, and the equinoctial points move ſlowly the contrary way, by propoſition 60. Conſequently, when the ſun has ſet out from the firſt of aries in the ecliptic, or from the vernal equinoctial point, and is moving round in the ecliptic, this equinoctial point will meet it, before it has got quite round, that is, the vernal equinox will return, before the ſun has performed one entire revolution in the ecliptic.

If B, Plat. XXI. fig. 3, is the vernal equinoctial point, and the ſun ſetting out from thence appears to deſcribe the ecliptic in the direction BCDA; then, ſuppoſing the equinoctial points, had no motion it muſt go quite round the ecliptic and return to B again, before the next vernal equinox. But if, whilſt the ſun is moving one way, the equinoctial point moves

moves the contrary way, if whilſt the ſun is moving from B through C, D, and A, the equinoctial point moves from B to Z, then the ſun will arrive at this equinoctial point before it gets to B, or the vernal equinox will return, before the ſun has gone through the whole ecliptic. Now the time, that paſſes between one vernal equinox and the next, is the tropical year; and the time, that paſſes, whilſt the ſun is completing an apparent revolution in the ecliptic, is the periodical year. But by the preceſſion of the equinoxes the vernal equinox returns, before the ſuns apparent revolution in the ecliptic is completed. Therefore the tropical year is completed and begins again ſooner than the periodical year.

66. *By the preceſſion of the equinoxes the poles of the world are made to deſcribe a circle from eaſt to weſt round the poles of the ecliptic.*

The axis of the ecliptic is perpendicular to the plane of the ecliptic, and the axis of the world is perpendicular to the plane of the equator. Now in the preceſſion of the equinoxes the plane, and conſequently the axis, of the ecliptic reſts, whilſt the plane of the equator revolves from eaſt to weſt, ſo as ſucceſſively to cut the ecliptic in different places. But if a plane revolves, any line perpendicular to that plane muſt revolve the ſame way: and conſequently, as the equator revolves, the axis of the world revolves. But if the equator in this revolution is always equally inclined to the plane of the ecliptic, then the axis of the equator, or axis of the world, will be always equally inclined to the axis of the ecliptic: and conſequently the poles of the world, or extremities of the axis of the world, will always be at the ſame diſtance from the poles of the ecliptic, or from the extremities of its axis. Therefore, ſince the poles of the world, revolve round the poles of the ecliptic, and yet the former are always at the ſame diſtance from the latter, it follows that this motion of the poles of the world round thoſe of the ecliptic is in a circle.

When the celeſtial globe is in the poſition that we have deſcribed under propoſition 60, the reader may eaſily repreſent this motion of the poles. We ſhould remember that the wooden horizon of the globe repreſents the ecliptic, and conſequently that the axis of the globe, as it is placed perpendicular to the horizon, does not repreſent the axis of the world, but the axis of the ecliptic. Upon Senex's celeſtial globe, the reader will find the poles of the ecliptic marked; and ſince, when the globe is rectifyed for the preſent purpoſe, the circle upon the globe, which is deſigned for the ecliptic, is put to repreſent the equator, the poles of the ecliptic upon the globe muſt repreſent the poles of the equator or of the world. Now as the globe is ſlowly turned from eaſt to weſt, theſe points, which

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in ſhewing this motion repreſent the poles of the world, will revolve the ſame way round the poles of the globe, which are here ſuppoſed to repreſent the poles of the ecliptic.

The axis of the world may revolve in this manner, though its inclination to the plane of the ecliptic is not altered. For, ſince the plane of the equator may be equally inclined to the plane of the ecliptic in the whole revolution of the equinoctial points, a line, which is perpendicular to the plane of the equator, as the axis of the world is, will not alter its poſition in reſpect of the plane of the ecliptic, but will in its whole revolution continue to make the ſame angle with it. This the reader will find in repreſenting this motion upon the celeſtial globe: for the points, which are put for the poles of the world, will in the revolution of the globe keep always at the ſame diſtance from the wooden horizon, which is here put for the ecliptic.

67. *The ſpots, which appear in the ſuns diſc, adhere to its ſurface.*

When a ſpherical body is near enough to appear of its true figure, this appearance is owing to the different ſhading upon different parts of it: for as a circular piece of board, when it is properly ſhaded by painting, will by this ſhading be made to appear like a ſphere, ſo a ſpherical body appears of its true ſhape for the ſame reaſon that the plane board appears ſpherical. But if the ſphere is at a great diſtance, this difference of ſhading cannot be diſcerned by the eye, and conſequently, the ſphere will appear of its true ſhape no longer, the ſhading is then loſt, and therefore the ſphere will appear like a flat circle.

This is the apparent ſhape of the ſun; it ſeems to be a bright flat circle. This flat circle is called the ſuns diſc. By the help of a teleſcope ſome dark ſpots have been found in this diſc. And the queſtion here to be decided is, whether theſe ſpots adhere to the ſurface of the ſun, or whether they are opake bodies, which move round the ſun, at ſome diſtance from it, as the planets do, and which, when they come between the ſun and us, intercept its light and appear like ſpots upon its diſc.

If one of theſe ſpots appears upon the eaſtern limb or edge of the ſuns diſc, it moves from thence towards the weſtern edge, and arrives at the weſtern edge in about 13 days $\frac{1}{2}$. Here the ſpot diſappears; and in about 13 days $\frac{1}{2}$ more, it is ſeen again upon the eaſtern edge; and ſo continues to go round completing its apparent revolution in 27 days, as long as the ſpot laſts. For theſe ſpots do not laſt very long in the ſame ſtate: ſome of them vaniſh, before they have performed one revolution, others divide themſelves into a number of ſmaller ſpots and others, that were ſmall

ſpots

ſpots at firſt run together, and form one that is larger. But ſome have been obſerved to continue long enough to complete two or three revolutions.

From this obſervation, that any ſpot in one revolution of it diſappears, or is hid behind the ſun, juſt as long as we ſee it upon the diſc, we may prove that the ſpot adheres to the ſuns ſurface. A ſpot completes an apparent revolution in about 27 days, for half that time we ſee it upon the diſc of the ſun, and for the other half it diſappears. This could not be the caſe, if theſe ſpots were opake bodies moving round the ſun at any diſtance from its ſurface: for then in one revolution they would diſappear for a longer time, and would be ſeen for a leſs time, than half the time of a revolution. Let A, Plat. XXII. fig. 5, be the center of the ſuns diſc, D its eaſtern and C its weſtern edge, HEG the orbit of an opake body moving round it, and B the eye of the ſpectator at the earth. If two lines BD and BC are ſuppoſed to be drawn from the ſpectators eye B ſo as to touch the ſun at D and C, then DBC, the angle contained between theſe lines is the angle, under which the ſuns diameter appears to a ſpectator on the earth. EG is the only part of the ſuppoſed bodys orbit, that is within this angle DBC, and conſequently, if the body was in any other part of its orbit, except EG, it would not appear upon the ſuns diſc. But EG is leſs than half its orbit; and the body would not take up half the time of a revolution to deſcribe EG. Therefore ſuch a body would not be ſeen upon the ſuns diſc, as the ſpots are, for half the time of a revolution. But if the orbit HEG is no bigger than LDFC or is cloſe to the ſun, that is, if the ſpot adheres to the ſuns ſurface, then half its orbit DFC, will be within the angle DBC, and conſequently the ſpot will appear upon the ſuns diſc during one half its revolution; but during the other half of its revolution, whilſt it deſcribes CLD, it will diſappear, becauſe then it will be behind the ſun and ſo will be hidden from us.

68. *The ſun is a ſpherical body, that revolves upon its own axis from weſt to eaſt, in about 25 days.*

The ſpots, that appear in the ſuns diſc adhere to its ſurface, by propoſition 63. And theſe ſpots revolve. Therefore the ſun revolves round its axis.

Whatever ſide of the ſun is turned towards the earth in this rotation, it always appears to be a flat bright circle. But all the ſides of it could not appear in this manner unleſs it was a ſphere. Therefore the ſun is a ſpherical body.

In the laſt propoſition we ſaid that an apparent revolution of a ſpot in the ſun is completed in about 27 days; and in this propoſition we ſay

that a rotation of the ſun is completed in 25 days. It may therefore be aſked whether if the ſpot adheres to the ſun, it is poſſible for the ſpot to take up more time to complete one revolution, than the ſun takes up to revolve once upon its axis? To this we anſwer that the time, in which a ſpot revolves once round, is juſt equal to the time of one rotation of the ſun round its axis. For as the ſun completes a rotation round its axis, ſo the ſpot completes a revolution in about 25 days. But then any ſpot in the ſun performs a revolution in leſs time, than it appears to us to have performed one. The ſpot revolves round the axis of the ſun once in about 25 days, but to us, who live on the earth, it does not appear to come to the ſame place in the ſuns diſc, or to complete a revolution, in leſs than 27 days. The motion of the earth in its orbit is the reaſon why the time of an apparent revolution of any ſpot is longer than the time of a real revolution. Let TSVW, Plat. XXII. fig. 1, be the ſun, RNAB the earths orbit, and A the place of the earth in its orbit. Let P be one pole of the ſun, or one extremity of the axis, round which it revolves, SV a diameter of the ſun, and V the place of a ſpot, which appears in the middle or center of the ſuns diſc, when the earth is at A. The ſun will have turned once round upon its axis and the ſpot will have revolved once, when the diameter SV has turned once round upon the point S, and is come into the ſame ſituation MSVA, that it was in at firſt. But then this ſpot will not appear to a ſpectator on the earth to have completed its revolution till it is ſeen again in the middle or center of the ſuns diſc. Now, during this rotation, the earth will have changed its ſituation, and will be at B: and conſequently the ſpot will not be ſeen in the center of the diſc, till the diameter SV has turned more than once round, and has gotten into the line TPWB.

A ſpot in the ſun moves round its axis from weſt to eaſt, in 25 days, and, after it has completed a revolution, it would appear to us to have completed it, if the earth was at reſt. But the earth in the mean time has moved forwards in its orbit from weſt to eaſt, or in the ſame direction that the ſpot moves, and conſequently the ſpot muſt move ſomething more than once round before it can come to the ſame ſituation in reſpect of the earth that it was in at firſt. But till it comes to the ſame ſituation in reſpect of the earth, it does not appear to have completed a revolution. Therefore the time of an apparent revolution of a ſpot, when ſeen from the earth, is longer than the time that the ſun takes up to revolve once round its axis.

In what manner we ſhew, by the ſpots moving from eaſt to weſt croſs the ſuns diſc, that theſe ſpots and the ſun with them turn round upon the ſuns axis from weſt to eaſt, ſhall be explaned hereafter.

69. *The*

69. *The axis of the ſun is inclined to the plane of the ecliptic.*

We have proved, in propoſition 68, that the ſun has a rotation round its axis. Now in ſuch a rotation every ſpot upon it muſt deſcribe a circle: and the plane of the circle ſo deſcribed muſt be perpendicular to this axis. A circle drawn round the ſun in the middle between the two poles of the ſun we may call the ſuns equator. Any ſpot, that is in the middle between the two poles, deſcribes this equator in one rotation of the ſun upon its axis. Any other ſpot deſcribes in the mean time ſome other circle, that is parallel to this equator, but is nearer to one of the poles than to the other. If the ſuns axis was perpendicular to the plane of the ecliptic, then the plane of the ſuns equator would be always in the plane of the ecliptic. But the earth, and conſequently the eye of a ſpectator upon the earth, is always in the plane of the ecliptic, by propoſition 28. Therefore if the axis of the ſun was perpendicular to the plane of the ecliptic, the eye of a ſpectator upon the earth would be in the plane of the ſpots motion, if the ſpot was in the ſuns equator, or elſe it would be in a plane parallel to the plane of the ſpots motion, if the ſpot was in any other part of the ſun. But if the eye is in the plane of a circle, or if the circle is very remote and the eye is in a plane, which is parallel to the plane of the circle, in either of theſe caſes, the circle will appear as if it was a right line. Therefore if the axis of the ſun was perpendicular to the plane of the ecliptic, all the ſpots upon it would at all times appear to deſcribe right lines. But theſe ſpots ſometimes appear to croſs the diſc of the ſun in curve lines. Therefore the axis of the ſun is inclined to the plane of the ecliptic.

If DEBC, Plat. XXII. fig. 6, is the diſc, and a ſpot appears upon the eaſtern limb or edge at D, ſuch a ſpot does not always go croſs the diſc in a right line DC: the ſpots are found ſometimes to deſcribe a right line DC, ſometimes a curve DFC, and ſometimes the ſame ſort of curve in a different ſituation DGC.

The inclination of the ſuns axis to the plane of the ecliptic will make the ſpots appear ſometimes to deſcribe curve lines. For if a line drawn from the ſpectators eye is perpendicular to the plane of a circle, as it is when we look at the circle ALGD, Plat. XXII. fig. 7, which is drawn upon the plane of the paper, then the circle appears of its true figure. If this line is oblique to the plane of the circle, it appears elliptical, as in fig. 8. And if the eye is in the plane of the circle it appears like a right line. All the ſpots on the ſuns ſurface deſcribe circles, that have their planes perpendicular to its axis: and conſequently, as the ſpectators eye is always

in the plane of the ecliptic, a line drawn from his eye cannot be perpendicular to the plane of a ſpots motion, unleſs it coincides with the axis of the ſun, that is, unleſs the axis of the ſun was in the plane of the ecliptic, and one end of it was turned towards the earth. But the axis of the ſun is not in the plane of the ecliptic. Therefore a line drawn from a ſpectators eye upon the earth can never be perpendicular to the plane of the circle, which any ſpot deſcribes: and conſequently no ſpot will appear to deſcribe a circle. But the axis of the ſun is inclined to the plane of the ecliptic at an angle of about 82 ½ degrees: and for this reaſon all circles upon the ſuns ſurface, that are perpendicular to its axis, are likewiſe inclined to the plane of the ecliptic; therefore ſince the ſpectators eye is always in the plane of the ecliptic, a line drawn from his eye to the ſun will be oblique to the plane of the circle, which any ſpot deſcribes. Therefore the circle, which the ſpot deſcribes, will appear elliptical. But only half of this ellipſis DGC, fig. 6, will be on the ſide of the ſun, that is towards us, or on the ſuns diſc, and the other half of it will be hidden by the ſun, becauſe this other half will be behind the ſun: ſo that the path of a ſpot acroſs the ſuns diſc will be a ſemi-ellipſis. If the plane of the circle continued would paſs below the ſpectators eye, then DGC, will be the path of the ſpot: but if the plane of it continued would paſs above the eye, then the ſpot will appear to deſcribe the ſemi-ellipſis DFC, But, notwithſtanding this inclination of the ſuns axis to the plane of the ecliptic, the ſpots upon its ſurface may ſometimes appear to deſcribe right lines. For ſince the ſuns equator, as it is inclined to the plane of the ecliptic, croſſes this plane in two points, if the earth, whilſt it revolves in its orbit, comes to either of theſe points, the eye of a ſpectator will be in the plane of the ſuns equator; and conſequently the ſpots will deſcribe circles, that are either in the ſame plane with the eye or have their planes parallel to that in which the eye is. Therefore in this ſituation of the earth the ſpots of the ſun will appear to deſcribe right lines.

70. *The ſpots in the ſun appear to move ſlower when they are at the edges of the diſc than when they are in the middle of it.*

The ſun moves uniformly round its axis, and therefore each ſpot deſcribes its reſpective circle uniformly, ſo that in the rotation it deſcribes equal ſpaces in equal times. But notwithſtanding this uniform motion of the ſpots, they will not appear to move uniformly, when they are ſeen from the earth. ADGL, Plat. XXII. fig. 7, is a circle divided into equal parts by the letters ABCDEFGHILMN. But if this circle is ſeen ob-

obliquely, theſe equal parts will appear to the eye to be unequal, as in fig. 8. For the parts about the middle *ilm* are ſeen by the eye at right angles, whereas thoſe at the ſides, *ghi*, *mna*, are inclined to the eye; and conſequently, by propoſition 129 of optics, the parts about the middle will appear longer than thoſe about the ſides. For this reaſon, if a ſpot is deſcribing the curve DGC; Plat. XXII. fig. 6, though it deſcribes equal ſpaces in equal times, and therefore moves uniformly; yet as theſe equal ſpaces ſeen from the earth appear to be ſhorter near the edges of the diſc and longer in the middle of it, the ſpaces, which the ſpot deſcribes in equal times will be apparently unequal, it will appear, in any given time, to deſcribe a ſhorter ſpace, when it is near the edge, than it appears to deſcribe, when it is in the middle of the diſc. But the apparent velocity of the ſpot is as the apparent length of the ſpaces deſcribed in equal times, by propoſition 9 of mechanics. Therefore the ſpot will appear to move ſlower, when it is near either edge, than it does, when it is in the middle of the diſc. The difference of apparent velocity of a ſpot at the edge and in the middle will be the ſame, if it appears to deſcribe a right line croſs the ſuns diſc. For then, if the circle, which the ſpot deſcribes, is divided into equal parts, the equal parts in the middle will be apparently longer than thoſe at the edges: and conſequently the ſpaces deſcribed in equal times will be apparently unequal, and the ſpot will ſeem to deſcribe ſhorter ſpaces in a given time, when it is near the edge of the diſc, than it does when it is the middle of it.

CHAP. III.

Of the inferior planets.

71. *Mercury and venus are called inferior planets.*

THESE two are called inferior planets in the Copernican ſyſtem; becauſe they are lower in the ſyſtem than the earth. They are lower in the ſyſtem than the earth, becauſe they are nearer to the ſun, which is the center of the ſyſtem.

72. *If we call the middle diſtance of the earth from the ſun* 1000, *the middle diſtance of mercury is* 387; *and its eccentricity is* 80. *The periodical time of mercury is* 87 *days*, 23 *hours*, 16 *minutes*.

Mercury is always ſo near to the ſun, that the great light of the ſun has prevented us from obſerving whether there are any particular ſpots on the ſurface of it. And for want of ſeeing ſuch ſpots it has not yet been determined what time mercury takes up to turn round its axis. Indeed

deed it is only from analogy that we can determine whether it has any rotation round its axis: only, as the other planets in the ſyſtem have ſuch a motion, we may conclude that this has too.

73. *If we call the middle diſtance of the earth from the ſun* 1000, *the middle diſtance of venus is* 724, *and its eccentricity is* 5. *The periodical time of venus is* 224 *days,* 16 *hours,* 49 *minutes: and it revolves round its axis in* 23 *hours.*

74. *The elongation of a planet is its apparent diſtance from the ſun.*

When a planet is in conjunction with the ſun, it has no elongation: when it is in oppoſition, its elongation is 180 degrees. If A, Plat. XXIII. fig. 1, is the place of the earth, S the ſun, FVPQRTD a circle in the concave ſphere of the heavens, and C the place of venus in its orbit: then to a ſpectator at A, the ſun appears in the heavens among thoſe ſtars that are at Q, and venus appears among thoſe that are at R, by propoſition 135 of optics. In this caſe the angle QAR is the apparent diſtance between the ſun and venus, and this is called the angle of elongation.

75. *The nodes of a planets orbit are thoſe two points where the orbit cuts the plane of the ecliptic.*

We obſerved, in propoſition 8, that the orbits of all the planets are inclined to the plane of the ecliptic, and conſequently croſs this plane: the two points where they croſs or interſect it are called the nodes.

76. *The line of the nodes of any planet is a line drawn from one node to the other.*

77. *The limits of a planets orbit are two points in the middle between the two nodes.*

If ABCD, Plat. XXI. fig. 3, is a circle in the plane of the ecliptic, and BGDE is the orbit of a planet; then B and D are the two nodes, a line drawn from B to D is the line of the nodes, G and E are the limits. Thus when the planet is at B, it is in the node; and as the orbit interſects the plane of the ecliptic at the node, the planet, whilſt it is in the node, is in the plane of the ecliptic. As it moves from B, it departs from the plane of the ecliptic, till it has moved through 90 degrees or $\frac{1}{4}$ of its orbit, and then it arrives at its greateſt diſtance from the ecliptic, or at the limit G: from thence it deſcends to the other node D, and from thence to the limit E, and returns to the node B.

74. *There*

78. *There are two different ſituations of an inferior planet, in either of which it will appear in conjunction with the ſun.*

Let A, Plat. XXIII. fig. 1, be the earths place in its orbit ABO, E the place of venus in its orbit EHG, S the ſun, and FVPQRTD an arc of a circle in the concave ſphere of the heavens. In this ſituation the ſun and venus are on the ſame ſide of the earth, and will appear in the ſame part of the heavens ſo as to be in conjunction. Or if the earth is at A, venus at G, and the ſun at S, in this ſituation likewiſe the ſun and venus are on the ſame ſide of the earth, and will appear in the ſame part of the heavens ſo as to be in conjunction. In the former ſituation, when the planet is at E, it is between the ſun and the earth, and in the latter ſituation, when the planet is at G, the ſun is between the earth and the planet.

79. *When either of the inferior planets appears in conjunction, if it is nearer to the earth than the ſun is, this is called its inferior conjunction.*

If the earth is at A, Plat. XXIII. fig. 1, the planet at E and the ſun at S, the planet is nearer to the earth than the ſun is, and appears in conjunction. This is the inferior conjunction of the planet.

80. *When either of the inferior planets appears in conjunction, if it is farther from the earth than the ſun is, this is called its ſuperior conjunction.*

If the earth is at A, Plat. XXIII. fig. 1. the planet at G, and the ſun at S, the planet is farther from the earth than the ſun is, and appears in conjunction. This is the ſuperior conjunction of the planet.

81. *An inferior planet is at its greateſt elongation, when a line drawn from the earth through the planet is a tangent to the planets orbit.*

If an inferior planet is at E, Plat. XXIII. fig. 1, the earth at A, and the ſun at S; the elongation is nothing, for the planet is then in its inferior conjunction. As the planet moves from E towards *y* its elongation encreaſes: for when it is at *y*, it appears in the line A*y*P, whilſt the ſun appears in the line ASQ, ſo that PAQ will be its elongation. When the planet is arrived at *x* it appears in the line A*x*V, which is a tangent to its orbit, and then the elongation is VAQ, which is the greateſt that it can be on that ſide of the ſun. For after this the elongation decreaſes: when the planet is at K, its elongation is PAQ: and when it arrives at G, its elongation is nothing, for then it is in its ſuperior conjunction. As

the

the planet moves on from G, its elongation encreaſes again: for when it comes to C, it appears in the line ACR, ſo that its elongation is RAQ: when the planet comes to H, a line AHT drawn from the earth through the planet is a tangent to the orbit, and the elongation is TAQ, which is the greateſt that it can be on the other ſide of the ſun. For after this the elongation decreaſes: when the planet is at *z*, its elongation is RAQ: and when it arrives at E, its elongation is nothing, as at firſt. In this propoſition we have ſuppoſed the planets orbit to be circular: and though the orbits are elliptical, yet as they differ little from circles we might make this ſuppoſition without any great error.

82. *The inferior planets are never in oppoſition to the ſun.*

When a planet is in oppoſition to the ſun, the planet is on one ſide of the earth, and the ſun is on the other ſide, or the earth is between the ſun and the planet; and conſequently the earth muſt be nearer to the ſun than the planet is. Now the earth cannot be nearer to the ſun than the planet is, unleſs the ſemidiameter of the earths orbit is leſs than the ſemidiameter of the planets orbit. But the ſemidiameter of the earths orbit is greater than the ſemidiameter of the orbit of an inferior planet, by propoſition 7. Therefore the earth cannot be nearer to the ſun than an inferior planet: and conſequently the earth can never be between the ſun and an inferior planet, or no inferior planet can be on one ſide of the earth, whilſt the ſun is on the other ſide, or no inferior planet can ever be in oppoſition.

If venus is at E, Plat. XXIII. fig. 1, it is ſeen in its inferior conjunction: as it moves on from thence the elongation encreaſes, till it comes to *x*, and then the elongation on one ſide of the ſun is the greateſt, by propoſition 77. But the elongation VAQ, when the planet is at *x*, is not 180 degrees, or VQ is not half a circle. Therefore the planet is not then in oppoſition, by propoſition 52. As the planet moves on from *x*, its elongation decreaſes, till it comes to its inferior conjunction at G. After this the planet appears on the other ſide of the ſun and the elongation encreaſes, till the planet comes to H: and then the elongation is the greateſt on this ſide, by propoſition 77. But the elongation TAQ, when the planet is at K is not 180 degrees, or TQ is not half a circle. Therefore, by propoſition 52, as the planet does not appear at the diſtance of half a circle from the ſun, it is not in oppoſition.

83. *A planet is in quadrature or in the quarters, when it is* 90 *degrees or a quarter of a circle diſtant from the ſun.*

80. *The*

84. *The inferior planets are never in quadrature.*

The elongation of a planet muſt be 90 degrees, if it is in quadrature. But the greateſt elongation of an inferior planet can never be 90 degrees or a right angle. For the greateſt elongation is the angle contained between ASQ a line drawn from the earth through the ſun, Plat. XXIII. fig. 1, and AHT a tangent to the planets orbit drawn from the earth through the planet: and as the orbit of the planet is leſs than the orbit of the earth, theſe two lines can never make a right angle with each other. If AQ and AT make a right angle with each other, then AT muſt be a tangent to ABO the earths orbit. Euc. b. III. prop. 18. Now AT is a tangent to the planets orbit, and conſequently cannot be a tangent to the earths orbit too, unleſs the planets orbit was equal to and coincided with the earths orbit. But the planets orbit is leſs than the earths orbit. Therefore AT can never make a right angle with AQ; and conſequently the planets elongation from the ſun can never be 90 degrees, or the planet can never be in quadrature.

From hence it appears that the inferior planets never appear very far from the ſun. When we ſee either venus or mercury, it is either in an evening in the weſt ſoon after the ſun has ſet, or in a morning in the eaſt not long before the ſun riſes. Venus is indeed bright enough ſometimes to be ſeen in the day time, but then it is not far from the ſun. The greateſt elongation of venus is found by obſervation to be about 48 degrees, and that of mercury about 33 degrees.

85. *As venus moves from the ſuperior conjunction to the inferior, it ſets after the ſun and is called the evening-ſtar; as it moves from the inferior conjunction to the ſuperior, it riſes before the ſun and is called the morning-ſtar.*

If the earth is at A, Plat. XXIII. fig. 1, and venus at G, venus is then in its ſuperior conjunction. Now if we conceive a line to be drawn from G the ſuperior to E the inferior conjunction, this line paſſes through the ſun and divides the orbit of venus into two parts GHE and E*x*G. The part GHE is to the eaſt, and the part E*x*G is to the weſt of the ſun. Therefore, as venus leaves the ſuperior conjunction at G and moves in its orbit from weſt to eaſt, it gets into the eaſtern part of its orbit GHE, and will be to the eaſtward of the ſun till it arrives in its inferior conjunction E. Whilſt venus is thus to the eaſtward of the ſun, the ſun muſt be to the weſtward of venus, and conſequently the ſun will ſet in the weſt before venus. Therefore venus, if it is far enough from the ſun, will be

ſeen in the weſt after the ſun is down: and in this caſe it is called the evening-ſtar.

But as venus leaves the inferior conjunction at E, it gets into the weſtern part of its orbit E*x*G, and will be to the weſtward of the ſun, till it arrives at its ſuperior conjunction G. Whilſt venus is thus to the weſtward of the ſun, it will ſet in the weſt before the ſun ſets; and conſequently will not be ſeen in an evening: for it ſeldom is bright enough to be ſeen before the ſun is down. But then as it ſets before the ſun in an evening, it will riſe before the ſun in a morning. Therefore venus, if it is far enough from the ſun, will be ſeen in the eaſt before the ſun riſes: and in this caſe it is called the morning-ſtar.

Now the motion of venus in its orbit is from weſt to eaſt. And therefore the reader may perhaps be at a loſs to know, why venus, when it ſets out from its inferior conjunction E, ſhould by moving from weſt to eaſt get to the weſtward of the ſun; ſince a motion from eaſt to weſt is more likely to carry it to the weſtward. This difficulty will clear up hereafter, when we ſhall ſhew that the inferior planets at the time of their inferior conjunction appear to a ſpectator on the earth to move in antecedentia or from eaſt to weſt. For when we ſay that the inferior and all the other planets revolve in their orbits in conſequentia, or from weſt to eaſt, we mean that their real motions are this way, and that, if they were ſeen by a ſpectator at the ſun, they would appear to move this way. But to a ſpectator on the earth, they appear to move ſometimes from weſt to eaſt, and ſometimes from eaſt to weſt.

86. *The greateſt elongation of an inferior planet on one ſide of the ſun is not always equal to the greateſt elongation of the ſame planet on the other ſide.*

If the orbits of theſe planets were circular their greateſt elongations on each ſide of the ſun would be equal to one another. Thus in the circular orbit *x*E*z*H, Plat. XXIII. fig. 1, QAT the greateſt elongation on one ſide is equal to QAV the greateſt elongation on the other ſide. Becauſe in a circular orbit the real diſtance of the planet from the ſun is the ſame on either ſide, when a line drawn from the earth through the planet is a tangent to the planets orbit, that is, at the time of the greateſt elongation: and conſequently, if the real diſtances from the ſun are equal, the apparent diſtances or elongations will be ſo too.

But the inferior planets do not move in circular but in elliptical orbits, and the ſun is in one of the focuſes. Therefore in the orbit NILM, the planet at the time of its greateſt elongation on one ſide may be in its aphe-

li-

ſion at L; and then at the time of its greateſt elongation on the other ſide it will be in ſome point N of its orbit, which is nearer to the ſun than the aphelion. Upon this account the real diſtances of the planet from the ſun at the time of the greateſt elongations will be unequal: and therefore the apparent diſtances ſeen from the earth will be unequal too, or the greateſt elongation on one ſide of the ſun will not be equal to the greateſt elongation on the other ſide.

87. *If the greateſt elongation of an inferior planet is known, the proportion which the ſemidiameter of the planets orbit bears to the ſemidiameter of the earths orbit may be found.*

Let PAQ be the angle of mercurys greateſt elongation, and ſuppoſe a line drawn from L the place of mercury at the time of its greateſt elongation to S the ſun. LS is the ſemidiameter of mercurys orbit, and AS is the ſemidiameter of the earths orbit. We are to determine the proportion, which LS bears to AS, from knowing the angle PAQ or LAS.

AP is a tangent to mercurys orbit, by propoſition 81: and ſince LS the ſemidiameter of the orbit is drawn from the point of contact L, this ſemidiameter LS is perpendicular to the tangent AL. Euc. b. III. prop. 18. Therefore LS is the ſine of the angle of elongation LAS, and AS is the radius. From hence then it follows, that by looking into any tables of ſines we may determine what proportion LS bears to AS: for LS the ſemidiameter of mercurys orbit is to AS the ſemidiameter of the earths orbit, as the ſine of the greateſt elongation is to the radius.

In the ſame manner the proportion between the ſemidiameter of venus's orbit and the ſemidiameter of the earths orbit may be determined.

Theſe ſemidiameters are the reſpective diſtances of mercury, venus, and the earth, from the ſun. But then the reader muſt obſerve, that we cannot by this method determine what thoſe diſtances are in any real meaſures, we cannot for inſtance determine what number of furlongs or miles, or ſemidiameters of the earth are contained in any of theſe diſtances. All that we find out from hence is the proportion, that the diſtances bear to one another. And the proportion is what has been already ſet down in propoſitions 68, 69, the reſpective diſtances of mercury, venus, and the earth from the ſun are as 387, 724, 1000.

88. *When venus is in the inferior conjunction, it is nearer to the earth than to the ſun: but when mercury is in the inferior conjunction, it is farther from the earth than from the ſun.*

If A, Plat. XXIII. fig. 1, is the place of the earth, E the place of venus in the inferior conjunction, and S the ſun; in this ſituation AE is the

diſtance of venus from the earth, and ES is the diſtance of venus from the ſun. Now AE is leſs than ES. For if we call AS 1000, ES is 724, and conſequently AE, which is the difference between AS and ES, is 1000—724=276, by propoſitions 72, 73. Therefore, when venus is in the inferior conjunction, its diſtance from the earth is to its diſtance from the ſun, as 276 to 724.

If I is the place of mercury, in its inferior conjunction; then, by propoſitions 72, 73, if we call AS 1000, IS is 387, and conſequently AI, which is the difference between AS and IS, is 1000—387=613. Therefore, when mercury is in the inferior conjunction, its diſtance from the earth is to its diſtance from the ſun as 613 to 387.

89. *The apparent velocity of the inferior planets is the greateſt at the two conjunctions.*

Venus, for inſtance, ſeen from the earth at A, Plat. XXIII. fig, 1, appears to move faſter when it is at the inferior or ſuperior conjunctions E and G, than when it is in any other ſituation. The eye of a ſpectator upon the earth is always in the plane of the ecliptic, by propoſition 8: and the orbit of venus is inclined to the plane of the ecliptic, by propoſition 8: therefore, the orbit of venus, if it was a viſible circle, would appear to a ſpectator upon the earth in the elliptical form *abcdefghilmn*, Plat. XXII. fig. 8. If the orbit of venus is divided into equal parts, theſe parts ſeen from the earth will appear unequal; by propoſition 129 of optics; for the equal parts about *d* and *l* are ſeen directly, and thoſe about *f*, *g*, *h*, *b*, *a*, *n*, are ſeen obliquely; ſo that the parts about *d* and *l* will appear longer than thoſe about *f*, *g*, *h*, *b*, *a*, *n*. The orbit of venus is not indeed a viſible circle; but from this ſuppoſition we may determine the apparent velocity of venus's motion. If venus's motion is uniform, it will deſcribe equal ſpaces in equal times. But the apparent length of equal ſpaces in venus's orbit, when ſeen from the earth, is unequal, and conſequently venus, when ſeen from the earth, will appear to deſcribe unequal ſpaces in equal times, or will appear to move ſometimes faſter and ſometimes ſlower. When venus is in the parts of its orbit about *d* and *l* it deſcribes in a given time ſpaces apparently longer, than thoſe which it deſcribes in an equal time, when it is in the parts of its orbit about *f*, *h*, *b*, *n*. But the apparent velocity of venus is as the apparent length of the ſpaces deſcribed in equal times, by propoſition 9 of mechanics. Therefore the apparent velocity of venus is greater, when it is about *d* or *l* in its orbit, than when it is about *f*, *h*, *b*, or *n*.

Thus we have ſhewn that the apparent velocity of venus is different, as the parts of its orbit, where it is moving are differently ſituated in reſpect

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of the ſpectator's eye. Its apparent velocity is the greateſt, when it is in ſuch parts of its orbit as the ſpectator ſees directly. It remains therefore to be ſhewn, that theſe parts of the orbit, where venus is at the two conjunctions, are ſeen directly from the earth. The orbit HExG, Plat. XXIII. fig. 1, is drawn circular, becauſe the paper, upon which it is drawn, is in the plane of it. But if A is the place of the earth, and the eye was at A, ſo as to view this circle obliquely, the circle would appear elliptical, the parts about E and G would be ſeen directly, for AE is perpendicular to a tangent at E, and the parts about H and x would be ſeen obliquely. But, if A is the place of the earth, then E and G are the places of the two conjunctions of venus. Therefore the parts of venus's orbit about the two conjunctions are ſeen directly, and the other parts are ſeen obliquely: and conſequently, from what has been ſaid, it follows, that the apparent velocity of venus, when it is in either conjunction, is greater, than when it is in any other ſituation.

We have here ſuppoſed that the eye at A ſees the plane of venus's orbit obliquely, from whence it follows that venus's orbit would appear elliptical. But it is poſſible that the eye of a ſpectator upon the earth may be in the plane of venus's orbit, as ſhall be ſhewn hereafter. In this caſe the orbit of venus will appear to be a right line, by what has been ſaid under propoſition 69. But in this caſe, as well as in the other, the parts about E and G the two conjunctions will be ſeen directly by the eye at A, and the other parts will be ſeen obliquely: and conſequently the apparent motion will be ſwifter in the two conjunctions than any where elſe.

90. *The apparent motion of a planet, if ſeen from the earth, is called its geocentric motion, and if ſeen from the ſun, is called its heliocentric motion.*

Theſe words *geocentric* and *heliocentric* are uſed in other inſtances, but in the ſame ſenſe. Thus the apparent place of a planet, if ſeen from the earth, is called its geocentric place, and if ſeen from the ſun, is called its heliocentric place. The latitude of a planet, if ſeen from the earth, is called its geocentric latitude, and if ſeen from the ſun, is called its heliocentric latitude.

91. *When the inferior planets are paſſing from their greateſt elongation on one ſide of the ſun through their ſuperior conjunction to their greateſt elongation on the other ſide, their geocentric motion is direct.*

In order to explane this propoſition we will ſuppoſe the earth to be at reſt at A, Plat. XXIII. fig. 1; and will correct this ſuppoſition hereafter

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by ſhewing that the apparent motion of venus or mercury ſeen from the earth is the ſame in this reſpect, whether the earth moves in its orbit or reſts at A. What we affirm then is this: as venus, for inſtance, moves from *x* its greateſt elongation on one ſide of the ſun, through G its ſuperior conjunction, to H its greateſt elongation on the other ſide, it will appear to a ſpectator upon the earth to move from weſt to eaſt, according to the order of the ſigns, that is, its geocentric motion will be direct.

The planets move round the ſun from weſt to eaſt: and conſequently if there was a ſpectator at the ſun, they would appear to him to move through the zodiac according to the order of the ſigns, that is, the heliocentric motion of venus or any other planet is direct. Now if the ſun S and the earth A are both on the ſame ſide of the planet, a ſpectator at the earth is in the ſame ſituation in reſpect of the planet and its motion, as if he had been at the ſun. Whilſt the planet is moving from *x* through G to H, a ſpectator either at A or at S is on the concave ſide of the planets orbit; and conſequently the planet will appear to move in the ſame manner, whichever of theſe two places the ſpectator views it from. But the apparent motion of the planet, when ſeen from the ſun, is direct; and conſequently its motion, when ſeen from the earth, will likewiſe be direct, whilſt it is moving from its greateſt elongation *x* on one ſide of the ſun, through its ſuperior conjunction G, to its greateſt elongation H on the other ſide.

When venus is at *x* it appears, to a ſpectator on the earth at A, to be in the line A*x*V, by propoſition 135 of optics, or is ſeen among the fixed ſtars at V. When venus has moved to K, it appears in the line AKP, or is ſeen among the fixed ſtars at P. When it has moved to G, it is in its ſuperior conjunction. When it has moved to C it appears among the fixed ſtars at R. And when it is come to K, it appears among the fixed ſtars at T. Thus whilſt venus has moved in its orbit from *x* its greateſt elongation on one ſide of the ſun, through G its ſuperior conjunction, to H its greateſt elongation on the other ſide, it appears to have deſcribed the arc VPQRT in the concave ſphere of the heavens. But the letters *x*KGCH lie from weſt to eaſt, becauſe they lie in the ſame direction that the planet moves round the ſun: and the letters VPQRT lie in the ſame direction with *x*KGCH. Therefore as the planet ſeems to a ſpectator on the earth to deſcribe the arc VPQRT, its apparent motion ſeen from the earth is direct, or from weſt to eaſt.

92. *As the inferior planets move from their greateſt elongation on one ſide of the ſun, through their inferior conjunction, to their greateſt elongation on the other ſide, their geocentric motion is retrograde.*

Whilſt

Whilſt venus, for inſtance, is moving from its greateſt elongation H, Plat. XXIII. fig. 1, through its inferior conjunction E to its other greateſt elongation *x*, it appears to a ſpectator upon the earth at A to move backwards, or from eaſt to weſt, or contrary to the order of the ſigns.

A ſpectator at the ſun is on the concave ſide of the planets orbit. But whilſt venus is moving from its greateſt elongation H on one ſide, through its inferior conjunction E, to *x* its greateſt elongation on the other ſide, a ſpectator upon the earth at A is on the convex ſide of its orbit. Therefore if a ſpectator at the ſun S would ſee the planet move one way, a ſpectator at the earth A will ſee it move the contrary way, or the geocentric motion of venus will in this caſe be contrary to its heliocentric motion. But its heliocentric motion is always direct, by propoſition 91. Therefore its geocentric motion will in this caſe be retrograde.

That two ſpectators, one on the earth and the other at the ſun, as they are on contrary ſides of the arc HE*x*, will for that reaſon ſee the planet apparently move contrary ways, may be underſtood from an eaſy inſtance of much the ſame ſort. If two men ſtand with their faces towards each other, and a bowl is rolled along upon the ground between them, this bowl will move from the right hand of one of the men towards his left, and from the left hand of the other towards his right. In like manner, if one man is at the earth A, and the other at the ſun S, then whilſt the planet deſcribes the arc HE*x*, which is between them, it will appear to move from the right hand of the man at S towards his left, and from the left hand of the man at A towards his right.

Whilſt venus's geocentric motion is direct, or whilſt it is deſcribing the arc *x*GH in its orbit, it appears to move among the fixed ſtars from V to T. But then after venus has been carryed in its orbit from H to *z*, it appears in the line A*z*R; and conſequently is ſeen among the fixed ſtars at R. When the planet comes to E in its orbit, it is ſeen among the fixed ſtars at Q. When it comes to *y*, it appears on the concave ſphere of the heavens at P. And when it comes to *x* its apparent place in the heavens is at V. Thus as the planet paſſes from its greateſt elongation H on one ſide of the ſun, through its inferior conjunction E, to its greateſt elongation *x* on the other ſide, it apparently runs back from T to V.

We may now ſee the reaſon why we ſhould conclude, in propoſition 68, that, becauſe the ſpots in the ſun appear to us to move from eaſt to weſt as they croſs its diſc, their real motion and the rotation of the ſun upon its axis is from weſt to eaſt. For as the geocentric motion of any inferior planet, will be retrograde, when the planet is in the part of its orbit, that is convex towards the earth; ſo for the the ſame reaſons a ſpot upon the ſurface of

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the ſun will be retrograde, when it is upon that hemiſphere of the ſun, which is towards the earth: the ſuns axis will then be, as it always is, on the concave ſide of the ſpots path, but a ſpectator at the earth will be on the convex ſide; and conſequently the heliocentric motion of the ſpot will be contrary to its geocentric motion: but its geocentric motion is from eaſt to weſt: therefore its motion round the ſuns axis is the contrary way.

When the ſpot is on the ſide of the ſun that is turned from the earth, its geocentric motion is in the ſame direction with its heliocentric motion, that is, the ſpot, if it could be ſeen from the earth, would appear to move from weſt to eaſt. Indeed in this part of the ſpots motion we cannot ſee it, becauſe it is behind the ſun, but we may be ſure that its geocentric motion, if we could ſee it, would be from weſt to eaſt, becauſe it firſt diſappears, upon the weſtern limb of the diſc, when it goes behind the ſun, and firſt appears again upon the eaſtern edge, when we ſee it again; conſequently, whilſt it was behind the ſun, its motion was from the weſtern towards the eaſtern edge.

93. *When the inferior planets are at their greateſt elongations, they appear ſtationary.*

Venus is ſtationary, or has no apparent motion for ſome time, when it is either at H or at ϰ, Plat. XXIII. fig. 1, that is, when its apparent place is either at T or V.

We have already proved in propoſition 91, that venus's geocentric motion is direct, whilſt it is moving from one of its greateſt elongations to the other through the ſuperior conjunction, and, in propoſition 92, that its geocentric motion is retrograde, whilſt it is returning from the latter of theſe elongations to the former through the inferior conjunction. We have alſo proved, in propoſition 89, that its apparent velocity is quicker near its conjunctions, than near its greateſt elongations. From hence then it follows, that the geocentric motion of venus changes gradually from direct to retrograde, and from retrograde to direct. Thus during the planets motion in its orbit from ϰ to G its geocentric motion is direct, and its apparent velocity is accelerated, but during its motion from G to H its geocentric motion is ſtill direct, and its apparent velocity is retarded. In like manner during its motion in its orbit from H to E, its geocentric motion is retrograde and its apparent velocity is accelerated, but during its motion from E to ϰ, its geocentric motion is ſtill retrograde, and its apparent velocity is retarded. Now as the velocity of the geocentric motion decreaſes, whilſt the planet approaches to either of its

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greateſt elongations, this motion at the greateſt elongation will be nothing, that is, the planet ſeen from the earth will for ſome time be ſtationary, or will appear not to move at all.

The planet, as has been ſaid, though it revolves in the orbit E*x*GH, appears to move forwards in the heavens upon the arc VQT, and then to move back again upon the ſame arc TQV. So that the motion of it, firſt to one ſide of the ſun and then to the other, is ſomething like the vibration of a pendulum. Its geocentric motion is accelerated, whilſt it moves from V to Q, and then is retarded, whilſt it moves from Q to T. And in its return, its geocentric motion is again accelerated, whilſt it moves from T to Q, and then is retarded, whilſt it moves from Q to V. But when its apparent place is either at V on one ſide or at T on the other ſide, it ſeems to ſtop there for ſome time and neither to move backwards nor forwards.

When either of the inferior planets, venus for inſtance, is at its greateſt elongation H or *x*, a line drawn from the earth through the planet as AHT or A*x*V is a tangent to the orbit, by propoſition 81. Now though a right line touches a circle only in one point, yet ſome part of the circle greater than a point is ſo near to the tangent as not to be diſtinguiſhable from it. Thus the arc *bd* ſo nearly coincides with the tangent AHT, that a ſpectators eye placed at the earth A could not diſtinguiſh the tangent from this part of the curve. Conſequently, whilſt the planet is deſcribing this arc, no other change will be made in its geocentric place, than would be made, if it was to move in the tangent THA. But the geocentric place of the planet would not be altered, if the planet was to move in the tangent: if it was to move directly from T towards A in the line THA, or directly from A towards V, the apparent place of it in the heavens during ſuch a motion would in one caſe be at T, and in the other caſe at V. Therefore, whilſt the planet is as its greateſt elongation, and is deſcribing a ſmall arc in its orbit, that nearly coincides with the tangent, its geocentric place does not alter, but it appears for ſome time in the ſame place in the heavens, or is ſtationary.

As an inferior planet approaches to its greateſt elongation, its apparent velocity decreaſes, by propoſition 89; becauſe the arc, which it deſcribes, is more and more oblique to the eye. But when it comes to its greateſt elongation, the end of the arc, that it deſcribes, is towards the eye at A; and conſequently the apparent length of that arc is nothing, by propoſition 129 of optics. For this reaſon, whilſt the planet is deſcribing this arc, it will have gone through a ſpace, which ſeen from the earth has no apparent length. Therefore the planet in this caſe will not appear to have moved at all, but will be ſtationary.

94. *The inferior planets do not always appear in the same part of the heavens, when they are in their conjunctions, or at their greateſt elongations, when they are direct, retrograde, or ſtationary.*

We have hitherto ſuppoſed the earth to be at reſt at A, Plat. XXIII. fig. 1, and upon that ſuppoſition have explained the progreſs and regreſs, the conjunctions and ſtations of the inferior planets. If this ſuppoſition was true, VT or the arc, which the planet at any time deſcribes in its progreſs, and TV the arc, which it deſcribes in its regreſs, would always be in the ſame part of the heavens. The planet, when in conjunction, would always appear at Q among the ſame fixed ſtars, and at its greateſt elongations, or when it is ſtationary, it would always appear among the ſame fixed ſtars at T on one ſide of the ſun and at V on the other ſide.

But this ſuppoſition is not true, for the earth revolves in its orbit ABO round the ſun. Now if the earth is at A, at the time of either conjunction, the planet at this conjunction would appear among the fixed ſtars that are at Q, and the arcs of greateſt elongation QV and QT would be on each ſide of thoſe ſtars. But if the earth is at B at the time of either conjunction, then the planet at the time of this conjunction will appear in the line BST; conſequently this conjunction will happen, when the planet is ſeen among the fixed ſtars, that are at T, and the arcs of greateſt elongation will be on each ſide theſe ſtars, that is, the conjunctions and greateſt elongations will happen in a different part of the heavens, when the earth is at B, from what they happen in, when the earth is at A.

The inferior planets always appear near the ſun. But by the earths revolution in its orbit, the ſun appears in different parts of the heavens at different times of the year, by propoſition 26. Therefore the inferior planets, as they are always near the ſun, will appear in different parts of the heavens at different times of the year. And conſequently their conjunctions and greateſt elongations will ſometimes happen, when they are in one part of the heavens, and ſometimes, when they are in another part.

Venus ſeen from the earth appears to vibrate in an arc VT, half of which is on one ſide of the ſuns apparent place Q, and half on the other ſide. But then, as the ſun is not always ſeen in the ſame part of the heavens, or as the ſuns apparent place is not always among the ſame fixed ſtars, this arc of the inferior planets apparent motion will not be always in the ſame part of the heavens.

The inferior planets will ſometimes be direct, ſometimes retrograde, and ſometimes ſtationary in the manner already deſcribed, notwithſtanding

ding the motion of the earth though in explaning thoſe appearances, we have neglected this motion. An inferior planets motion is direct as it paſſes through its ſuperior conjunction, it is retrograde as it paſſes through its inferior conjnnction; and the planet appears ſtationary, when it is at its greateſt elongations, by propoſitions 91, 92, 93. Now an inferior planet will have its ſuperior and inferior conjunctions, and will come to its greateſt elongations, not only if the earth is at reſt at A, as we before ſuppoſed, but if it revolves in its orbit ABC. Therefore the direct and retrograde motions, and the ſtations of theſe planets will happen, either if the earth is at reſt at A, or if it revolves in its orbit. An inferior planet will appear in conjunction, if a line AESGQ drawn from the earth to the planet paſſes through the ſun. But if the periodical time of the planet is leſs than that of the earth, though both of them revolve the ſame way round the ſun, yet the planet will ſometimes overtake the earth or come even with it: and when the planet thus overtakes the earth or comes even with it, a line drawn from the earth to the planet will paſs through the ſun, and conſequently the planet will appear in conjunction. In like manner an inferior planet is at its greateſt elongation, when a line drawn from the earth to the planet is a tangent to the planets orbit. And if the planet revolves round the ſun in a leſs time than the earth revolves, the planet will ſometimes be ſo far from the ſun that a line drawn from the earth to the planet will be a tangent to the planets orbit: and conſequently the planet, notwithſtanding the earths motion, will then be at its greateſt elongation. Or otherwiſe. The apparent ſituations of a planet in reſpect of the earth at different times will be the ſame, whether the planet and earth both move round the ſun, the planet with a greater and the earth with a leſs velocity, or whether the earth is at reſt and the planet alone revolves in its proper orbit with the difference of thoſe velocities: for the planet will depart from or approach to the earth equally upon either ſuppoſition. But we have ſhewn, in propoſitions 91, 92, 93, that, if the earth is at reſt and the planet alone revolves, then the planet will be ſometimes direct, ſometimes retrograde, and ſometimes ſtationary. Therefore theſe appearances will be the ſame, whilſt the earth and planet revolve in their reſpective orbits.

95. *The heliocentric latitude of an inferior planet is the greateſt, when the planet is in one of its limits.*

This follows from the definition of a limit, propoſition 77: for the limits are thoſe points in a planets orbit, which are moſt remote from the ecliptic: and conſequently, when a planet is in its limit, it will be at the

greateſt diſtance from the ecliptic, or will have the greateſt latitude: and the heliocentric latitude, or latitude ſeen from the ſun, will be greater or leſs, in proportion as the planet is farther from or nearer to the ecliptic.

96. *The geocentric latitude of an inferior planet is directly as its heliocentric latitude, and inverſely as the earths diſtance from it.*

The geocentric latitude of a planet is the apparent length of a line drawn from the planet to the plane of the ecliptic. Now the apparent length of ſuch a line is directly as its real length and inverſely as the diſtance of the ſpectators eye, by propoſitions 126, 127, of optics. But the real length of a line drawn from the planet to the ecliptic is as its heliocentric latitude, by propoſition 95, and the ſpectators eye is at the earth. Therefore the geocentric latitude is as the heliocentric latitude directly, and as the diſtance of the earth from the planet inverſely.

Let NBAN, Plat. XXIII. fig. 2, be a circle drawn in the plane of the ecliptic, N*v*VN the orbit of venus inclined to this plane, S the ſun, T and *t* different places of the earth in its orbit at different times. If the planet is at *v* a line *v*B drawn from the planet to the ecliptic is its latitude; or if it is at V, then the line VA is its latitude. Such a line viewed from the ſun is the heliocentric latitude: thus if the planet is at V, the line AV ſeen from the ſun ſubtends the angle ASV. Now if the ſpectators eye is at S, as the planet moves round the ſun, it will always be at the ſame diſtance from S, and conſequently, the apparent length of any line, as VA *v*B, drawn from the planet to the ecliptic will always be as its real length, by propoſition 127 of optics. Therefore the planets heliocentric latitude will be directly as the real length of ſuch a line. But the real length of ſuch a line is nothing when the planet is at either node NN, and is greateſt, when it is in a limit or in the middle between the two nodes. Therefore the planets heliocentric latitude will be greateſt, when it is at one of the limits, as was affirmed in propoſition 95. But the geocentric latitude will vary not only as the diſtance of the planet from the ecliptic, but likewiſe as the diſtance of the earth from the planet varys. If the earth is at T, and the planet is at V, its latitude VA ſubtends the angle ATV, but if the planet is in the ſame place and the earth at *t* the ſame latitude VA ſubtends only the angle A*t*V. The real length of the line VA is as the heliocentric latitude, by propoſition 95. But the apparent length of this line ſeen from the earth is as its real length directly, and as the diſtance of the earth or ſpectators eye inverſely, by propoſitions 126, 127, of optics: and the apparent length of this line is the planets geocentric latitude. Therefore the geocentric latitude is as the heliocentric latitude directly, and as the diſtance of the earth from the planet inverſely.

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From hence it follows that if the earth is at T and the planet moves from V to ϑ the geocentric latitude will decreaſe upon a double account; firſt becauſe the planet approaches the node, and conſequently its heliocentric latitude decreaſes; and ſecondly becauſe its diſtance from the earth encreaſes, and conſequently its latitude ſeen from the earth muſt decreaſe.

97. *When venus is in the inferior conjunction, its heliocentric latitude is leſs than its geocentric latitude.*

When venus is in its inferior conjunction, it is farther from the ſun than from the earth, by propoſition 88. Therefore its latitude ſeen from the ſun will be leſs than its latitude ſeen from the earth, by propoſition 96, or propoſition 126 of optics.

98. *When mercury is in its inferior conjunction, its heliocentric latitude is greater than its geocentric latitude.*

When mercury is in its inferior conjunction, it is nearer to the ſun than to the earth, by propoſition 88. Therefore its latitude ſeen from the ſun will be greater than its latitude ſeen from the earth, by propoſition 126 of optics.

99. *The ſun enlightens only half a planet at once.*

Let ABCD, Plat. XXIII. fig. 3, be an opake ſphere, ſuch as the planets are, and let the ſun at E ſhine upon this planet. The ſun is at a very great diſtance from any of the planets; and conſequently two rays, which diverge from E and touch the planet at A and B, may be looked upon as parallel to one another upon account of the remoteneſs of the radiant E, by propoſition 28 of optics. For the ſame reaſon any other ray EC, which comes from the ſame radiant, and would if it was continued, paſs through the center of the planet, is likewiſe parallel either to EA or to EB. Let a line AB be drawn from the point of contact A to the other point of contact B, this line AB will be perpendicular to EC, becauſe the ſun ſhines from E upon the planet juſt as far towards A as it ſhines towards B. Now becauſe EC is perpendicular to AB, and EA and EB are parallel to EC, EA and EB will likewiſe be perpendicular to AB. Euc. b. XI. prop. 8. But if the tangents EA and EB are perpendicular to the line AB drawn from one point of contact to the other, AB is a diameter of the ſphere; Euc. b. III. prop. 18: and conſequently if a circle was drawn upon the planet paſſing through the two points A and B, the plane of this circle would paſs through the planets center and would divide the planet into two hemiſpheres ACB, and ADB. The hemiſphere ACB is all of it towards the

ſun

ſun, and conſequently will be enlightened. But the other hemiſphere ADB is all of it turned from the ſun ſo that no rays can fall upon it.

100. *We can ſee only half a planet at once.*

If ABCD, Plat. XXIII. fig. 3, is a ſpherical body, ſuch as a planet is, and from any two of the extreme points A and B, that are viſible to the eye of a ſpectator at E, two tangents AE and BE are drawn, the rays that deſcribe theſe tangents are parallel to one another, upon account of the great diſtance between the planet and the point E to which thoſe rays converge, by propoſition 29 of optics. If a line CE is drawn from the center of the planet to the eye at E, this line will, for the ſame reaſon be parallel either to AE or to BE. From one point of contact A to the other point B, draw AB. EC, which paſſes through the center of the planet, is perpendicular to AB, or E will be juſt as far from A as it is from B; becauſe A and B are the limits of what the ſpectator can ſee upon the ſurface of the planet; and his view will extend equally both ways, juſt as far towards A as towards B. But if EC is perpendicular to AB, then EA and EB, which are parallel to EC, are likewiſe perpendicular to AB. Euc. b. XI. prop. 8. Therefore EAB is a diameter of the planet; becauſe the two tangents EA and EB are perpendicular to it. Euc. b. III. prop. 18. From hence it follows that, if a circle was drawn round the planet paſſing through the two points A and B, the plane of this circle would paſs through the planets center and would divide the planet into two equal parts or hemiſpheres ACB and ADB. The hemiſphere ACB is towards the eye at E, and all rays that come from it are within the two tangents AE and BE; ſo that theſe rays will come freely to the ſpectators eye, and the hemiſphere ACB will for that reaſon be viſible. The other hemiſphere is turned from the earth and no rays, that come from it, will paſs to the ſpectators eye at E: and upon that account the hemiſphere ADB will be inviſible.

101. *The hemiſphere of a planet, which is towards the earth, is called the diſc of the planet.*

When a ſphere is placed before the eye, at no great diſtance from it, the picture of the ſphere upon the retina is a circle properly ſhaded. Now the picture upon the retina would be juſt the ſame, if a circular piece of board properly ſhaded was placed before the eye. Therefore, ſince the picture upon the retina is the ſame in both caſes, the appearance to the eye will be the ſaine in both caſes, by propoſition 113 of optics, and conſequently a ſphere, when its diſtance is but ſmall, appears like a circle

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properly ſhaded. But if the ſphere is very remote, as the planets are, this ſhading will not be perceived: and for this reaſon in any planet the hemiſphere, that is towards us, appears only like a flat circle, as has been obſerved already, in propoſition 67, where we were ſpeaking of the ſun. This flat circle is called the planets diſc.

102. *The inferior planets are inviſible, when they are in their inferior conjunction, and their whole diſc is illuminated, when they are in their ſuperior conjunction.*

The inferior planets are opake ſpherical bodies, they have no light of their own, and are made viſible to us only by the light, which they receive from the ſun and reflect to us. Now, if the earth is at I, Plat. XXIII. fig. 4, venus at A, and the ſun at S, venus is then in its inferior conjunction: the ſun enlightens only one hemiſphere of venus at once, by propoſition 99: and in this ſituation the enlightened hemiſphere of venus is turned from the earth. Therefore the hemiſphere of venus which is towards the earth, as it receives no light from the ſun, will be inviſible to us. But when venus is in the ſuperior conjunction at E, the ſame hemiſphere is turned both towards the ſun and towards the earth. Therefore as the entire hemiſphere of venus, which we ſee, is enlightened, venus in this ſituation will appear round, or will ſhine with a full orb, as the moon does at the full.

Mercury revolves round the ſun juſt as venus does in an orbit leſs than the earths orbit: ſo that what has been here ſhewn in relation to one of theſe planets is equally true of the other.

103. *If an inferior planet is in one of its nodes at the time of its inferior conjunction, it will appear like a black ſpot in the diſc of the ſun.*

The ſun is always in the plane of the ecliptic, by propoſition 28. Both venus and mercury deſcribe orbits, that are inclined to the plane of the ecliptic. Therefore the inferior planets will be in the ſame plane with the ſun, unleſs they are in one of the nodes. If ABCD, Plat. XXI. fig. 4, is a circle drawn in the plane of the ecliptic, the ſun will be always in the ſame plane with this circle. But the orbit of venus is inclined to the plane of the ecliptic, as EFGH is inclined to ABCD. Therefore, unleſs venus is at one of the nodes B or D, it will not be in the ſame plane with the ſun. Now if either of the inferior planets, venus for inſtance, appears in the ſuns diſc, it muſt be in the ſame plane with the ſun. Conſequently, if venus at the time of the inferior conjunction paſſes directly between the ſun and the earth, it muſt either be in one of the nodes or very near to one of the nodes at that time.

Venus, or mercury, at the time of the inferior conjunction, does not always paſs directly between the earth and the ſun, ſo as to intercept the ſuns light, and appear like a black ſpot upon its diſc. Either of theſe planets, venus for inſtance, at the time of the inferior conjunction may have ſome latitude; and conſequently may be either above the plane of the ecliptic, as at G, or below it as at E. But the ſun is always in the plane of the ecliptic. Therefore venus at the time of the inferior conjunction may be either ſo far above the ſun or ſo far below it as not to croſs its diſc. But if the planet at its inferior conjunction is in or very near to one of the nodes, it is then in the ſame plane with the ſun: and for that reaſon it will paſs directly between the ſun and the earth, and by intercepting the ſuns light as it paſſes, it will appear like a black ſpot in the diſc of the ſun.

In like manner we might ſhew, that an inferior planet at the ſuperior conjunction is not always directly behind the ſun. For if venus for inſtance, at the time of the ſuperior conjunction is not in the ſame plane with the ſun, that is, if it is not in or very near to one of the nodes, it will be either above the ſun or below it, and not directly behind it.

104. *The inferior planets go through all the changes that we ſee in the moon.*

We have already ſhewn, in propoſition 95, that the inferior planets at their inferior conjunction become inviſible, as the moon is at the new, and that at their ſuperior conjunction their whole diſc is bright as the moons is at the full. Somewhere between the two conjunctions, as at C and G, Plat. XXIII. fig. 4, only half the ſide, which is towards the ſun at S and is enlightened by its rays, will be towards a ſpectator upon the earth at I. Therefore in this ſituation, only half the diſc of the planet, which we ſee from the earth, will be enlightened, and the other half will be dark; ſo that the planet will appear as the moon appears at the quarters. At D or at F, the whole enlightened hemiſphere of the planet will not be turned towards the earth, as it is when the planet is at E: but more than half this hemiſphere will be towards the earth: and conſequently more than half the planets diſc will be enlightened, whilſt the reſt is dark, or the planet will appear gibbous, as the moon appears before and after the full. At B or H, the whole enlightened hemiſphere of the planet will not be turned from the earth, as it is when the planet is at A: but more than half this hemiſphere will be turned from the earth: and conſequently leſs than half the planets diſc will be enlightened, whilſt the greateſt part is dark, or the planet will appear horned as the moon appears before and after the new.

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In the ſeveral ſituations ABCDEFGH, the diſc of the planet, as it is ſeen from the earth, appears in the manner that is repreſented at *abcdefgh*. When the planet, which is inviſible at *a* its inferior conjunction, moves from thence, through *b*, *c*, *d*, it is firſt horned, then halved, then gibbous; till at laſt, at its ſuperior conjunction *e*, it is in the full: then, as it moves through *f*, *g*, *h*, it becomes firſt gibbous, then halved, then horned, till it diſappears again at *a*.

105. *A circle, ſuppoſed to be drawn upon the ſurface of a planet, which ſeparates the enlightened hemiſphere of the planet from the dark one, we ſhall call the circle of light.*

106. *A circle, ſuppoſed to be drawn upon the ſurface of the planet, which ſeparates the viſible hemiſphere of the planet from the inviſible one, we ſhall call the circle of the diſc.*

107. *If the centers of the ſun, the earth, and an inferior planet are joyned by lines, which are drawn from one to the other ſo as to form a triangle, the external angle of this triangle at the planet, is equal to the angle contained between the circle of light, and the circle of the diſc.*

Let S be the ſun, Plat. XXIII. fig. 5; T the earth; and *mrnq* either of the inferior planets, ſuppoſe venus. Imagine a line SO to be drawn from the center of the ſun to the center of venus, another line TO to be drawn from the center of the earth to the center of venus, and a third line ST to be drawn from the center of the ſun to the center of the earth. Theſe lines SO, TO, and ST joyn the three centers of the ſun, the earth, and the planet, and form the triangle STO. If the ſide TO of this triangle is continued to P, SOP is the external angle at the planet.

Draw the line *rq* perpendicular to SO a line drawn from the center of the ſun to the center of the planet. This line *rq*, as we look at the figure, will repreſent the plane of a circle perpendicular to SO; and conſequently *rq* is the circle of light, by propoſition 99. Draw the line *nm* perpendicular to TO a line drawn from the center of the earth to the center of the planet. This line *nm*, as we look at the figure, will repreſent the plane of a circle perpendicular to TO; and conſequently *nm* is the circle of the diſc, by propoſition 100, *r*O*n* or *m*O*q* is the angle, which the circle of light makes with the circle of the diſc.

What we are to prove therefore is, that POS the external angle at the planet is equal to *m*O*q* the angle contained between the circle of light and the circle of the diſc.

SO*r* is a right angle by the conſtruction; becauſe SO is perpendicular to *rq*. And TO*m* is likewiſe a right angle by the conſtruction; becauſe TO is perpendicular to *nm*. Conſequently SO*r* is equal to TO*m*. But PO*r* is equal to *q*OT, becauſe they are vertical angles. Euc. b. 1. prop. 15. Therefore, when PO*r* is taken from SO*r* and *q*OT is taken from TO*m* the remainders will be equal, or SO*r*—PO*r*=TO*m*—*q*OT. But SO*r*—PO*r*=POS and TO*m*—*q*OT=*m*O*q*. Therefore POS=*m*O*q*.

The repreſentation in this figure will not ſhew us how the diſc of the planet will appear to us, when we ſee it from the earth. The plane of the paper, as we are looking at the figure, is perpendicular to a line drawn from our eye. Whereas, when we look at venus in the heavens, the plane of the diſc, which *nm* repreſents, is perpendicular to a line drawn from our eye: and conſequently, when we look at venus in the heavens, the appearance will be ſuch as this figure would ſhew, if *rmqn* was a ſphere, *nm* and *rq* two circles drawn round it, and the ſpectators eye was placed at T.

In order to repreſent the planet, either venus or mercury, as it will appear to us, when we, who live upon the earth, ſee it in the heavens, let the circle *obcp*, Plat. XXIII. fig. 6, be the diſc of the planet, or circle of the diſc; and if the external angle at the planet is leſs than a right one, draw *bfc* a ſemiellipſis, in ſuch a manner that the angle *fbp* may be equal to the external angle at the planet. This ſemiellipſis will repreſent the circle of light. For firſt by the conſtruction the ſemiellipſis *bfc* and circle of light make the ſame angle with the circle of the diſc. Secondly, as the plane of the diſc is perpendicular to the eye, and the circle of light is inclined to the plane of the diſc, the circle of light muſt be inclined to the eye, and conſequently it will appear elliptical. Thirdly, as half the circle of light is before the planet and half behind it, only half the circle of light will be upon the diſc; and therefore as much of the circle of light as is to be repreſented upon the diſc will be only a ſemiellipſis. This ſemiellipſis divides the diſc into two parts *bfcp* and *bfco*. But this ſemiellipſis divides the light part of the planet from the dark part. Therefore one of theſe parts will be light and the other will be dark. All the changes made in the ſhape of an inferior planet may be explaned upon this principle.

Firſt; let A, fig. 5, be the place where the planet is at its inferior conjunction: in order to determine the phaſe or appearance of the planet in this ſituation, conceive the whole ſphere *mrnq*O to move from O to A and the line SO to turn round in the mean time upon S as upon a center; in this caſe it is plane that, when O the center of the planet arrives at A, the line SO will concide with SA, and the line TO or TP will coincide

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with TA. Therefore TP and OS will lie in the ſame right line, and conſequently POS the external angle at the planet will be nothing. From hence it follows that the circle of light will make no angle with the circle of the diſc; ſo that theſe two circles will coincide. And the diſc at that time is the dark hemiſphere: therefore the planet at that time, as it reflects no light to us upon the earth, will not be ſeen by us.

Secondly, if the planet is at its greateſt elongation, the line TOP will be a tangent to the planets orbit, by propoſition 8. Therefore in this ſituation of the planet the angle, which SO a ſemidiameter of the orbit, makes with the tangent TP, or the external angle SOP will be a right one. Conſequently in this ſituation the circle of light makes a right angle with the circle of the diſc. The circle *bocp*, fig. 6, is the circle of the diſc having its plane perpendicular to the eye. Therefore the plane of the circle of light, which makes right angles with the diſc, muſt paſs through the eye. But if the plane of a circle paſſes through the eye, that circle will appear like a right line; and conſequently, in this ſituation of the planet, the line *bc* dividing the diſc into two equal parts will repreſent the circle of light. From hence it follows that, when the planet is at its greateſt elongation, half the diſc will be enlightened by the ſun and half of it will be dark; and the planet will appear as the moon appears at the quarters.

Thirdly; if the angle POS is leſs than a right one, as it will be when the planet is between its greateſt elongations and its inferior conjunction, the circle of light will make a leſs angle than a right one with the circle of diſc, and will divide the diſc into two unequal parts; thus, in fig. 6, *bfc* the circle of light makes a leſs angle than a right one with *bpc*; and as only ſo much of the diſc as included between *bfc* and *bpc* is enlightened, the bright part of the diſc will be horned like a creſcent.

Fourthly, if the planet is any where between its greateſt elongations and the ſuperior conjunction E, the external angle POS will be greater than a right one: and conſequently the circle of light will make a greater angle than a right one with the circle of the diſc, and will divide the diſc into two unequal parts; thus in fig. 6, *bfc* makes a greater angle than a right one with *boc*; and ſince ſo much of the diſc is enlightened as is included between *bfc* and *boc* the planet in this ſituation will be gibbous.

Fifthly, the external angle POS, which is a right one, when the planet is at its greateſt elongations, becomes greater than a right one as the planet approaches to its ſuperior conjunction E; and when the planet is arrived at this ſuperior conjunction it is equal to two right ones. Therefore the circle of light makes two right angles with the circle of the diſc,

but theſe two circles can no otherwiſe make two right angles with one another than by coinciding; and conſequently the circle of light coincides with the circle of the diſc at the planet's ſuperior conjunction: and the diſc at that time is the enlightened hemiſphere. Therefore the planet at that time will appear round like the full moon.

108. *When venus is the evening-ſtar, its horns are turned towards the eaſt; when it is the morning-ſtar, its horns are turned towards the weſt.*

When venus is the evening-ſtar the ſun ſets before it or is to the weſt of it, by propoſition 85. But the horns of venus are always turned towards the dark part of it, as will appear from propoſition 107 and an inſpection of Plat. XXIII. fig. 6. Therefore when venus is the evening-ſtar, or when the ſun is to the weſt of it, its horns are turned towards the eaſt.

When venus is the morning-ſtar the ſun riſes after it or is to the eaſt of it by propoſition 85. But the horns of venus are always turned from the ſun. Therefore, when venus is the morning-ſtar or when the ſun is to the eaſt of it, its horns are turned towards the weſt.

109. *Venus does not appear the brighteſt to us at the time of the ſuperior conjunction.*

If the brightneſs of venus, as we ſee it from the earth, depended only upon the quantity of the diſc that is enlightened, venus muſt neceſſarily appear brighter at the ſuperior conjunction than at any other time: becauſe at the ſuperior conjunction the whole diſc is enlightened, and at all other times the part enlightened is leſs than the whole diſc, by propoſitions 107, 102, 103. But the brightneſs of venus depends upon the quantity of light, which it reflects to the earth. And the quantity of light reflected to us depends partly upon the quantity of the diſc that is enlightened, and partly upon the diſtance of venus from the earth. If the diſtance of venus is given, then it is evident, that the quantity of light, which it reflects to the earth, will be directly as the quantity of the diſc that is enlightened; when the whole diſc is enlightened we ſhould receive twice the light from it, that we receive, when but half of it is enlightened, and four times the light that we receive, when but a quarter of it is enlightened; provided that in all theſe circumſtances venus was at the ſame diſtance from us. But if a given part of the diſc is enlightened, the light that will come to the earth from this given part, will be inverſely as the ſquare of venus's diſtance from the earth, by propoſition 22 of mechanics and 34 of optics. Thus if the diſtance of venus decreaſes from 4 to 1,

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the light, which we receive from it, will encreaſe from 1 to 16; if at the diſtance 4 we call its light 1, at the diſtance 1 its light will be 16; provided that at theſe different diſtances an equal quantity of the diſc was enlightened.

When venus is at E, Plat. XXIII. fig. 5, in the ſuperior conjunction, where the whole diſc is enlightened, let us call its light one: when venus is at O and its elongation 40 degrees, only $\frac{1}{4}$ of its diſc will be enlightened: and yet venus, when at O, will appear brighter to us than when at E. The light indeed, which we receive, has decreaſed from 1 to $\frac{1}{4}$ upon one account; for the quantity of the diſc enlightened, when the planet is at E, is to the quantity enlightened, when it is at O, as 1 to $\frac{1}{4}$. But then venus, when at E is 4 times farther from us, than when at O, and conſequently upon this account the light, which we receive, when venus is at E is to the light which we receive, when it is at O, as 1 to 16. Therefore upon both accounts together, the light, that we receive from venus, when at E, is to the light, that we receive from it, when at O, as 1 to $\frac{1}{4}$ and as 1 to 16, or as 1×1 to $\frac{1}{4} \times 16$, or as 1 to $\frac{16}{4}$ or as 1 to 4; that is venus, though but a quarter of its diſc is enlightened, when at O, will appear 4 times brighter than when at E; becauſe though the light, which we receive is diminiſhed, as leſs of the diſc is enlightened, yet it will be encreaſed in a greater proportion upon account of the planets diſtance being diminiſhed.

This propoſition is equally true of mercury: but venus was here made choice of; becauſe the brightneſs of venus, when a quarter of its diſc is enlightened, is very remarkable: for in this ſituation it is ſo bright, that it may be ſeen in the day-time, when the ſun is up. It has been farther obſerved, that venus in this ſituation is bright enough to caſt a ſhadow in the night-time, and that, none of the other heavenly bodies will do this except the ſun and the moon. But venus, when it is leſs bright than this, will caſt a ſhadow; and I have ſeen the planet jupiter caſt a ſhadow about the time of its oppoſition.

110. *The changes made in the diſcs of the inferior planets do not prove the truth of the Copernican ſyſtem.*

We are told that, when Copernicus firſt publiſhed his account of the ſolar ſyſtem, it was objected to him that it could not be true, becauſe, if it was, the inferior planets muſt have different phaſes, according to their different ſituations with reſpect to the ſun and the earth; whereas they always appear round to us. The anſwer made by him is ſaid to be, that they appear round to the eye by reaſon of their great diſtance; but that, if

if we could have a nearer and more diſtinct view, we ſhould ſee in them the ſame variety of ſhapes as we do in the moon. And here it is commonly obſerved, that as the invention of teleſcopes has ſhewn us this variety of ſhapes, it has verifyed Copernicus's ſolution of the objection, and at the ſame time has confirmed the truth of his ſyſtem.

The inferior planets do indeed always appear round to the naked eye upon account of their great diſtance and indiſtinctneſs; and the variety of their ſhapes cannot be ſeen without the help of a teleſcope. But it is very ſtrange that a defender of Ptolemy's ſyſtem ſhould make ſuch an objection, as this is, to Copernicus's ſyſtem, and not leſs ſtrange that Copernicus ſhould make this anſwer.

A defender of Ptolemys ſyſtem would ſcarce have made this objection: becauſe the ſame objection might be urged to more advantage againſt his own ſyſtem. For if Ptolemys ſyſtem is true, and the inferior planets are opake ſpheres, they muſt have different phaſes according to their different ſituations with reſpect to the ſun, and could never appear round, unleſs they were to be ſeen in oppoſition. This the reader may underſtand by conſidering the Ptolemaic ſyſtem, Plat. XXI. fig. 4. The earth is here ſuppoſed to be at *s* in the middle of the ſyſtem, and to have no motion at all. Now ſuppoſe the ſun to be at *b*, the earth at *s*, and the moon at *n*; I would aſk how the moon would appear? the reader, I imagine, is aware that in this ſituation, the moons enlightened ſide is turned from the earth, and conſequently that the dark hemiſphere is towards the earth: therefore the moon would be inviſible to us, as it is at the new. I would aſk farther how the moon would appear, if the ſun was at *b* and the moon was at *r*, or at the diſtance of a quarter of a circle from the ſun? the moons enlightened hemiſphere is always towards the ſun, and in this ſituation half that enlightened hemiſphere would be ſeen from the earth: therefore half of the moons diſc would be bright, and the other half would be dark, which is the known appearance of the diſc at the quarters. I ſuppoſe it is evident from hence, that, as all the diſc is dark, when it is at *n*, and half of it is dark, when at *r*, in the intermediate places between *n* and *r*, more than half the diſc will be dark, or the moon will appear horned. I ſhould next aſk, where the moon muſt be in order to be full or to appear round? it would certainly appear in this manner no otherwiſe than by being in oppoſition at *a* whilſt the ſun is at *b*: for, in this ſituation only, the whole enlightened hemiſphere would be towards the earth. Now if the moons diſc is half light, when at *r*, and is all light, when at *a*, the conſequence is, that in the intermediate ſituations the moon would appear gibbous.

Let

Let us apply what has been here ſaid of the moon to either of the inferior planets. The orbits of theſe planets in the Ptolemaic ſyſtem are juſt in the ſame ſituation in reſpect of the earth and ſun that the moons orbit is; their orbits, like that of the moon, paſs between the earth and the ſuns orbit. Conſequently the ſhape of theſe planets muſt change juſt as the moons does. If the ſun is at *b*, when venus is in conjunction at *b*, the bright ſide of venus will be turned from the earth; and therefore venus will be inviſible, as the moon is at the new. If, whilſt the ſun is at *b*, venus was to be at *a*, its diſc muſt appear halved. If, whilſt the ſun is at *b*, venus was to be at *c*, its diſc would be round like the full moon. If, whilſt the ſun is at *b*, venus was to be in any intermediate place between *b* and *a*, its diſc muſt appear horned. And if, whilſt the ſun is at *b*, venus was to be in any intermediate place between *a* and *c*, its diſc muſt appear gibbous. We ſee therefore how little reaſon a defender of Ptolemy could have to make any objection to the ſyſtem of Copernicus from the conſtantly round appearance of the inferior planets: ſince in the Ptolemaic, as well as in the Copernican ſyſtem, the ſhape of theſe planets ought to change juſt as the moons does.

Or if any defender of Ptolemy had made this objection, it is moſt likely that Copernicus's anſwer to it would not have been conjectural: he would ſcarce have ſaid, that perhaps future aſtronomers may diſcover the ſame phaſes in the inferior planets, which we obſerve in the moon, that they appear round to us by reaſon of their great diſtance, but that, if we could have a nearer and more diſtinct view, we ſhould ſee in them the ſame variety of ſhapes that we ſee in the moon. It would have been a properer and more convincing anſwer to a defender of Ptolemy, if Copernicus had ſhewn him, that the ſame objection might be made to both ſyſtems, ſince the inferior planets, whichever of theſe ſyſtems was true, ought to change their ſhapes as the moon does. Nay the objection would be ſtronger againſt the Ptolemaic than againſt the Copernican ſyſtem: becauſe venus in the Copernican ſyſtem may appear round without being in oppoſition to the ſun, whereas in the Ptolemaic ſyſtem venus in order to appear round muſt be in oppoſition. But in fact venus is never ſeen in oppoſition to the ſun. Therefore if the Ptolemaic ſyſtem was true, venus ought never to appear round.

We have thus ſeen that the mere change of ſhape in the inferior planets is an argument, which in the common way of urging it, proves nothing at all as to the truth or falſhood of the Copernican ſyſtem. Venus changes its ſhape as the moon does, therefore the Copernican ſyſtem muſt be true, is no juſt concluſion; ſince venus would change its ſhape in the ſame manner in the Ptolemaic ſyſtem.

If

If, beſides the changes of ſhape made in the inferior planets, we conſider the ſituation of the planets in reſpect of the ſun, when theſe changes happen, this indeed will ſhew us that the Ptolemaic ſyſtem is falſe. Venus, in the Ptolemaic ſyſtem, could not be halved, unleſs it was 90 degrees diſtant from the ſun, nor gibbous, unleſs it was more than 90 degrees diſtant, nor full, unleſs it was in oppoſition. Whereas in fact venus undergoes all theſe ſhapes, though its elongation does not exceed 48 degrees, though it is never in oppoſition, nor even in quadrature. Therefore the Ptolemaic ſyſtem does not agree with fact or is not true.

The Copernican ſyſtem ſhews us how theſe changes may happen in the ſhapes of the inferior planets, though they are never either in oppoſition, or in quadrature; and conſequently the Copernican ſyſtem may be true: this argument will not prove that it muſt be true, becauſe the ſame changes would happen in the ſame ſituations of the planets as well in the Tychonic as in the Copernican ſyſtem. Thus, Plat. XXI. fig. 5, if *t* is the earth, *a* the ſun, and *esdr* the orbit of venus; when venus is at its inferior conjunction *s*, its dark ſide will be towards the earth, and its diſc will be inviſible: when it is at its ſuperior conjunction *r*, its enlightened hemiſphere will be to towards the earth, and its diſc will be all bright as the full moon is: when venus is at *e* or *d* its greateſt elongations on each ſide of the ſun, half its enlightened hemiſphere will be towards the earth, and its diſc will appear as the moon does at the quarters: when venus is any where between the greateſt elongations and the inferior conjunction, it will appear horned: and when it is any where between the greateſt elongations and the ſuperior conjunction, it will appear gibbous.

CHAP. IV.

Of the ſuperior planets.

111. *Mars, jupiter, and ſaturn are called ſuperior planets.*

THESE three are called ſuperior planets, in the Copernican ſyſtem; becauſe they are higher in the ſyſtem, or farther from the center of it than the earth is.

112. *If we call the middle diſtance of the earth from the ſun* 1000, *the middle diſtance of mars is* 1523, *and its eccentricity is* 141. *The periodical time of mars is* 686 *days,* 23 *hours: and it revolves round its axis in* 24 *hours* 40 *minutes.*

113. *If*

PLATE XXI.

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

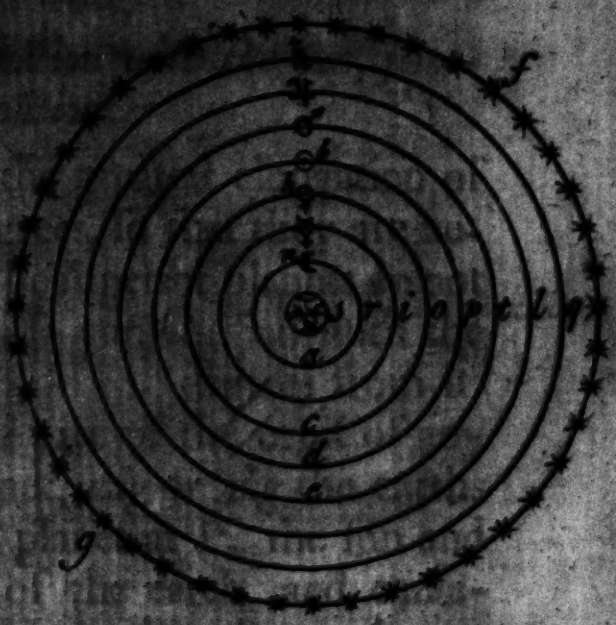


Fig. 5.

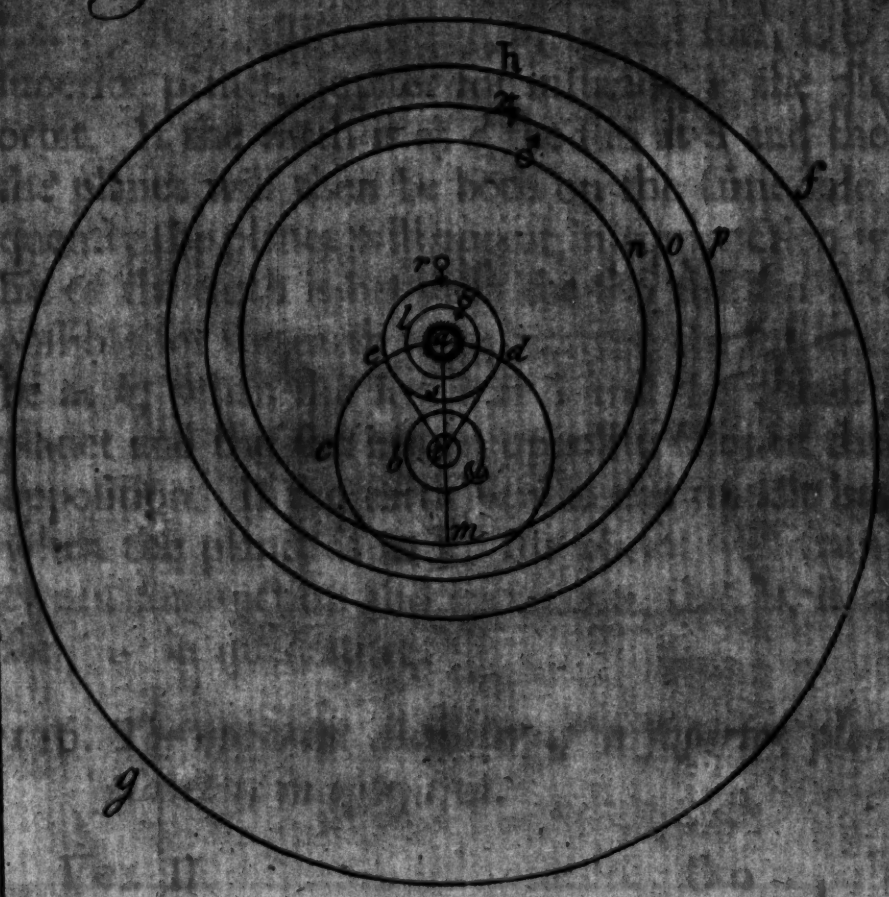
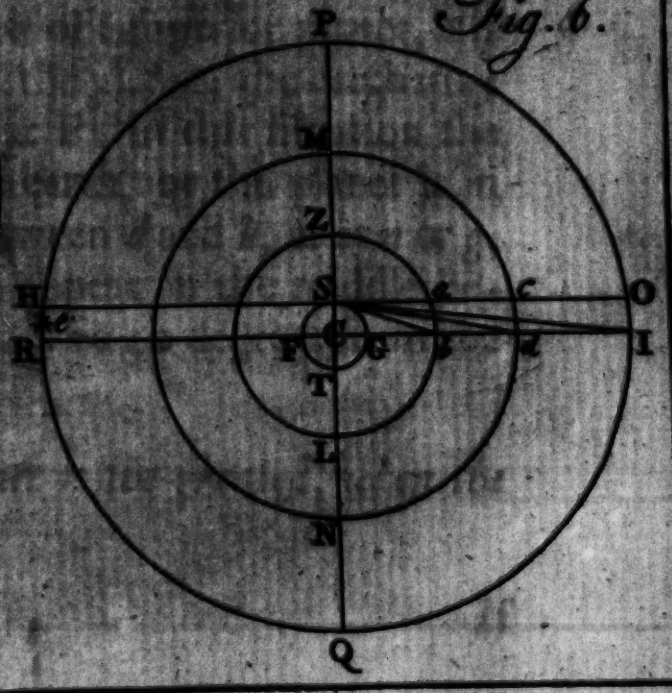
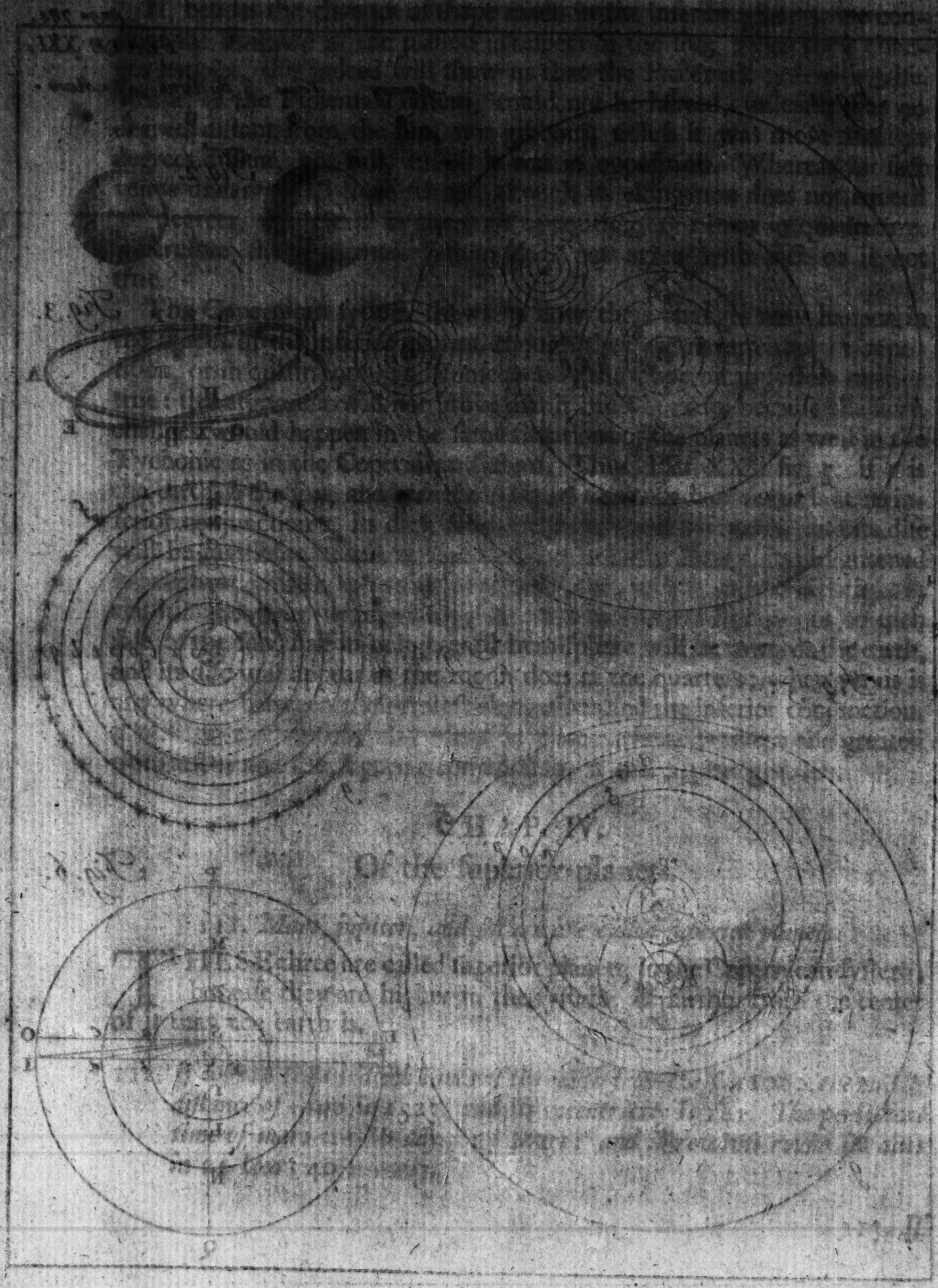


Fig. 6.





113. *If the middle diſtance of the earth from the ſun is called* 1000, *the middle diſtance of jupiter is* 5201, *and its eccentricity is* 250. *The periodical time of jupiter is* 4332 *days*, 12 *hours: and it revolves round its axis in* 9 *hours*, 56 *minutes.*

114. *If the middle diſtance of the earth from the ſun is called* 1000, *the middle diſtance of ſaturn is* 9538, *and its eccentricity is* 547. *The periodical time of ſaturn is* 10759 *days*, 7 *hours.*

Saturn is ſo remote from the earth that no particular ſpots have been diſcovered upon its diſc: as we can ſee no ſuch ſpots, the time that it takes up to turn round upon its axis cannot be determined; and indeed it is only by analogy that we can prove it to have any rotation upon its axis; only, ſince the other planets have ſuch a rotation, we conclude that as ſaturn is like them in other reſpects, it is like them in this reſpect too and turns round upon its axis as they do.

115. *The ſuperior planets are ſometimes in conjunction with the ſun, ſometimes in quadrature, and ſometimes in oppoſition.*

We have obſerved, in propoſition 84, that the greateſt elongation of either of the inferior planets is leſs than 90 degrees, ſo that they are never far from the ſun but conſtantly attend it. The ſuperior planets do not keep ſo near to it. They are indeed ſometimes in conjunction with the ſun; but then they are ſometimes in oppoſition or 180 degrees from it. Let S, Plat. XXIII. fig. 7, 8, be the ſun; QO a part of the orbit of any ſuperior planet, jupiter for inſtance, P the planet, *adg* or *nkg* the earths orbit. If the earth is at *d* the ſun at S and the planet at P, the ſun and the planet will then be both on the ſame ſide of the earth, and conſequently the planet will appear in conjunction with the ſun. But as the orbit of the earth is between the ſun and the orbit of a ſuperior planet, it is poſſible for the earth to be between the ſun and the planet; the earth may be at *k*, when the ſun is at S and the planet at P: in this ſituation the planet and the ſun are on oppoſite ſides of the earth, or the planet is in oppoſition. If the earth was in the middle between *d* and *k*, as at *n* or *g*, when the planet is at P, the planet will then appear in the middle between conjunction and oppoſition, that is, it will be 90 degrees from the ſun, or in quadrature.

116. *The apparent diameter of a ſuperior planet is the greateſt, when the planet is in oppoſition.*

If any one of the ſuperior planets is in conjunction, the diſtance of the earth from the planet is equal to the ſemidiameter of the planets orbit added to the ſemidiameter of the earths orbit: If jupiter is at P, Plat. XXIII. fig. 7, 8, the ſun at S, and the earth at *d*, jupiter is then in conjunction, by propoſition 115: and the diſtance of jupiter at P from the earth at *d* is P*d*, or PS the ſemidiameter of jupiters orbit added to S*d* the ſemidiameter of the earths orbit. If any one of the ſuperior planets is in oppoſition, the diſtance of the earth from the planet is equal to the ſemidiameter of the planets orbit after the ſemidiameter of the earths orbit is ſubſtracted from it: if jupiter is at P, the ſun at S, and the earth at *k*, jupiter is then in oppoſition, by propoſition 115: and the diſtance of jupiter at P from the earth at *k* is P*k*, or PS the ſemidiameter of jupiters orbit after S*k* the ſemidiameter of the earths orbit is taken from it. It is evident from what has been ſaid, that the earth is much nearer to a ſuperior planet at the time of the planets oppoſition than at the time of its conjunction, and that the difference between thoſe two diſtances *k*P and *d*P is *d*S added to S*k*, or a diameter of the earths orbit. It is evident likewiſe that the earth cannot be farther from the planet than it is at the planets conjunction, nor nearer than at the planets oppoſition. But the planets apparent diameter, when ſeen from the earth, is inverſely as its diſtance from the earth, by propoſition 126 of optics. Therefore a ſuperior planets apparent diameter will be leaſt of all at its conjunction, and greateſt of all at its oppoſition.

117. *Venus is nearer to the earth than any other planet.*

The orbit of the earth is between thoſe of mars and venus. But venus is nearer to the earth than mars. If the diſtance of the earth from the ſun is called 1000, we have ſeen, in propoſition 88, that the diſtance of venus from the earth at the time of venus's inferior conjunction is 276. If the middle diſtance of the earth from the ſun or *ks*, Plat. XXIII. fig. 8, is called 1000, then, ſuppoſing mars to be at P in its oppoſition, PS or mars's middle diſtance from the ſun is 1523; and conſequently P*k* or PS—S*k* is 1523—1000=523, which is the diſtance between mars and the earth, when they are the neareſt to one another, that is, by propoſition 116, when mars is in oppoſition.

118. *If a ſuperior planet had no motion of its own; its apparent geocentric motion, in any given time, ariſing from the earths motion in its orbit would be proportional to the angle, which the arc deſcribed in that time by the earth would ſubtend, if that arc was to be viewed by an eye placed at the planet.*

If

If P is a ſuperior planet, Plat. XXIII. fig. 7, 8, which we will ſuppoſe to have no motion of its own: then as the earth moves in its orbit *adg* or *nkg*, the planet will appear to move to a ſpectator, who is upon the earth. This apparent motion we here call the planets geocentric motion ariſing from the motion of the earth in its orbit. Let AG or GN be an arc in the concave ſphere of the heavens. When the earth is at *a*, the planet P appears in the right line *a*PA, and ſeems to us to be among the fixed ſtars at A, by propoſition 135 of optics. After the earth has moved from *a* to *b* the planet is then ſeen in the right line *b*PB, or ſeems to us to be among the fixed ſtars at B. Therefore whilſt the earth has been moving through the arc *ab* in its orbit, the planet appears in conſequence of this motion to have deſcribed the arc AB in the heavens. The arc AB is proportional to the angle APB ſubtended by it. But this arc is what the planet deſcribes by its geocentric motion whilſt the earth is deſcribing *ab* in its orbit. Therefore the geocentric motion of the planet, whilſt the earth is deſcribing the arc *ab* is proportional to the angle APB or to the angle *a*P*b*; for theſe two angles are equal to one another, becauſe they are vertical. Euc. b. I. prop. 15. Now the angle *a*P*b* is the angle which the arc *ab* would ſubtend to an eye placed at the planet P. Therefore the geocentric motion of the planet P, whilſt the earth is deſcribing the arc *ab* in its orbit, is proportional to the angle, which this arc *ab* would ſubtend, if an eye was placed at the planet and was to view this arc from thence.

This may be ſhewn in the ſame manner during the motion of the earth through any other arc in its orbit.

119. *The geocentric velocity of a ſuperior planet is the greateſt at the conjunction and oppoſition of the planet.*

If the earth is about *cde*, Plat. XXIII. fig. 7, the planet P appears in conjunction, if the earth is about *lki*, fig. 8, the planet appears in oppoſition. Now the geocentric motion of the planet will be ſwifter in theſe two ſituations than in any other. Arcs of a given length about *cde* or *lki* in the earths orbit would be ſeen directly, and equal arcs in any other part of the orbit, as at *lm* or *bc*, would be ſeen obliquely by an eye placed at P. Therefore arcs of a given length in thoſe parts of the orbit, where the earth is at the time of the planets conjunction and oppoſition, would ſubtend greater angles than any other arcs, if they were to be viewed by an eye placed in the planet, by propoſition 129 of optics. But the apparent geocentric velocity or motion of the planet is proportional to thoſe angles, by propoſition 118. And conſequently the geocentric velocity of the planet is greater, or the planet ſeen from the earth ap-

pears to move ſwifter, at the time of its conjunction and oppoſition than at any other time.

120. *When the earth is in ſuch a ſituation that a line drawn from a ſuperior planet to the earth is a tangent to the earths orbit, the planet appears ſtationary.*

If the earth is either at *a* or *g*, Plat. XXIII. fig. 7, 8, a line P*a* or P*g* drawn from the planet to the earth is a tangent to the earths orbit: and when the earth is in this part of its orbit, the planets geocentric motion is nothing, that is, the planet ſeen from the earth appears for ſome time to ſtand ſtill in the ſame part of the heavens, or is ſtationary.

Though the tangent P*a* touches the earths orbit only in one point, yet the orbit for ſome little ſpace on each ſide the point of contact is very near to the tangent, ſo that, as to ſenſe, the motion in the orbit will not differ from a motion in the tangent. But, whilſt the earth is moving in the line P*a*, it is moving directly away from the planet, and the planet will appear all the time in the ſame line *a*PA. Therefore whilſt the earth is in this part of its orbit, which as to ſenſe coincides with the tangent, the apparent place of the planet in the heavens will be at A; that is, the planet will not appear to change place but will be ſtationary. In like manner when the earth is at *g* in the tangent *g*P, though it is moving from thence to *k*, yet for ſome time its motion in its orbit will, as to ſenſe, be the ſame with a motion in the tangent. But if the earth was to move directly towards the planet in the tangent *g*P, the planet during this motion would be always ſeen in the line *g*PG, and conſequently its apparent place in the heavens would continue to be at G. Therefore, whilſt the earth is moving in that part of its orbit, which as to ſenſe does not differ from the tangent, the planet will continue to appear at G, or will be ſtationary.

The apparent motion of a ſuperior planet in any given time is proportional to the angle, which the arc deſcribed by the earth in that time would ſubtend, if it was to be viewed by an eye placed at the planet, by propoſition 118. But if the earth is at *a* or *g* in a line, which is a tangent to its orbit, the end of the arc, which it is then deſcribing, is towards the planet P, and conſequently the arc ſeen from the planet would ſubtend no angle, by propoſition 129 of optics. Therefore, when the earth is in that part of its orbit, the apparent motion of the planet will be nothing, or the planet will be ſtationary.

As the earth moves from *a* to *d*, the geocentric velocity of a ſuperior planet encreaſes; as the earth moves from *d* to *g*, the geocentric velocity of the

the planet decreaſes; as the earth moves from *g* to *k* the geocentric velocity of the planet encreaſes again; and as the earth moves from *k* to *n*, the geocentric velocity of the planet decreaſes again. Since therefore the geocentric velocity of the planet is the greateſt when the earth is at *d* or *k*, and decreaſes as the earth approaches to the tangent P*n* on one ſide, or P*g* on the other, it follows, that the geocentric velocity will be nothing or the planet will be ſtationary, when the earth comes to either of thoſe tangents.

121. *When a ſuperior planet is paſſing from one of its ſtations to the other through the conjunction, its geocentric motion is direct.*

Whilſt the earth is moving from *a*, Plat. XXIII. fig. 7, through *d* to *g*, a ſuperior planet P appears to move in the heavens from one of its ſtations A through its conjunction D to its other ſtation G by propoſitions 120, 115. In this caſe the apparent motion of the planet ſeen from the earth is direct or in conſequentia, that is the planet appears to move from weſt to eaſt, or according to the order of the ſigns.

The motion of the earth round the ſun S, makes the ſun appear to move in conſequentia, by propoſition 26. But whilſt the planet is moving from one of its ſtations to the other through its conjunction, or whilſt the earth is moving from *a* through *d* to *g*, the ſun and the planet are both on the ſame ſide of the earth. Therefore the motion of the earth will make both the ſun and the planet appear to move the ſame way; and conſequently the planets motion will be direct or in conſequentia.

In order to explane the retrograde motion of a ſuperior planet in the following propoſition, we may here obſerve, that a line *a*PA is ſuppoſed to be drawn from the earth through the planet to the concave ſphere of the heavens: then as one end of this line attends the earth in its motion through *abcdefg* and the middle of it turns round upon the planet P as upon a center, the other end A will mark out the planets apparent motion in the heavens; ſo that ABCDEFG will be the arc that the planet appears to deſcribe in the heavens whilſt its motion is direct, or will be the arc of the planets progreſs: and conſequently theſe letters in the order ABCDEFG are in conſequentia, or are diſpoſed according to the order of the ſigns.

122. *When a ſuperior planet is paſſing from one of its ſtations to the other through the oppoſition, its geocentric motion is retrograde.*

Whilſt the earth moves from *g*, Plat. XXIII. fig. 8, to *n* through *k*, the planet P appears to move from one of its ſtations G to the other ſtation

tion N through its oppoſition K, by propoſitions 120, 115. In this caſe the apparent motion of the planet ſeen from the earth is retrograde or in antecedentia, that is, the planet appears to move from eaſt to weſt or contrary to the order of the ſigns.

The motion of the earth round the ſun S makes the ſun appear to move in conſequentia, by propoſition 26. But whilſt the planet is moving from one of its ſtations to the other through its oppoſition, or whilſt the earth is moving from *g* through *k* to *n*, the ſun and the planet are on contrary ſides of the earth. Therefore the motion of the earth will make the ſun and planet appear to move contrary ways; and conſequently, as the ſuns geocentric motion is direct or in conſequentia, the planets geocentric motion will be retrograde or in antecedentia.

If the end *g* of the line *g*PG attends the earth in its motion through *ghiklmn*, and the middle of this line turns round upon the planet at P, the other end G, which marks out the planets motion in the heavens, will deſcribe the arc GHIKLMN. Now theſe letters in this order lie in a contrary direction to the letters ABCDEFG, fig. 7. But the arc ABCDEFG is the arc of the planets progreſs, as was obſerved under propoſition 121. Therefore GHIKLMN will be its arc of regreſs.

123. *Half the arc of a ſuperior planets regreſs is proportional to the angle, which a ſemidiameter of the earths orbit would ſubtend, if it was viewed from the planet.*

NG, Plat. XXIII. fig. 8, is the arc of the planets regreſs, if the planet is at P, and NK is half the arc of regreſs. This arc NK is proportional to the angle NPK, becauſe it ſubtends the angle NPK. But NPK is equal to its vertical *n*PS. Euc. b. I. prop. 15. Therefore NK half the arc of regreſs is proportional to the angle *n*PS. Now *n*PS is the angle, which *n*S a ſemidiameter of the earths orbit would ſubtend, if it was viewed by an eye placed at the planet P; and conſequently half the arc of a ſuperior planets regreſs is proportional to the angle, which a ſemidiameter of the earths orbit would ſubtend, if it was viewed from the planet.

124. *Mars's arc of regreſs is greater than jupiters, and jupiters is greater than ſaturns.*

Mars is nearer to the earth than jupiter, and jupiter is nearer than ſaturn. Therefore a ſemidiameter of the earths orbit would ſubtend a greater angle, if it was viewed from mars, than if it was viewed from jupiter; and a greater, if it was viewed from jupiter, than if it was viewed from ſaturn, by propoſition 126 of optics. Now half the arc, and conſequently double this

this half, or the whole arc of regreſs of a ſuperior planet is proportional to the angle, which the ſemidiameter of the earths orbit would ſubtend, if it was viewed from the planet, by propoſition 123. Therefore mars's arc of regreſs is greater than jupiters, and jupiters is greater than ſaturns.

125. *If the arc of regreſs of a ſuperior planet is known, the proportion, which the ſemidiameter of the earths orbit bears to the ſemidiameter of the planets orbit, may be determined.*

When the arc of regreſs GN, Plat. XXIII. fig. 8, is found by obſervation, half this arc, or NPK, and conſequently the angle NPK ſubtended by the arc NK, will be known. Therefore the angle *n*PS will likewiſe be known, becauſe it is equal to the angle NPK. Now as the planet P is ſtationary, when the earth is at *n*, and the planets geocentric place is as at N the end of the arc of regreſs, by propoſition 122, it follows, that P*n* is a tangent to the earths orbit, by propoſition 120. Therefore if from the point of contact *n* a ſemidiameter *n*S of the earths orbit is drawn, this ſemidiameter will be perpendicular to P*n*; and conſequently *n*S is the ſine of the angle *n*PS or half the arc of regreſs. From hence then we may conclude, that as the ſine of half the arc of regreſs is to the radius, in the tables of ſines, ſo is *n*S the ſemidiameter of the earths orbit to PS the ſemidiameter of the planets orbits.

By this means the proportion between the reſpective diſtances of the earth, mars, jupiter, and ſaturn from the ſun are found, as they are laid down in propoſitions 112, 113, 114. But then we muſt remember that this method only determines the proportion, which *n*S bears to PS, it does not determine the length either of *n*S or PS in any real meaſure; it does not determine how many miles or leagues or ſemidiameters of the earth, there are either in *n*S the diſtance of the earth from the ſun or in PS the diſtance of the planet from the ſun.

126. *The ſuperior planets do not always appear in the ſame part of the heavens at their conjunctions and oppoſitions, in their progreſs, their regreſs, and their ſtations.*

In explaning the conjunctions and oppoſitions, the progreſs, regreſs and ſtations of the ſuperior planets, we have hitherto ſuppoſed the planet itſelf to be at reſt. Upon this ſuppoſition the conjunctions and oppoſitions of the planet would always happen, when the planet appears in the ſame part of the heavens. If A, Plat. XXIV. fig. 1, is a ſuperior planet, ETIFPM the earths orbit, S the ſun, and GONHRKG a circle in the concave ſphere of the heavens; then ſuppoſing the planet A to have no motion

of

of its own, it will always be in oppoſition, when the earth is at E, and in conjunction, when the earth is at F. But when the earth is at E the planet A appears in the line EAG, ſo that the planet will be in ♎ at the time of oppoſition. And when the earth is at F, the planet appears in the line FAG, and conſequently it will be in ♎ again, at the time of oppoſition.

The conjunctions and oppoſitions, the progreſs, regreſs, and ſtations of any ſuperior planet will happen in the ſame manner, whether the earth and the planet both revolve in their reſpective orbits, or the planet is at reſt and the earth only revolves in its orbit with the difference of their true velocities; except only, that if the planet is at reſt, the conjunctions, and oppoſitions, the progreſs, regreſs, and ſtations will always happen, when the planet is in the ſame parts of the heavens; whereas, if the planet revolves as well as the earth, they will happen ſometimes when the planet is in one part of the heavens, and ſometimes when it is in another.

If the planet continues at A, its oppoſition happens whenever the earth comes to E, ſo as to be even with the planet, and on the ſame ſide of the ſun; and its conjunction happens whenever the earth comes to F, ſo as to be even with the planet and on the contrary ſide of the ſun. But if the planet revolves in its orbit ALBCDQ, but revolves ſlower than the earth, in this caſe the earth will overtake the planet, and come even with it ſometimes on the ſame ſide of the ſun and ſometimes on the contrary ſide. Therefore in this caſe likewiſe the planet will ſometimes be in conjunction and ſometimes in oppoſition. Thus when the planet is at D, the earth may be at P, and then the planet will appear in oppoſition; for the earth is even with the planet and they are both on the ſame ſide of the ſun: the ſun will appear in the line PSO, and the planet in the line PDR; therefore the ſuns geocentric place O is oppoſite to the planets geocentric place R. Or when the planet is at D, the earth may be at T, and then the planet will appear in conjunction; for the earth is even with the planet and they are on contrary ſides of the ſun: the ſun and the planet will therefore appear on the ſame ſide of the earth in the line TSDR. In like manner if the earth is at M, when the planet is at Q the ſuns geocentric place among the ſtars will be N, and the planets geocentric place will be in the oppoſite point K. Or if the earth is at I, when the planet is at Q, the geocentric place both of the ſun and planet will be on the ſame ſide of the earth at K, that is, the planet will be in conjunction. If the planet is at C, its conjunction will happen, when the earth is at E, and its oppoſition, when the earth is at F. Its geocentric place at ſuch a conjunction or oppoſition would be in the line EFCH or at ♈.

From

From hence it appears that by the planets motion in its orbit, it is seen sometimes in one part of the heavens, and sometimes in another, at the times of its oppositions and conjunctions. And it is evident, that these oppositions and conjunctions will happen just as frequently, whether the planet revolves in its orbit and the earth likewise revolves in a less time, or whether, as we supposed in the foregoing propositions, the planet stands still and the earth alone revolves with a velocity, which is the difference between its own and the planets true velocity: for upon either supposition the earth will overtake the planet or will come even with it just as often.

Now we have proved in propositions 121, 123, 120, that a superior planet, if it has no real motion of its own, as it passes through its conjunction, appears to move in consequentia; and that, as it passes through its opposition, it appears to move in antecedentia; and that whilst its motion is changing from direct to retrograde, it appears stationary. But if it has a real motion, we have now shewn, that its conjunctions and oppositions will happen in the same manner, as if it had no real motion. Therefore what has been said already concerning the progress, regress, and stations of the superior planets, upon supposition they had no real motion, is equally applicable to them, though they do revolve in their respective orbits. For since upon either supposition they will sometimes be in conjunction and sometimes in opposition, their apparent motions will sometimes be direct and sometimes retrograde; and as their motion changes from direct to retrograde, they will appear stationary. Only as the revolution of the superior planets is the reason why their conjunctions and oppositions happen in different parts of the heavens, so for the same reason their progresses, regresses, and stations must likewise happen in different parts of the heavens.

127. *The heliocentric latitude of a superior planet is the greatest, when the planet is in one of its limits.*

The latitude of a planet is the length of a line drawn from the planet to the plane of the ecliptic. The heliocentric latitude is the apparent length of this line viewed from the sun. Now as the sun is in or near the center of the planets orbit, an eye placed at the sun will always be at the same distance from the planet, and consequently at the same distance from that line, the length of which is its latitude. Therefore the apparent length of this line seen from the sun will always be as its real length, by proposition 127 of optics; that is, the heliocentric latitude will always be as the planets distance from the ecliptic. But the orbit of any superior pla-

net is inclined to the plane of the ecliptic, and croſſes that plane at the two nodes. Therefore, when the planet is in the nodes, it is in the ecliptic; and when it is at the limits, it is at the greateſt diſtance from the ecliptic: conſequently, when the planet is at the limits, its heliocentric latitude will be the greateſt.

128. *The geocentric latitude of a planet is directly as its heliocentric latitude and inverſely as the earths diſtance from the planet.*

The latitude of a planet is the length of a line drawn from the planet to the plane of the ecliptic. The geocentric latitude is the apparent length of this line viewed from the earth. Now the apparent length of this line will be directly as the real length and inverſely as the diſtance of the ſpectators eye, by propoſition 126, 127, of optics. But the heliocentric latitude is as the real length, by what has been proved under propoſition 127, and the ſpectators eye is ſuppoſed to be upon the earth. Therefore the apparent length of that line, or the planets geocentric latitude, is directly as its heliocentric latitude and inverſely as the earths diſtance from it.

129. *When a ſuperior planet is in conjunction its heliocentric latitude is greater than its geocentric latitude.*

When a ſuperior planet is in conjunction the ſun is between the earth and the planet, that is, the ſun is nearer to the planet than the earth is; and conſequently the latitude of the planet, if viewed from the ſun, muſt be apparently greater, than if viewed from the earth, by propoſition 126 of optics.

130. *When a ſuperior planet is in oppoſition its geocentric latitude is greater than its heliocentric latitude.*

When a ſuperior planet is in oppoſition, the earth is between the ſun and the planet, that is, the earth is nearer to the planet than the ſun is; and conſequently the latitude of the planet, if viewed from the earth, muſt be apparently greater than if viewed from the ſun, by propoſition 126 of optics.

131. *Mars appears gibbous, when it is in quadrature; but is full or round in conjunction and oppoſition.*

If the earth is at E, Plat. XXIV. fig. 1, mars, when at A, is in oppoſition, and, when at C, in conjunction. In either of theſe ſituations the ſame hemiſphere of mars is towards both the ſun at S and the earth at E. Therefore the whole viſible hemiſphere of mars will be enlightened,

and

and consequently its whole disc will be bright, as the moons is at the full. When the earth is at E and mars is at Q or L, it is in quadrature: for in either case the apparent distance of mars from the sun is 90 degrees, because mars appears either in the line EQ or in the line EL, and the sun in the line ES; and either EQ or EL makes a right angle or an angle of 90 degrees with the line ES. Now if mars is at Q the plane or circle of the disc *xz* makes a right angle with EQ, and the plane of the circle of light makes a right angle with the line SQ. But as SQ and SE do not coincide with one another, the circle of light will not coincide with the circle of the disc: and consequently the whole disc will not be enlightened, but the planet through a telescope will appear gibbous.

But though a part of the dark hemisphere of mars is towards the earth, when it is in quadrature, which makes it appear gibbous, yet, when it is at B or D, so little of this dark hemisphere is towards us, that it will not be sensible, and the planets disc will appear round.

132. *Jupiter and saturn always appear round.*

The enlightened hemisphere of these and all the other planets is towards the sun, and consequently, if the eye of the spectator was at the sun, these or any other planets would always appear round like the full moon. But EQ, Plat. XXIV. fig. 1, the distance of jupiter or saturn from the earth, is so great that ES bears a very small proportion to it, and consequently either of these planets will appear the same as to sense, whether it is viewed from the sun S or from the earth E. Therefore jupiter and saturn seen from the earth will always appear round like the full moon.

The circle or plane of jupiters disc *xz* is perpendicular to the line EQ, if jupiter is at Q; and the plane of the circle of light is perpendicular to SQ. But if EQ is so great that ES bears little or no proportion to it, that is, if in respect of the planets distance ES is nothing, then EQ coincides with SQ, and consequently the circles, whose planes are perpendicular to EQ and SQ coincide with one another, that is, the circle of light coincides with the circle of the disc. Therefore the whole disc will be enlightened, or jupiter will always appear round. And by the same method we might shew that saturn will always appear in the same manner.

CHAP. V.

Of the ſecondary planets.

133. *The middle diſtance of the moon from the earths center is equal to 60 ½ ſemidiameters of the earth; its eccentricity is 3 ½ of the earths ſemidiameters; its orbit makes an angle of about 5 ½ degrees with the plane of the ecliptic; its periodical time is 27 days 7 hours 43 minutes; and the time of a rotation round its axis is equal to its periodical time.*

THE eccentricity of the moon is variable, it is ſometimes greater and ſometimes leſs, than what is here laid down. And the inclination of its orbit is likewiſe variable. The truth of both theſe aſſertions ſhall be made out in the proper place.

134. *The moons higher apſis is called its apogee; its lower apſis is called its perigee.*

The moons orbit, which it deſcribes round the earth, is elliptical, and the earth is placed in one of its focuſes. When the moon is at its greateſt diſtance from the earth, or in the higher apſis, it is ſaid to be in apogee; and when it is at its leaſt diſtance from the earth, or in the lower apſis, it is ſaid to be in perigee.

135. *The new moon is the moon in conjunction with the ſun, and the full moon is the moon in oppoſition.*

136. *The ſyzygies of the moon is a common name to expreſs both its conjunction and oppoſition.*

137. *The moons aſcending node is called dragons head, and its deſcending node is called dragons tail.*

The moons orbit is inclined to the plane of the ecliptic, as has been obſerved already, and the two points, where the orbit cuts this plane, are its nodes. Now when the moon is in either node it is in the plane of the ecliptic. As it paſſes from one of the nodes to one of the limits, it comes to the north or to our ſide of the ecliptic, and, as it then aſcends towards us, this is called the aſcending node. As the moon paſſes from the other node to the other limit, it goes to the ſouth or from our ſide of the ecliptic, and as it then deſcends from us, this is called the deſcending node.

The aſcending node or dragons head is expreſſed by this mark ☊, and the deſcending node or dragons tail by this ☋.

138. *A periodical month is the time that the moon takes up to deſcribe its orbit, and contains 27 days, 7 hours, 43 minutes.*

139. *A ſynodical month is the time that paſſes between any new moon and the next that follows it: this month contains 29 days, 12 hours, 44 minutes, 3 ſeconds.*

140. *A ſynodical month is longer than a periodical month.*

The moon takes up a longer time to paſs from one conjunction to the next, than to deſcribe its whole orbit, or the time between one new moon and the next that follows it is longer than the moons periodical time. The difference between thoſe two months is about 2 days, 5 hours.

The moon revolves round the earth from weſt to eaſt; and the ſun apparently revolves round the earth the ſame way, by propoſition 26. Now at the new moon, or when the ſun and moon are in conjunction, they both ſet out from the ſame place to move the ſame way round the earth, the moon in its orbit and the ſun in the ecliptic. But the moon moves round much faſter than the ſun, and conſequently the moon will overtake the ſun: and when the moon does overtake the ſun, it will be a new moon again. If the ſun had no apparent motion in the ecliptic, the moon would come even with it, or be in conjunction again, after it had gone once round in its orbit. But as the ſun moves forwards in the ecliptic, whilſt the moon is going round, the moon muſt move a little more than once round, before it comes even with the ſun, or before it comes to conjunction. Therefore the time between one conjunction and the next that follows it is ſomething more than the time that the moon takes up to go once round its orbit, or a ſynodical month is longer than a periodical one.

Let S, Plat. XXIV. fig. 2, be the ſun; CF a part of the earths orbit; MD a diameter of the moons orbit, when the earth is at A; and *md* parallel to MD the ſame diameter, when the earth is at B. Whilſt the earth is at A, if the moon is at D, the moon will be in conjunction; and if the earth was to continue at A, when the moon had gone once round its orbit from D through M ſo as to return to D again, it would again be in conjunction. Therefore upon this ſuppoſition, that the earth has no motion in its orbit, the periodical and ſynodical months would be equal to one another. But as the earth does not continue at A, it will move forwards in its orbit, during this revolution of the moon, from A to B, and as the moons orbit moves with it, the diameter MD will then be in the poſition

ſition *nd*. Therefore, when the moon has deſcribed its orbit, it will be at *d* in this diameter *nd*. But if the moon is at *d* and the ſun at S, the moon will not be in conjunction; and conſequently the periodical month is completed, before the ſynodical. The moon in order to come to conjunction, when the earth is at B, muſt be at *e* in the diameter *ef*, or beſides going once round in its orbit, it muſt deſcribe the arc *de*. The ſynodical month is therefore longer than the periodical by the time that the moon takes up to deſcribe this arc *de*.

141. *The moon at the conjunction is not ſeen by us: the moon at the oppoſition has its whole diſc enlightened.*

Let QTL, Plat. XXIV. fig. 3, be a part of the earths orbit, S the ſun, T the earth, ACEG the moons orbit. The moons orbit is here ſuppoſed to be circular, and the earth to be in its center: for though it is in fact elliptical and the earth is in one of its focuſes, yet there is no neceſſity to take notice of this in explaning ſome of the moons appearances. The orbit is here indeed repreſented by an ellipſis, but this is not deſigned to repreſent the true figure of that orbit: it is deſigned to repreſent a circular orbit inclined to the plane of the paper, upon which it is drawn, in the ſame manner as the moons orbit is inclined to the plane of the ecliptic.

Now if the moon is at E in its orbit, it will be on the ſame ſide of the earth with the ſun, or in conjunction with the ſun. In this ſituation the ſun is beyond the moon and conſequently does not ſhine upon the hemiſphere of the moon, which is towards us. For this reaſon, as the moon has no light of its own, and the hemiſphere, which is towards us, has no light from the ſun, the diſc of the moon will be dark; as it is repreſented at *e*, and becauſe it is dark, that is, becauſe it neither has any light of its own to emit nor receives any from the ſun to reflect, we ſhall not ſee it.

But if the moon is at A in its orbit, the moon will be on one ſide of the earth and the ſun on the other ſide, or the moon will be in oppoſition to the ſun. In this ſituation the ſun and the earth are both on the ſame ſide of the moon, and conſequently the hemiſphere, which is viſible to us, who live upon the earth, will be the ſame hemiſphere that the ſun ſhines upon. Therefore the whole diſc of the moon will be enlightened, or the moon will be full, as it is repreſented at *a*.

142. *When the moon is near the quadratures, its diſc is half enlightened; when it is between the quadratures and the conjunction, the enlightened part of its diſc is horned; when it is between the quadratures and the full, the enlightened part of its diſc is gibbous.*

When

When the moon is at C or G in its orbit, is apparent diſtance from the ſun S is 90 degrees: for the line TC or TG, in which the moon appears, as we view it from the earth T, makes a right angle with the line TS, in which the ſun appears. The moon in either of theſe ſituations is in the quarters or quadratures. At E it is new; when it has moved from E the conjunction to G, it is in quadrature, and this is called the firſt quarter of the moon. At A it is full, when it has moved from A the oppoſition to C, it is in quadrature again, and this is called its laſt quarter.

Now at either quadrature, as the moon is in the middle between the conjunction and the oppoſition, the ſtate of its diſc will be the middle between its ſtates at the new and at the full. At the new its whole diſc is dark, and at the full its whole diſc is enlightened. Therefore in the middle ſtate it will be half dark and half enlightened, as it is repreſented at *g* and *c*. When the moon is in the quarters, half the dark and half the enlightened hemiſphere will be towards the earth; therefore the hemiſphere, which we ſee, or the diſc of the moon, will be half light and half dark.

When the moon is any where between the new and the firſt quarter, ſuppoſe at F, or any where between the laſt quarter, and the new, ſuppoſe at D, its diſc will be in an intermediate ſtate between what it is at the quarters and what it is at the new. But at the quarters it is half light, and at the new it is all dark. Therefore in the intermediate ſtates it will neither be half light nor all dark, but between both: ſo that the enlightened part, being leſs than half the diſc, will be horned or will have the form of a creſcent; as it is repreſented at *f* and *d*. In this ſituation of the moon the hemiſphere, which we ſee, or the diſc, conſiſts of more than half the dark hemiſphere and of leſs than half the enlightened one.

When the moon is any where between the firſt quarter and the full, ſuppoſe at H, or any where between the full and the laſt quarter, ſuppoſe at B, its diſc will be in an intermediate ſtate between what it is at the quarters and what it is at the full. But at the quarters it is half dark, and at the full it is all light. Therefore in the intermediate ſtates it will neither be half dark nor all light, but between both: ſo that the enlightened part, being more than half the diſc, will appear gibbous; as it is repreſented at *b* and *b*. In this ſituation of the moon the hemiſphere, which we ſee, or the diſc, conſiſts of more than half the enlightened hemiſphere and of leſs than half the dark one.

143. *The angle of the moons elongation is nearly equal to the angle contained between the circle of light upon the moons ſurface and the circle of the diſc.*

The

The moons elongation is its apparent diſtance from the ſun. What we mean by the circles of light and of the diſc has been explained already, in propoſitions 105, 106, where we were ſpeaking of the inferior planets.

Suppoſe the moon to be at F, Plat. XXIV. fig. 3. the earth at T, and the ſun at S. The angular diſtance of the moon from the ſun, or the moons elongation, is FTS the angle contained between the line TF, in which the moon appears, and the line TS, in which the ſun appears, when they are both of them viewed from the earth. If a line *Tr* is ſuppoſed to be drawn from the center of the earth to the center of the moon, and a circle is likewiſe ſuppoſed to be drawn round the ſpherical ſurface of the moon with its plane perpendicular to *Tr*; the plane of this circle will paſs through our eye, not indeed if the eye was at T the earth, but as we are looking at the paper where this figure is drawn. Conſequently the line *oz* perpendicular to *Tr* will repreſent this circle. And this is the circle of the diſc, by propoſition 106. If another line is drawn from the center of the ſun to the center of the moon, and a circle is drawn round the moon at *mn* with its plane perpendicular to *Sr*, this will be the circle of light, by propoſition 105, and as its edge, when we are looking at the paper, is towards our eye, the line *mn* will repreſent the plane of this circle. Now the angle *mro*, which theſe two circles make with one another, is nearly equal to *STr*. If the triangle *TrS* is completed by drawing a line TS from the center of the earth to the center of the ſun, the external angle *Trx* at the moon is equal to *mro*, as may be proved by the ſame reaſoning that we have already made uſe of in propoſition 107. But the external angle *Trx* is equal to the two internal and oppoſite angles to *STr*+*TSr*, Euc. b. I. prop. 32. Therefore *mro* is equal to *STr*+*TSr*. Now *STr* is the moons elongation: and from what has been ſaid it appears that *mro* the angle contained between the circle of light and the circle of the diſc is greater than *TSr* the moons elongation. The propoſition does not affirm that they are exactly, but only that they are nearly equal to one another. The difference between them is the angle *TSr*. This angle *TSr* is the angle, which *Tr* the ſemidiameter of the moons orbit would ſubtend if it was viewed from the ſun, and we ſhall have occaſion to obſerve in another place that this angle is ſo ſmall that we may neglect it or conſider it as equal to nothing. Therefore there will be no ſenſible difference between *mro* and *STr*.

From this propoſition we may ſhew how the moons diſc will appear, in the ſame manner as we ſhewed, under propoſition 107, how the diſcs of the inferior planets appear, in different ſituations. Let *bocf*, Plat. XXIII. fig. 6, be the circle of the diſc. When the moon is in conjunction, its elon-

elongation is nothing, and consequently the circle of light will coincide with the circle of the disc, and as the moon has then its dark hemisphere towards the earth the whole disc will be dark. When the moon is in quadrature its elongation is 90 degrees; and consequently the circle of light will make a right angle with the circle of the disc. Therefore as we look directly at the disc *bcf* the plane of the circle of light which is perpendicular to that of the disc will pass through our eye. In this case the plane of the circle of light will be apparently a right line *bc* dividing the disc into two equal parts, one of which is half the enlightened and the other is half the dark hemisphere of the moon; so that half the disc will be light and half of it dark. When the moons elongation is less than 90 degrees, or when the moon is between the quarters and the conjunction, the circle of light will make a less angle than a right one with the circle of the disc, and its plane will be oblique to our eye; just as the semicircle *bfc*, which is oblique to the eye and therefore appears elliptical, makes an angle less than a right one with the side of the disc *bpc*. In these circumstances *bfcp* the enlightened part of the disc will be a crescent. When the moons elongation is more than 90 degrees, but less than 180, that is, when the moon is between the quarters and the opposition, the circle of light will make a greater angle than a right one with the circle of the disc and its plane will be oblique to our eye, whilst we look at the disc; as the semicircle *bfc*, which is oblique to the eye, and therefore appears elliptical, makes an angle greater than a right one with the side of the disc *bpc*. In these circumstances *bfcp* the enlightened part of the disc will appear gibbous. When the moons elongation is 180 degrees, that is, when the moon is in opposition, the circle of light makes an angle of 180 degrees with the circle of the disc, or, which is the same thing, these two circles coincide with one another, and as the hemisphere towards the earth is the enlightened one, the whole disc of the moon will be enlightened.

143. *The horns of the moon, after the new, are turned towards the east, and before the new, towards the west.*

The moon, just after the new, is seen in the west after the sun is down. Now as the sun at this time sets in the west before the moon, the sun is planely to the west of the moon. But the horns are always turned towards the dark part of the disc or from the sun, for the bright edge of the disc, where the horns are, is what the sun enlightens. Therefore the horns at this time are turned towards the east.

The moon, just before the new, is seen in the east before the sun rises. But the moon in this situation must have risen above the eastern horizon

before the ſun; from whence we may readily underſtand that the ſun is to the eaſt of the moon; and conſequently, as the moons horns are always turned from the ſun, they will be turned towards the weſt.

144. *When the moon is horned, that part of the diſc, upon which the ſun does not ſhine, has light enough to make it viſible.*

It has been a queſtion whether the moon, beſides the light which it receives from the ſuns ſhining upon it, does not emit a faint light of its own. Thoſe who have imagined the moon to be a faintly luminous body, have been led into this opinion partly by the appearance mentioned in the propoſition. When the moon is horned or when *bfcp*, Plat. XXIII. fig. 6, is the part which the ſuns rays enlighten, the other part of the diſc *bfco*, upon which the ſun does not ſhine, has a dim light, that is ſufficient to make it viſible. So that, juſt before the new and juſt after it, we may ſee the whole diſc; *bfcp* appears very bright, but *bfco* is very faint. This faint or ſecondary light, which comes from the part *bfco*, where the ſun does not ſhine upon the diſc, has by ſome been thought to be a light emitted by the moon itſelf. We may be ſure that this is a miſtake; becauſe if the moon had any light of its own, this light would make that part of the moons diſc, which the ſun does not ſhine upon, appear as plane to us at all times, when the moon for inſtance is in the quarters, or when it is gibbous, as when it is horned.

In order to account for this ſecondary or faint light upon that part of the new moons diſc, where the ſun does not ſhine, we ſhould obſerve that as the moon reflects the ſuns light to the earth and by that means ſhines upon the earth, ſo the earth will reflect the ſuns light and will ſhine upon the moon. When the moon is near the new, it is at D or F, Plat. XXIV. fig. 3, nearly on the ſame ſide of the earth with the ſun S: and conſequently at that time the enlightened hemiſphere of the earth will be towards the moon. Therefore the earth will ſhine full upon that part of the moons diſc, where the ſun does not ſhine. The light thus viewed from the earth is the faint light which we were here to account for. When the moon is in the quarters as at G or C, or when it is gibbous, as at B or H, the enlightened hemiſphere of the earth is not directly towards the dark part of the moon, and conſequently at theſe times the part of the diſc, which the ſun does not ſhine upon, has little or no light from the earth.

145. *The moon always turns the ſame ſide towards the earth.*

This is a matter of fact and can only be proved by obſervation. At the full we ſee the whole diſc of the moon: and ſome parts of it are much bright-

brighter than the reſt: the darker parts we will call ſpots. Soon after the new, only a ſmall part at the weſtern edge of the diſc is enlightened by the ſun; and at the firſt quarter, the weſtern half of the diſc is enlightened. In either of theſe caſes the ſpots upon the weſtern part of the moon are of exactly the ſame ſhape and in exactly the ſame ſituation, that they are upon the weſtern part of the diſc, when the moon is at the full and we can ſee the whole diſc. But the ſpots upon the weſtern part of the moon could not appear to us of the ſame ſhape and in the ſame ſituation, at the full, ſoon after the new, and at the firſt quarter, unleſs the moon was in the ſame ſituation in reſpect of the earth at all theſe different times, that is, unleſs the ſame ſide of it or the ſame hemiſphere was towards the earth.

In like manner a little before the new, only a ſmall part of the eaſtern edge of the diſc is enlightened by the ſun, and at the laſt quarter the eaſtern half of the diſc is enlightened. In either of theſe caſes the ſpots are the ſame as to ſhape and ſituation with what we ſee upon the eaſtern part of the diſc at the full. But this could not happen, unleſs at all theſe times the moon was in the ſame ſituation in reſpect of the earth or had the ſame hemiſphere towards us.

What is affirmed in this propoſition, is to be underſtood with ſome reſtriction: for though the diſc of the moon, as to far the greateſt part of it, is the ſame at all times; yet about the edges of it, there is ſome little alteration; it is nearly the ſame, though not exactly the ſame hemiſphere of the moon, which is always turned towards the earth. Whence this little alteration ariſes ſhall be ſhewn hereafter.

146. *The time that the moon takes up to perform a rotation round its axis is equal to the time that it takes up to perform one revolution in its orbit.*

The ſame ſide of the moon, or nearly the ſame ſide, is always towards the earth, by propoſition 145. I ſhall not make uſe of this obſervation to prove that the moon has a rotation round its axis: but, upon ſuppoſition that it has ſuch a rotation, I ſay that the time of a rotation upon its axis muſt be equal to the time of a revolution in its orbit, becauſe it always turns the ſame ſide towards the earth. Let *t*, Plat. XXIV. fig. 4, be the earth, ABCD the moons orbit, *rpsq* the moon, and F a fixed ſtar. If the moon is at A and revolves upon an axis that paſſes through *o*, then the ſeveral ſides of it, *r*, *p*, *s*, and *q*, will in their turns be towards the fixed ſtar F. Now if the moon had no other motion beſides this, the ſeveral ſides *s*, *q*, *r*, *p*, would likewiſe in their turns be towards the earth

But the moon revolves in its orbit ABCD, and always turns the same side towards the earth. From hence then we are to prove that, if the moon has a rotation round its axis, the time of its rotation is equal to the periodical time or time of a revolution in its orbit. If the moon was to continue at A, it is plain that, when the side *p* is towards the star F, a quarter of a rotation is completed; when the side *r* is towards the star F half a rotation is completed, when the side *q* is towards the star F, three quarters of a rotation is completed: and when the side *r* returns to its first situation and is towards the star F, a whole rotation is completed. But suppose, that in the time taken up to make a quarter of a rotation, the moon has moved through a quarter of its orbit from A to B: then because a quarter of a rotation is completed the side *p* of the moon is towards the star F; and whilst the moon is at B with its side *p* towards the star F, the side *s*, which is towards the earth, is the same that was towards the earth, whilst the moon was at A. Suppose farther that in the time of half a rotation the moon has moved through half its orbit and is come to C: then, because the moon has turned half round upon its axis, the side *r* will be towards the star F, and this same side *s*, which was towards the earth, whilst the moon was at A, will likewise be towards the earth, when it is come to C. Suppose again that in the time taken up to perform three quarters of a rotation upon the axis, the moon has gone through three quarters of its orbit and is come to D: then, because the moon has turned three quarters round its axis, the side *q* will be towards the star E; and whilst the moon is at D with the side *q* towards the star F, the same side *s* will be towards the earth, that was towards it when the moon was at A. When the revolution is completed and the moon is returned to A, if the rotation round the axis is likewise completed, the moon will in all respects be in the same situation as at first, the side *r* will be towards the star, and the side *s* towards the earth.

Thus we have seen that if the moon, whilst it revolves in its orbit ABCD, turns round upon its own axis *c*, that passes through it, the same side *s* will always be towards the earth *t*, provided the time of a rotation upon the axis is equal to the time of a revolution in the orbit.

I have not here endeavoured to prove that the moon has a rotation round its axis from what has been observed, in proposition 145, about its turning always the same side towards the earth: because this appearance does not sufficiently prove that the moon has such a rotation. For the same side of it would always be towards the earth, either if the moon has a rotation round an axis within itself or round an axis without itself, whether it turns once round its own axis in the time of a revolution, or turns round an axis that passes through the earth.

We

We have already ſhewn, that if the moon turns once round upon its own axis or even round an axis within itſelf in the time of a revolution, the ſame ſide of it will always be towards the earth. It remains for us to ſhew that the ſame thing will happen, if the moon's motion is referred to an axis that is without itſelf and paſſes through the earth. In order to ſhew this, we will illuſtrate it by ſome inſtances of a motion of the ſame ſort. While a cart-wheel turns round, a nail in the rim of the wheel revolves with it. Such a nail has its point always towards the axle. But though the ſame ſide of the nail is always turned towards the axle, the nail has no rotation round an axis of its own; it does not turn upon an axis that is within itſelf or that paſſes through it; but its motion is round the axle of the wheel, which is an axis without itſelf. If a cylinder or rolling-ſtone was laid at the earths equator with its axis parallel to the earths axis, then, as the earth turns round its axis, this rolling-ſtone would turn round with it, and the ſame ſide of the ſtone would be always towards the earths center, in the ſame manner as the ſame ſide of the moon is always towards the earth. But this rolling-ſtones rotation would be round the earths axis and not round its own; its motion muſt be referred to an axis without itſelf and not to an axis within itſelf. Suppoſe that *orſs*, Plat. XXIV. fig. 4, was a ſmall round leaden weight with a ſtaple fixed in it at *s*; then if a ſtring was faſtened to this ſtaple, and a man ſtanding at *t* had hold of the other end of the ſtring, the man by turning himſelf round quick might make the weight whirl round in the circle ABCD. In the mean time the ſtaple and conſequently the ſame ſide of the weight would be always towards the man at *t*; and yet the weight would not turn upon any axis of its own. The axis of this motion would be the man at *t* and not any line paſſing through the weight. In like manner though the moon does always turn the ſame ſide towards the earth, yet it does not appear from hence that the moon has a rotation round its own axis: the axis of this motion may as well paſs through the earth as through the moon, notwithſtanding the ſame ſide of it is always towards the earth. All that we can prove from hence is, that if the moon has a rotation round its axis independent of the motion in its orbit, then the time of a rotation is equal to its periodical time. That the moon has ſuch a motion round an axis of its own independent of the motion in its orbit, ſhall be ſhewn in the two following propoſitions.

147. *The moon appears to librate upon a line perpendicular to its axis: this is called its libration in latitude.*

Upon ſuppoſition that the moon has a rotation upon its own axis, we may explane the libration in latitude: and if this ſuppoſition will be found

found necessary to explain the libration, we may come to this conclusion; the moons doubtless is hollow, therefore it have rotation round an axis of its centre.

Let CE, Plat. XXIII. fig. 3, represent the plane of the moons orbit, let S represent the earth in the middle of this orbit, and let AGBC be the moon. We here suppose the moon to have a rotation round an axis within itself, let PR be this axis: call P the moons north pole and R its south pole. AGB is the visible hemisphere, and AB is the plane of the disc, by propositions 99, 100. In this situation of the moon its north pole P is within the disc, and may be seen from the earth S. But though the same hemisphere of the moon, as to much the greatest part of it, is always the same, by proposition 145; yet some little variations are made in this visible hemisphere about the edges of the disc. Sometimes the pole P is in the visible hemisphere, and then the pole R is not seen within the disc; at other times the pole R is in the visible hemisphere, and then the pole P is not seen within the disc. Now as each pole of the moon is thus alternately shewn to us and hidden from us, the moon appears to librate or oscillate upon a line drawn through I perpendicular to its axis PR. The pole P appears to swing back beyond A the edge of the disc, and this brings the pole R forwards beyond B the opposite edge of the disc: then the pole P appears to swing forwards beyond A and comes into sight again, and the other pole R at the same time swings back beyond B and is hidden from us. But this vibration of the two poles P and R, or of the axis PR upon the line I is not real, but apparent: for when the moon is in the same part of its orbit, the same hemisphere is always seen by us. I have explained this libration as if it might happen whilst the moon is at C; but whilst it is at C the pole P will always be in the visible and the pole R in the invisible hemisphere. The pole P gets into the invisible hemisphere and the pole R into the visible one by the moons moving half round its orbit from C to E. Indeed the moons disc is not always exactly the same at the full; sometimes the pole P will be seen in the disc and sometimes the pole R: the visible hemisphere, which constitutes the disc, is always the same, when the moon is in the same part of its orbit; but the moon, when at the full, is not always in the same part of its orbit, by proposition 139.

This apparent libration, which we have been explaining, arises from the inclination of the moons axis to the plane of its orbit. When the moon is at C, if the axis PR is inclined to EC the plane of its orbit so as to make with it the angle PIG; then to a spectator at the earth S AGB will be the visible hemisphere, and the pole P will be within the disc. But when

the

the moon is at E the opposite point of its orbit, the axis PR being always parallel to itself, will make the angle PeE = FeQ with the plane of its orbit, and the visible hemisphere will be AHB. This hemisphere, by proposition 145, is in all respects the same with AGB, except only that here the pole R will be within the disc instead of the pole P. Thus from the inclination of the moons axis to the plane of its orbit it follows, that sometimes its north pole and sometimes its south pole will be towards the earth: and this is the occasion of its apparent libration in latitude.

From this libration we may prove that the moon has a rotation upon an axis of its own independent of its revolution round the earth. For if the earth was the only axis of the moons motion, any diameter of it, as PR, would always be in the same situation in respect of the earth: so volution of the moon AGBC round an axis that passes through S the earth, will not alter the position of PR in respect of S, through which the axis of this revolving motion passes; if the end P of this diameter is towards S the supposed axis of motion, when the moon is at C, the same end P of this diameter would still be towards S, when the moon has gone half round its orbit and is come to E. Therefore if the only axis of the moons motion passed through the earth, or if the moon had no rotation upon an axis, that passes through itself, independent of its revolution round the earth, exactly the same hemisphere of the moon would always be turned towards the earth and there would be no libration in latitude. But the moon does appear to librate upon a line drawn through e: and consequently it has a rotation round an axis within itself.

148. *The moon appears to librate upon its axis: this is called its libration in longitude.*

We will here proceed as in proposition 147: we will first suppose that the moon does turn round upon its axis in order to explane this libration, and from thence we shall see, that, as this libration can be explaned upon no other principle, it must follow that, because the moon does librate in this manner, it must have a rotation round its own axis independent of the revolution in its orbit.

Suppose the moon to be at A, in its orbit, Plat. XXIV. fig. 5, and the earth to be at T; and let a line passing through the moon at *c* be its axis. T*c* is a line drawn from the earths center to the moons center, and consequently, if a circle, whose plane is perpendicular to T*c*, encompasses the moon at *ds*, this will be the circle of the disc, and *des* will be the visible hemisphere, by propositions 100, 106. But this disc is not always exactly

eaſtern edge becomes viſible. When *fg* is the circle of the diſc and *fxg* the viſible hemiſphere, we ſee a part of the eaſtern edge of the moon, which we are not uſed to ſee; and an equal part of the weſtern edge becomes inviſible. This apparently ſwinging or libraring motion of the moon upon its axis *c* from eaſt to weſt and from weſt to eaſt alternately is called its libration in longitude.

In explaining what this libration is, we have ſuppoſed theſe changes in the moons diſc to happen, whilſt it is at the ſame place A in its orbit. But in ſhewing the cauſe of this apparent libration, we ſhall find that theſe differences in the diſc will happen in different parts of the moons orbit; however as the moon at the full is not always in the ſame part of its orbit, this libration will make the diſc at the full to be not always exactly the ſame.

If the moon has a rotation round its axis, one rotation will be completed in the moons periodical time, by propoſition 145. And as the moons rotation round its axis is uniform, a quarter of a rotation will be completed in a quarter of its periodical time. Now the moons orbit is elliptical, and the earth is not placed in C the center of its orbit, but at T one of the focuſes. From whence it follows, that the moon moves ſlower in the apogee A than in the perigee P: and that its velocity conſtantly encreaſes, whilſt it is moving from apogee to perigee, and conſtantly decreaſes, whilſt it is moving from perigee to apogee, by propoſition 161 of mechanics. When the moon is at A, *dxs* is the viſible hemiſphere, and *ds* perpendicular to T*c* is the circle of the diſc. After it has moved from A to B or through a quarter of its orbit, if it had turned juſt a quarter round its axis, *ds* would ſtill be perpendicular to T*c* and would be the circle of the diſc. But it turns a quarter round its axis in a quarter of its periodical time, and it takes up more than a quarter of its periodical time to move through the firſt quarter of its orbit from A to B; becauſe in this quarter it is moving with its leaſt velocity. Therefore when the moon comes to B, *ds* will not be perpendicular to T*c*; but the point *s* on the moons ſurface will be turned from weſt to eaſt more than a quarter round the axis *c*. Thus the point *s*, which could be ſeen from the

PLATE XXII.

Fig. 1.

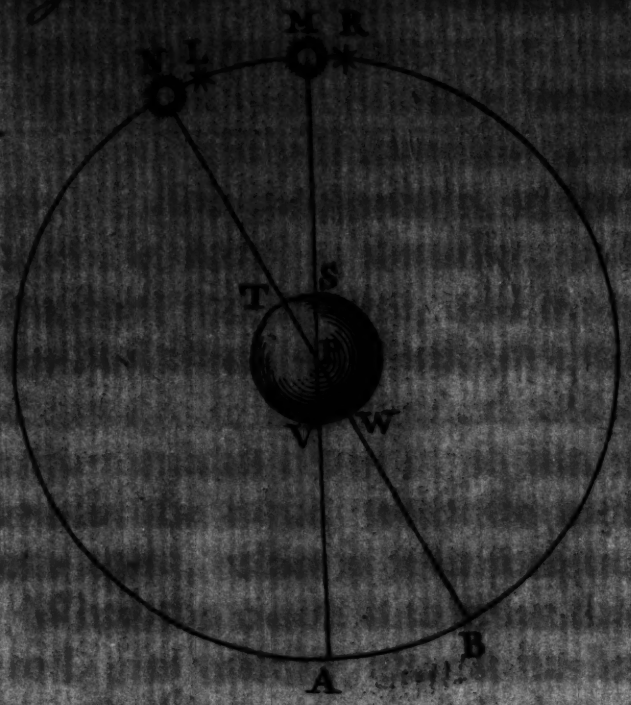


Fig. 2.

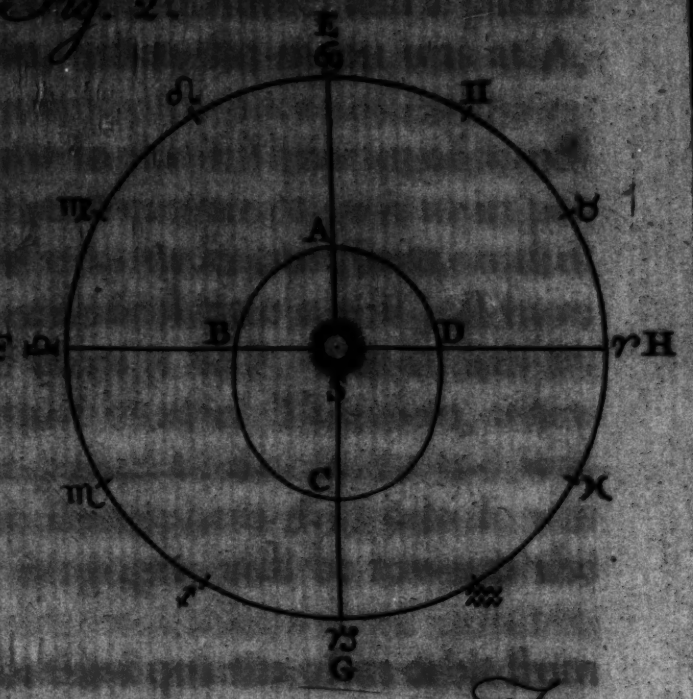


Fig. 3.

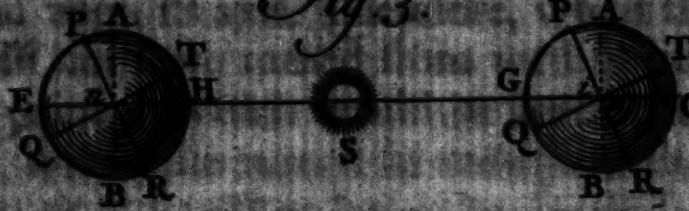


Fig. 4.

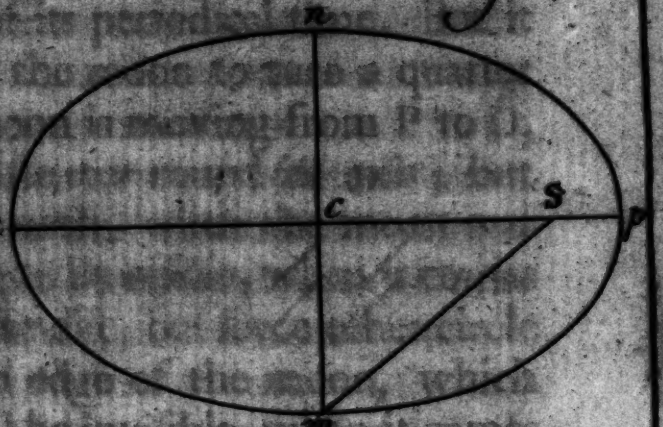


Fig. 5.

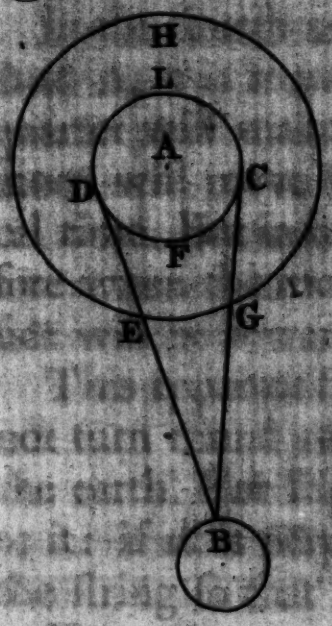


Fig. 6.

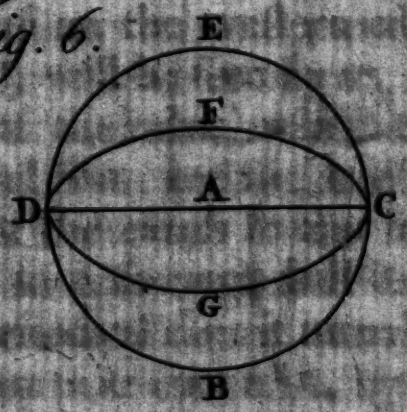


Fig. 7.

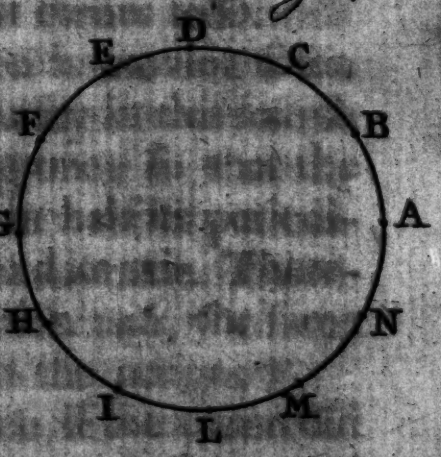
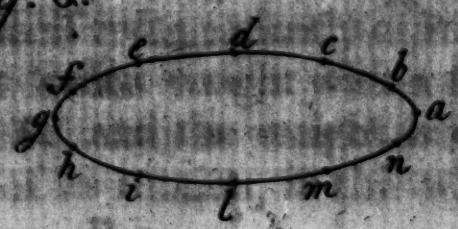


Fig. 8.



earth, when the moon was at A, will be too far turned to the eaſt to be ſeen, when it is at B: *ds* will then be no longer the circle of the diſc, but ſome other circle *bl*: a part *bs* upon the weſtern edge, which we could ſee when the moon was at A, will then become inviſible, and a part *dl* upon the eaſtern edge, which was inviſible, when the moon was at A, will then be ſeen by us.

But as much as the moon moved too ſlow in the firſt quarter of its orbit, ſo much it will move too faſt in the ſecond quarter from B to its perigee P: and conſequently, as the exceſs of velocity in one quarter makes up for the defect in the other, in half its periodical time it will juſt have completed half its orbit, and have moved from A to P. But in half its periodical time it will have revolved half round its axis. Therefore, when the moon is in its perigee, *ds* will again be perpendicular to T*c* and will be the circle of the diſc: ſo that the ſame hemiſphere *dxs*, which was towards the earth, when the moon was in apogee, will be towards the earth again, when it is in perigee.

When the moon is to move through the third quarter of its orbit from P to D; its velocity, after it has left the perigee, is ſo great as to carry it through this quarter in leſs than a quarter of its periodical time. But it takes up a quarter of the periodical time for the moon to turn a quarter round upon its axis. Therefore, whilſt the moon is moving from P to D, it will not have turned from weſt to eaſt a quarter round its axis; but any point *d*, which would be ſeen from the earth, when the moon was at P, will not be turned far enough to the eaſt to be viſible, when it comes to D: thus *ds* will no longer be the circle of the diſc, but ſome other circle *bl*; by which means a part *dl* on the eaſtern edge of the moon, which we could ſee, when the moon was at P, will be inviſible, when it comes to D, and an equal part *sb* on the weſtern edge, which was inviſible, when the moon was at P, will be ſeen by us, when it comes to D.

In the fourth quarter of the orbit, whilſt the moon is moving from D to A, it is in its retarded ſtate, and the defect of velocity in this fourth quarter will make up for the exceſs in the foregoing one; ſo that the moon will move from P to A through half its orbit, in half its periodical time. But in half its periodical time it turns half round its axis. Therefore when the moon comes to its apogee A, by propoſition 146, the ſame ſide will be towards the earth, that was towards it in the perigee P.

This apparent libration of the moon could not happen, if the moon did not turn round an axis within itſelf independently of its revolution round the earth. For ſuppoſe A was a leaden weight with a ſtring T*c* faſtened to it: if *ds* is perpendicular to T*c*, and the weight is whirled round by the ſtring ſo that T is the axis of its motion, in this whole revolution *ds*

will continue perpendicular to *Tc*, and conſequently the ſame hemiſphere *dxs* of the weight will always be towards T. In like manner, if the earth was the only axis of the moons motion, and the moon had no rotation round an axis within itſelf, *dt* would always be the circle of the diſc, for it would always be perpendicular to *Tc*, and the ſame hemiſphere *dxs* would be always towards the earth. But exactly the ſame hemiſphere of the moon is not always towards the earth; for the moon has a libration in longitude. Therefore the earth is not the only axis of the moons motion, but the moon has an axis of its own, which paſſes through itſelf, and upon which it performs a rotation.

149. *The moons ſurface is not ſmooth, but uneven and mountainous.*

If the moons ſurface was exactly ſmooth, the limit between the enlightened and the dark parts of the diſc would be exactly defined, when the moon is horned, or halved, or gibbous. For theſe limits are the circle of light, by propoſition 142; and a circle drawn round a ſphere that is exactly ſmooth will be every where regular. If *bocp*, Plat. XXIII. fig. 6, is the moons diſc, *bfcp* the enlightened part, and *bfco* the dark part; then, upon ſuppoſition that the moons ſurface was exactly ſmooth, theſe parts would be exactly defined, or the line *bfc*, which ſeparates one from the other, would be an even regular curve. But by looking through a teleſcope the limit between the enlightened and dark parts is found to be jagged and uneven. Many bright ſpots appear in the dark part of the diſc ſtanding out at different diſtances from the common boundary. Theſe bright ſpots in a little time grow larger and ſpread, till they become big enough to joyn themſelves to the enlightened part of the diſc. Now ſince theſe bright ſpots are beyond the regular limit between the bright and dark parts of the diſc, they muſt receive the ſuns light upon them, ſooner than they would do, if they were upon a level with the moons ſurface: for they appear bright at a time when they are on what ought regularly to be the dark ſide of the limit. Therefore theſe ſpots are above the level of the ſurface: they are the tops of mountains, upon which the ſun ſhines ſooner than it would do, if they were not raiſed ſo high above the level. But though the ſun ſhines firſt upon the tops of theſe mountains, yet as it riſes higher above the plane of the diſc, its light will gradually deſcend down their ſides, and thus theſe bright ſpots are enlarged, till at laſt the ſun ſhines upon the valleys between them, and then they joyn themſelves to the enlightened part of the diſc.

That part of the moon, round which the circle of the diſc paſſes, is undoubtedly as mountainous as the reſt: but yet the circle of the diſc appears regular and even, the edges of the diſc are exactly defined and do

not

not appear uneven and jagged. I ſhall explane this in the words of that excellent mathematician and philoſopher Dr. Smith. 'If the ſurface of the moon had but one row of mountains placed round the limb of the diſc, the bright part of it would appear irregularly indented; but ſince the viſible limb muſt be conſidered as a large zone poſſeſſed by many rows of mountains lying behind one another with reſpect to the obſervers eye, the mountains in ſome rows, being oppoſite to the valleys in others, will fill up the inequalities in the viſible limb; eſpecially being all ſo remote and uniformly enlightened, that the inequalities of their diſtances from the eye cannot be perceived: juſt as a great number of mountains upon the earth, viewed from a great diſtance, or the waves in the ſea, how great ſoever, do yet compoſe an horizon apparently ſmooth and circular.'

150. *The diſtance of jupiters innermoſt ſatellite from the center of jupiter is* 5,667 *ſemidiameters of jupiter: the diſtance of the ſecond is* 9,017 *ſemidiameters: the diſtance of the third is* 14,384 *ſemidiameters; and the diſtance of the fourth is* 25,299 *ſemidiameters.*

151. *The periodical time of jupiters innermoſt ſatellite is* 1 *day* 18 *hours* 27 *minutes* 34 *ſeconds: the periodical time of the ſecond is* 3 *days* 13 *hours* 13 *minutes and* 42 *ſeconds: the periodical time of the third is* 7 *days* 3 *hours* 42 *minutes and* 36 *ſeconds: the periodical time of the fourth is* 16 *days* 16 *hours* 32 *minutes and* 9 *ſeconds.*

152. *Saturn is encompaſſed with a thin ring, the plane of which is inclined to the plane of the ecliptic at an angle of about* 31 *degrees: the diſtance of this ring from the planet is equal to its own breadth: and the ſemidiameter of it is to the ſemidiameter of ſaturn as* 9 *to* 4. The planet and its ring are repreſented Plat. XXIV. fig. 6.

153. *The diſtance of ſaturns innermoſt ſatellite from the center of ſaturn is* 1,93 *ſemidiameters of the ring: the diſtance of the ſecond is* 2,47 *ſemidiameters: the diſtance of the third is* 3,45 *ſemidiameters: the diſtance of the fourth is* 8 *ſemidiameters: the diſtance of the fifth is* 23,35 *ſemidiameters.*

154. *The periodical time of ſaturns innermoſt ſatellite is* 1 *day* 21 *hours* 18 *minutes* 27 *ſeconds: the periodical time of the ſecond is* 2 *days* 17 *hours* 41 *minutes* 22 *ſeconds: the periodical time of the third is* 4 *days* 12 *hours* 25 *minutes* 12 *ſeconds: the periodical time of the fourth is* 15 *days* 22 *hours* 41 *minutes* 14 *ſeconds: the periodical time of the fifth is* 79 *days* 7 *hours* 48 *minutes.*

155. *Any ſatellite is at its greateſt elongation from its primary, when a line drawn from the earth through the ſatellite is a tangent to the ſatellites orbit.*

This and the following propoſitions are expreſſed in general terms ſo as to include the ſatellites both of jupiter and ſaturn. We will explain them in the inſtances of jupiters ſatellites, and the reader will eaſily infer that they are equally true of ſaturns.

Let FIE, Plat. XXIV. fig. 7, be a part of jupiters orbit, AXBT the earths orbit, S the ſun, FOLMN the orbit of jupiters outermoſt ſatellite. If the earth is at X, and the ſatellite is at L or N, ſo that a line XL or XN drawn from the earth to the ſatellite is a tangent to the ſatellites orbit; then this ſatellite ſeen from the earth will appear at a greater diſtance from the primary I, than it can do in any other ſituation. This is proved to be true of the ſatellites in reſpect of their primaries in the ſame manner that we proved propoſition 81, to be true of the inferior planets in reſpect of the ſun.

156. *Any ſatellite appears in conjunction with its primary, either when the ſatellite is between the earth and the primary, or when the primary is between the ſatellite and the earth: the former is the ſatellites ſuperior conjunction with its primary, and the latter is its inferior conjunction.*

If the earth is at X, Plat. XXIV. fig. 7, and jupiter is at I, the outermoſt ſatellite will be in conjunction with jupiter, when they both appear in the ſame line XMIV. The ſatellite will therefore be in its inferior conjunction, when it is at M between the earth and jupiter, and in its ſuperior conjunction when it is at V, ſo that jupiter is between the ſatellite and the earth.

157. *The apparent motion of any ſatellite is direct, as it paſſes from its greateſt elongation on one ſide of its primary, through the ſuperior conjunction, to its greateſt elongation on the other ſide.*

As the outermoſt ſatellite of jupiter paſſes from N, Plat. XXIV. fig. 7, its greateſt elongation on one ſide of jupiter through V its ſuperior conjunction to L its greateſt elongation on the other ſide, its geocentric motion, that is, its apparent motion, when ſeen from the earth at X, is from weſt to eaſt or in conſequentia. This is ſhewn to be true of the ſatellites moving round their reſpective primaries, in the ſame manner that we have ſhewn it to be true, in propoſition 91, of the inferior planets moving round the ſun.

158. *The*

158. *The apparent motion of any ſatellite is retrograde, as it paſſes from its greateſt elongation on one ſide of its primary, through the inferior conjunction, to its greateſt elongation on the other ſide.*

As the outermoſt ſatellite of jupiter paſſes from L, its greateſt elongation on one ſide of jupiter, Plat. XXIV. fig. 7, through M its inferior conjunction, to N its greateſt elongation on the other ſide, its geocentric motion, or its motion ſeen from the earth at X, is from eaſt to weſt or in antecedentia. This is proved in the caſe of a ſatellite moving round its primary, in the ſame manner as we proved it, in propoſition 92, of an inferior planet moving round the ſun.

159. *The ſatellites are ſometimes to the weſt and ſometimes to the eaſt of their reſpective primaries.*

If from the earth X, Plat. XXIV. fig. 7, we conceive a line XIV to be drawn through jupiter, this line divides the orbits of jupiters ſatellites into two parts, one of which as MNV is on the weſtern ſide of the planet, and the other VLM is on the eaſtern ſide. Therefore as any ſatellite is paſſing from its inferior conjunction M through MNV the weſtern half of its orbit to V its ſuperior conjunction, it will appear on the weſtern ſide of its primary; and as it is paſſing from its ſuperior conjunction V through VLM the eaſtern half of its orbit to M its inferior conjunction, it will appear on the eaſtern ſide of its primary.

160. *The greateſt elongation of a ſatellite on one ſide of its primary is equal to its greateſt elongation on the other ſide.*

If the earth is at X, Plat. XXIV. fig. 7, and a ſatellite, which deſcribes the orbit LMNPO round the primary I, is either at L or N; it will then be at its greateſt elongation, by propoſition 157. The angle of elongation LXI on one ſide of the primary I is equal to the angle of elongation NXI on the other ſide. This is equally true of any other ſatellite. From hence then it follows that the two points of greateſt elongation L and N are always at the ſame or nearly the ſame diſtance from the primary: and conſequently the orbit of this or any other ſatellite is either exactly or very nearly circular, and the primary is either in or very near to the center of the ſatellites orbit. See propoſition 86.

161. *The ſatellites appear to move in right lines, and, when we ſee them, they are all in the ſame right line.*

The orbits of the ſatellites encompaſs the primary. But if the eye is in the planes of their orbits, as their orbits, if they were viſible, would upon

on account of their ſituation in reſpect of the eye appear to be right lines, ſo a body that revolves in ſuch an orbit will appear to deſcribe a right line. Now the ſpectators eye is always in the plane of the ecliptic. And, ſince the ſatellites appear to deſcribe right lines, the eye is likewiſe in the plane of their orbits. Therefore the planes of their orbits lie in the plane of the ecliptic. But, ſince the planes of the orbits of all the ſatellites, which move round the ſame primary, are in the plane of the ecliptic, all theſe ſatellites muſt move in one and the ſame plane: and conſequently they will always appear in the ſame right line.

162. *A ſatellite cannot be ſeen in its ſuperior conjunction, and can ſeldom be diſtinguiſhed from the primary planet in its inferior conjunction.*

If the earth is at B, Plat. XXIV. fig. 7, and jupiter at J, a ſatellite when it is at P is in its ſuperior conjunction. In this ſituation the ſatellite is behind the primary and is hidden by it.

When the ſatellite is at its inferior conjunction R, it paſſes between jupiter and the earth, and in this ſituation it is ſeldom to be diſtinguiſhed from jupiter. Sometimes indeed a dark ſpot different from the ſhadow of the ſatellite has been obſerved to croſs the diſc of jupiter in the ſame path, that the ſatellite is deſcribing. Therefore at theſe times the ſatellite muſt be more obſcure than it is at other times. For if it was always equally bright, it would be always alike undiſtinguiſhable from the primary. Conſequently theſe ſatellites muſt have ſome parts darker than others; they appear bright when that ſide is towards us, which reflects the moſt light, and are more obſcure when the other ſide is towards us. From hence we may conclude that, as theſe ſatellites turn different ſides towards us at different times, they have a rotation round their reſpective axiſes.

163. *Saturns ring cannot be ſeen, when the ſun is in the plane of it, or when the earth is in the plane of it, or when its plane is between the ſun and the earth.*

The edge of ſaturns ring does not reflect light enough to make it viſible. Now if the ſun is in the plane of the ring, that is, if the edge of the ring is directly towards the ſun, the rays of the ſun will fall only upon the edge: few or none can fall upon the plane or flat part of the ring on either ſide: becauſe the ſun is juſt even with this plane, ſo that its rays will paſs nearly parallel to it. But when the rays fall only upon the edge of the ring, as this reflects little or no light, the ring will not be viſible to us, and ſaturn will appear round, like the other planets.

When

When the earth is in the plane of the ring, that is, when the edge of the ring is towards the earth, we can receive no light, that is reflected from the flat part of it; and the edge reflects little or no light. Therefore in this situation the ring will be invisible to us.

When the plane of the ring lies between the sun and the earth, the sun must be on one side of the ring and the earth on the other. Consequently the sun will not shine upon that side of the ring, which is towards the earth: and as the ring is an opake body, there will in this situation be no light come from it to us, for which reason it will not be seen.

164. *Saturns ring will be seen, when both the sun and the earth are on the same side of its plane.*

In this situation the sun shines upon that side of the ring, which is towards us, and the light, which it receives from the sun and reflects to us, will make it visible.

165. *When we see saturns ring, it appears like two arms or handles to the planet.*

If the plane of the ring was perpendicular to the plane of the ecliptic, we might then sometimes see it as a broad bright circle having the planet in its center. For in this case a line drawn from the eye, which is always in the plane of the ecliptic, might be perpendicular to the plane of the ring. But the plane of the ring is inclined to the plane of the ecliptic, and consequently when we see it, the plane of it is oblique to the eye. Therefore the ring will appear elliptical, as in Plat. XXIV. fig. 6. But then we shall not see the whole ellipsis; for that part of it, which is behind the planet, is hidden by the planet; and that part, which is before, cannot be distinguished from the planet. Therefore all that we see is so much of the ring as lies on each side of the planet towards *a* and *b*: and each of these parts will appear only like two arms or handles.

CHAP. VI.

Of eclipses.

166. *The moon is eclipsed by the earths shadow.*

AS the earth is an opake body, it will intercept the suns light, and consequently in the space behind the earth there will be no light from the sun. This dark space is the earths shadow. When the moon,

as

as it revolves in its orbit, comes into this shadow, it is eclipsed at its light fails, because in this situation it can receive no light from the sun.

167. *The moon can never be eclipsed, but when it is at the full.*

The space, where the suns light is intercepted by the earth, is in respect of the sun directly behind the earth; or the shadow of the earth, like the shadow of any other opake body, is always opposite to the sun. Therefore unless the moon is opposite to the sun, it can never be in the earths shadow. But the moon is at the full, when it is opposite to the sun, by proposition 140; and the moon can never be eclipsed, but when it comes into the earths shadow. Therefore the moon can never be eclipsed, but when it is at the full.

168. *The moon, when it is at the full, will not be eclipsed, unless it is in or near one of the nodes.*

The earths shadow is always directly opposite to the sun; and consequently it lies in the same plane with the earth and the sun. But both the earth and the sun are in the plane of the ecliptic, by proposition 28. Therefore the earths shadow is always in the plane of the ecliptic. From hence it follows that if the moon, at the time of the full, is not in or near the plane of the ecliptic, it will not be eclipsed. But the moons orbit is inclined to the plane of the ecliptic and does not coincide with it. Therefore the moon may be at the full, without being in the plane of the ecliptic; it may be either above the ecliptic or below it, that is, it may be either to the north or to the south of the ecliptic; and in either of these cases it will not fall into the shadow, but will be above it in one case and below it in the other.

If ABCD, Plat. XXI. fig. 3, is a circle drawn in the plane of the ecliptic, the sun and the earth, and consequently the shadow cast by the earth opposite to the sun, will always be in this plane. But the moons orbit makes an angle of more than 5 degrees with the plane of the ecliptic, by proposition 133: therefore let FGHE represent this orbit. Now if the sun at the time of the full moon is at A, the earths shadow will be at C directly opposite to the sun. The moon in opposition or at the full must be even with the earths shadow; but then it will not fall into the shadow, because if the shadow is at C, the moon, when even with it, will be at G, and consequently above the shadow. In like manner, if the sun at the time of the full moon is at C, the shadow of the earth will be opposite to it, and in the same plane with the sun; therefore the shadow will

be

be at A in the plane of the ecliptic. Now the moon does not move in the plane of the ecliptic ABCD; it moves in an orbit EFGH, which is inclined to the plane of the ecliptic. But at the full it may be oppoſite to the ſun, though it is not in the ſame plane with the ſun: and if the ſun is at C, the moon in oppoſition will be at E: therefore in this ſituation it will be below the ſhadow.

If it was neceſſary at the oppoſition of the moon and the ſun that the moon and the ſun ſhould be in oppoſite points of the heavens, then at every full moon there muſt be an eclipſe. But the moon and the ſun are in oppoſition, when they have oppoſite longitudes, or when, if a circle is drawn through them from weſt to eaſt, they are in oppoſite points of this circle, or half of this circle lies between them. In the mean time it is not neceſſary that as the ſun has no latitude, ſo the moon in oppoſition ſhould have none; the ſun is always in the ecliptic, but the moon at the full may be far enough either to the north or to the ſouth of the ecliptic, not to come within the earths ſhadow; for the ſhadow is always in the ecliptic.

But if the moon at the time of oppoſition has little or no latitude, that is, if it is in or near the plane of the ecliptic, it will then be eclipſed. If the moon at the time of oppoſition is at D or B, which are the moons nodes, it will then come within the earths ſhadow. The nodes are the points where the moons orbit croſſes the plane of the ecliptic: and conſequently, when the moon is in one of its nodes it is in the plane of the ecliptic as well as in the plane of its own orbit, becauſe it is in the common interſection of theſe two planes. But the earths ſhadow is always oppoſite to the ſun and in the plane of the ecliptic. Therefore if the moon at the time of oppoſition is in one of the nodes, it will be eclipſed by the earths ſhadow.

What we have here ſaid about the earths ſhadow, that it is always in the plane of the ecliptic, is to be underſtood of the center of this ſhadow: for the ſhadow has ſome breadth, ſo that part of it will be above the ecliptic and part below it. For this reaſon though the moon is not in one of the nodes, at the time of oppoſition, yet if it is near one of the nodes it will be eclipſed. If the ſun is at P, and conſequently, the center of the earths ſhadow is at Q the oppoſite point of the plane of the ecliptic; though the moon at the oppoſition is at H, yet the earths ſhadow will fall upon it, if HQ the moons latitude is leſs than half the breadth of the ſhadow: for as half the earths ſhadow is above the ecliptic, if the moon is near one of its nodes D, it will be near the plane of the ecliptic, and may be near enough for the earths ſhadow to fall upon it.

169. *The ſun is bigger than the earth; and the earths ſhadow is a cone, which ends in a point at ſome diſtance from the earth.*

The ſhadow of the earth does not extend ſo far as the orbit of mars: for mars is never eclipſed by the earth, when it is in oppoſition to the ſun. This ſhews us that the ſhadow of the earth does not reach to an infinite diſtance. But if it was of any other figure, beſides a cone ending in a point, like EFC, Plat. XXIV. fig. 8, it would reach to an infinite diſtance. The ſun and the earth are both of them ſpherical bodies. Let us firſt ſuppoſe that the ſun is leſs than the earth. Let AB, Plat. XXIV. fig. 9, be the ſun, and EF the earth. Now if lines, ſuch as AEC, BFD, are drawn all round the ſun and the earth, ſo as to touch each of their ſurfaces on the ſame ſide, the ſhadow of the earth will be included within theſe lines: for no light from the ſun can paſs beyond the earth into the ſpace between EC and FD. But ſuppoſing the ſun to be leſs than the earth, theſe lines will be farther aſunder at the earth than, they are at the ſun; and ſtill farther aſunder as they paſs on beyond the earth, that is, they will diverge from one another: therefore the ſhadow, which is between them, will be broader the farther it is from the earth. Such a ſhadow as this will be an encreaſing truncate cone; there is no vertex to the cone towards the ſun, for it begins from a circle drawn round the earth at EF; but the extent of it the other way towards CD beyond the earth will be infinite: for inſtead of coming to an end, it grows wider the farther it is extended. A ſhadow of ſuch a length would reach to the orbit of mars, or jupiter, or ſaturn, and would eclipſe any of theſe planets, when the earth comes between the ſun and the planet. But the earth never eclipſes any of theſe planets. Therefore this is not the ſhape of its ſhadow; and conſequently the ſun is not leſs than the earth.

Secondly, if the earth was equal to the ſun, then tangents, ſuch as AEC BFD, Plat. XXIV. fig. 10, drawn all round from the ſuns ſurface to the earths ſurface on the ſame ſide, would be at the ſame diſtance from one another at AB the ſun, at EF the earth, and at any other place CD beyond the earth, that is, theſe tangents would be parallel to one another. The ſhadow is included within theſe tangents, and a ſhadow ſo terminated would be a cylinder, the baſe of which would be a circle drawn round the earth at EF. Such a ſhadow as this would continue of the ſame breadth at all diſtances beyond the earth, that it has at EF, and conſequently it would reach to an infinite diſtance, ſo that mars, or jupiter, or ſaturn might be eclipſed by it. But, as before, theſe planets are never eclipſed by the earth. Therefore this is not the ſhape of the ſhadow, and conſequently the earth is not equal to the ſun.

Third-

Fig. 2.

Fig. 3.

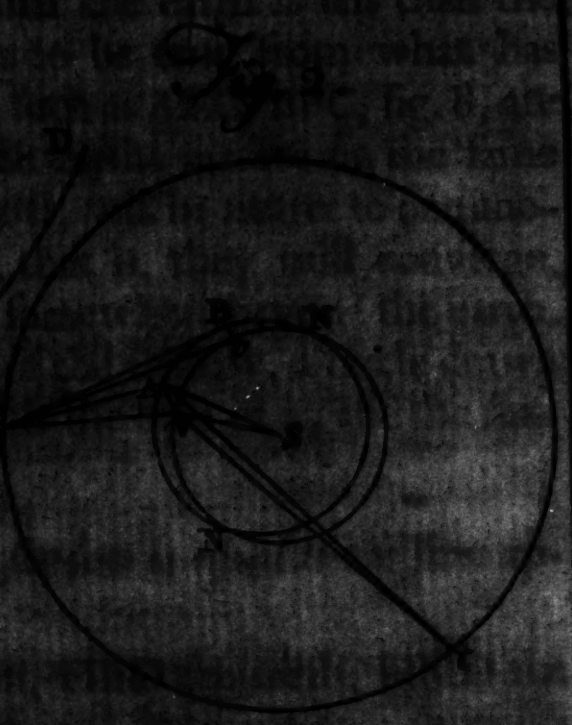


Fig. 4.

Fig. 5.

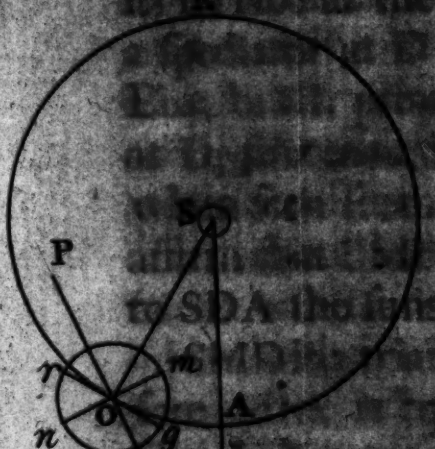


Fig. 6.

Fig. 7.

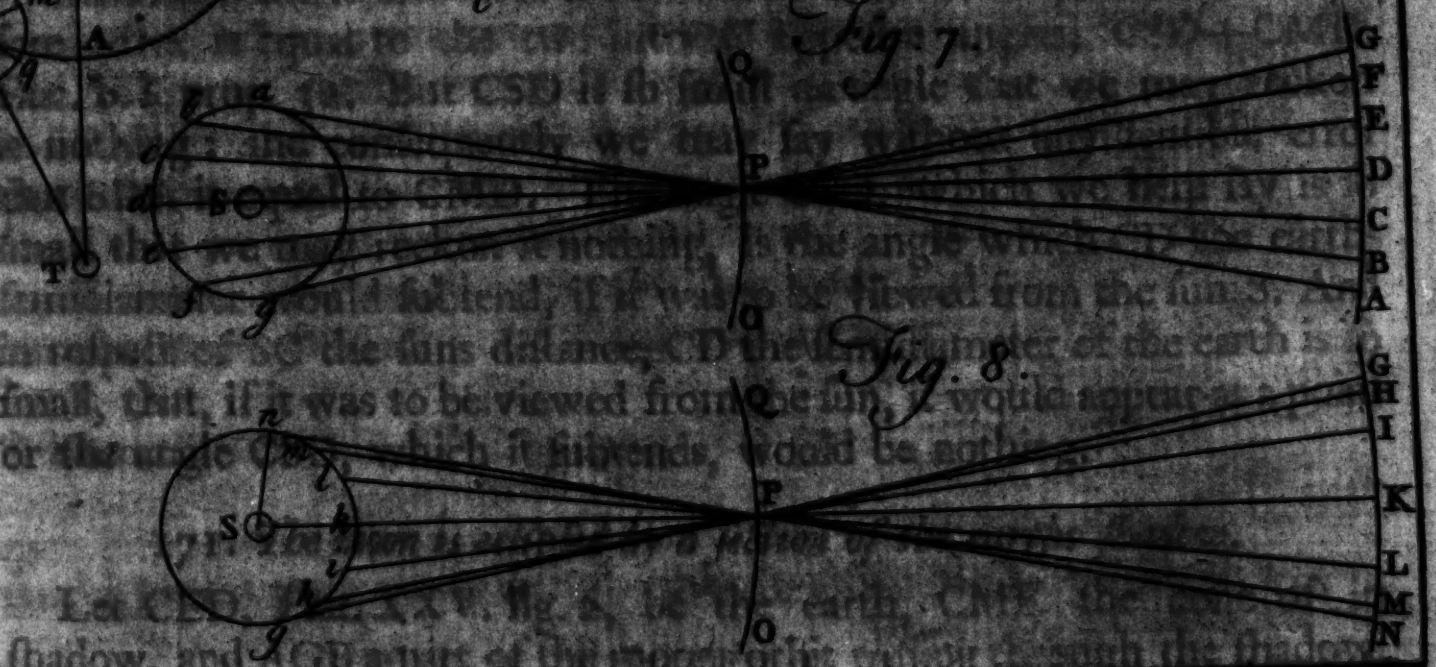
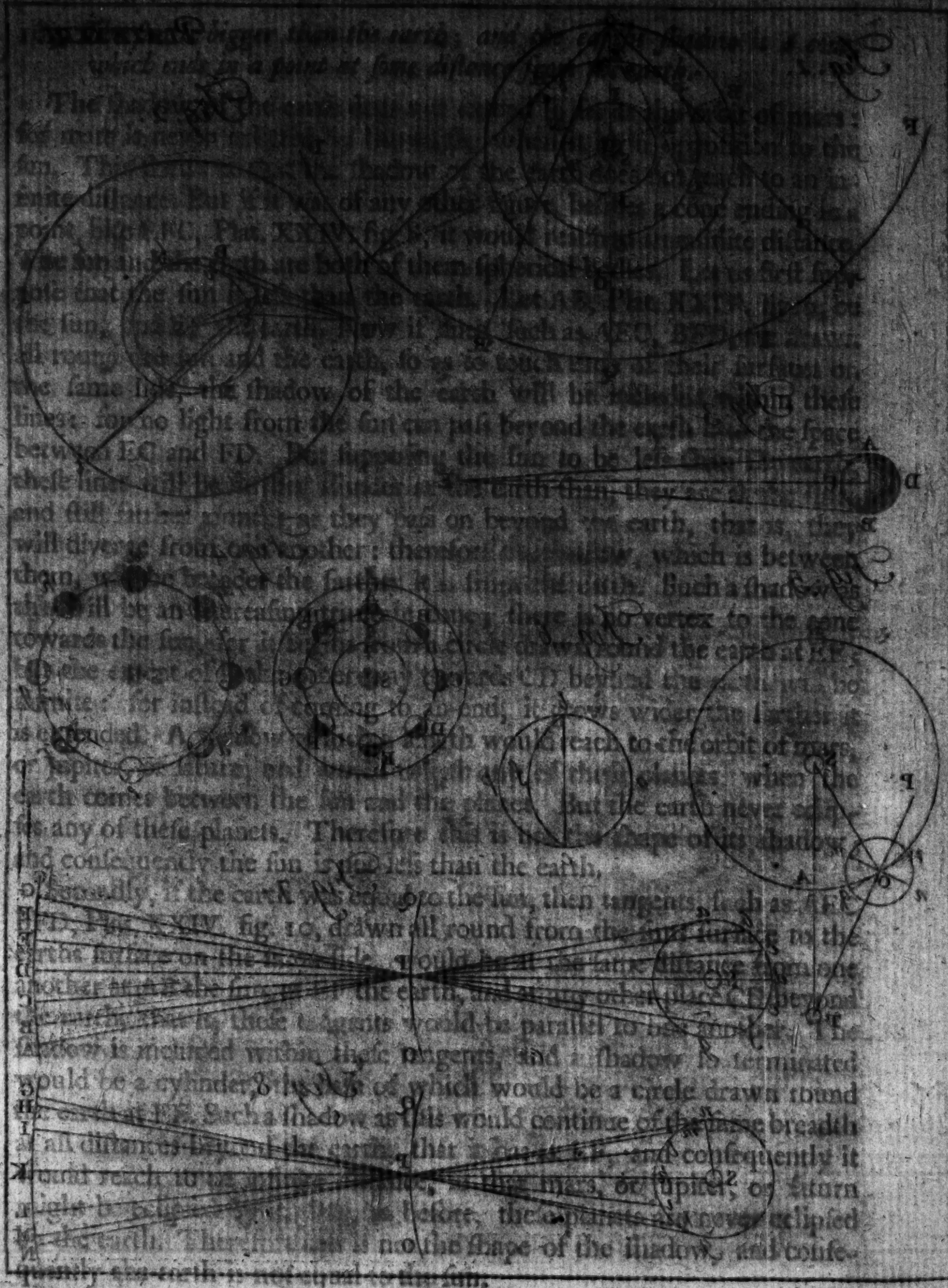


Fig. 8.



Third-

Thirdly, ſince we have proved that the earth can neither be bigger than the ſun nor equal to it, we may conclude that the earth is leſs than the ſun. From this ſuppoſition, which appears to be true from what has been ſaid already, it follows that if tangents, ſuch as AEC, BFC, fig. 8, are drawn all round from the ſuns ſurface to the earths ſurface on the ſame ſide, theſe tangents, which include the ſhadow, will be nearer to one another at the earth than they are at the ſun, that is, they will converge, and conſequently will meet in a point as C ſomewhere beyond the earth. The point C where they meet is the vertex of the ſhadow, a circle drawn round the earth at EF the points of contact is the baſe of it, and the ſhadow itſelf is a cone.

170. *The ſemiangle of the cone of the earths ſhadow is equal to the ſuns apparent ſemidiameter.*

Let AFBG, Plat. XXV. fig. 1, be the ſun, HED the earth, HMD the cone of the earths ſhadow: the angle contained between two tangents BHM, ADM drawn from the edges of the ſun to the edges of the earth, or the angle HMD, is the angle of this cone, and CMD, which is half HMD, is the ſemiangle of the cone. SA drawn from the ſuns center to its ſurface at the point of contact A, is a ſemidiameter of the ſun, which a ſpectator at D will ſee directly, becauſe DA is perpendicular to SA. Euc. b. III. prop. 18. This ſemidiameter, when it is ſeen from the earth at D, ſubtends the angle SDA; ſo that the ſuns apparent ſemidiameter, when ſeen from the earth, is proportional to the angle SDA. Now we affirm that CMD the ſemiangle of the cone of the earths ſhadow is equal to SDA the ſuns apparent ſemidiameter.

SMD is a triangle, and SDA is the external angle at the baſe MD. Therefore SDA is equal to the two internal oppoſite angles, CSD + CMD. Euc. b. I. prop. 32. But CSD is ſo ſmall an angle that we may reckon it nothing: and conſequently we may ſay without any ſenſible error that SDA is equal to CMD. The angle CSD, which we here ſay is ſo ſmall that we may reckon it nothing, is the angle which CD the earths ſemidiameter would ſubtend, if it was to be viewed from the ſun S. And in reſpect of SC the ſuns diſtance, CD the ſemidiameter of the earth is ſo ſmall, that, if it was to be viewed from the ſun, it would appear as a point or the angle CSD, which it ſubtends, would be nothing.

171. *The moon is eclipſed by a ſection of the earths ſhadow.*

Let CED, Plat. XXV. fig. 2, be the earth, CME the cone of its ſhadow, and AGB a part of the moons orbit paſſing through the ſhadow.

As the moon deſcribes this part of its orbit, it paſſes through a circular ſection of the ſhadow. FNHL repreſents this circular ſection; it is here drawn elliptical becauſe upon the paper we have a perſpective view of this circle, and the plane of it is ſuppoſed oblique to the readers eye. In fig. 7, we have a direct view, where A, or B, or D, or E repreſents this ſection of the earths ſhadow, through which the moon paſſes at the time of its eclipſe.

172. *The ſemidiameter of the ſection of the earths ſhadow, which eclipſes the moon, is equal to the difference between the moons horizontal parallax and the ſuns apparent ſemidiameter.*

We ſhall explane more particularly hereafter what is meant by the moons horizontal parallax: at preſent it will be ſufficient to inform the reader, that the moons horizontal parallax is equal to the angle, which a ſemidiameter of the earth would ſubtend, if it was to be viewed directly from the moon. Thus let CED, Plat. XXV. fig. 2, be the earth, CME the cone of its ſhadow, and CT a ſemidiameter of the earth drawn from the point of contact C. This ſemidiameter is perpendicular to the tangent CM. Euc. b. III. prop. 18. Therefore if a ſpectator was at F, the ſemidiameter CT would be ſeen directly, and would ſubtend the angle CFT. But AFHB is the lunar orbit. Therefore if the moon was at F, the ſemidiameter of the earth ſeen directly from the moon would ſubtend the angle CFT. This angle is what we here call the moons horizontal parallax. FMG is the ſemiangle of the cone of the earths ſhadow, which, by propoſition 171, is equal to the ſuns apparent ſemidiameter. FNHL is the ſection of the earths ſhadow, through which the moon paſſes at an eclipſe. FH is the diameter of this ſection, FG is its ſemidiameter, and FTG is the angle, which this ſemidiameter will ſubtend, when it falls upon the the moon and is ſeen by us from the earth. We affirm then that FTG is the difference between CFT and FMG. TFM is a triangle; CFT is the external angle at the baſe, and conſequently is equal to the two internal oppoſite angles taken together, that is, to FMG+FTG. Euc. b. I. prop. 32. But ſince FMG with the addition of FTG is equal to CFT, it follows that FTG is the difference between CFT and FMG.

FTG+FMG=CFT. Euc. b. I. prop. 32. Therefore FTG=CFT —FMG.

173. *The ſection of the earths ſhadow, by which the moon is eclipſed, is broader than the moon.*

The

The moons horizontal parallax is about 61′, and the ſuns apparent ſemidiameter is about 16′. The ſemidiameter of the ſection of the earths ſhadow, through which the moon paſſes at an eclipſe, is the difference between them, or 61′—16′=45′, by propoſition 172, and conſequently its diameter is about 90′. But the apparent diameter of the moon is ſomething more than 31′. Therefore the diameter of that ſection of the earths ſhadow, through which the moon paſſes at an eclipſe, is about three times as long as the diameter of the moon.

174. *The moons eclipſe is a partial one, if only a part of the moon is covered by the earths ſhadow.*

175. *The moons eclipſe is a total one, if the whole moon is covered by the earths ſhadow.*

176. *The moons eclipſe is a central one, if the center of the earths ſhadow falls upon the center of the moons diſc.*

177. *The moon, when it is at the full, will not be eclipſed, if its latitude is equal to or greater than the ſum of its own ſemidiameter added to the ſemidiameter of the earths ſhadow.*

In a perſpective view of two circles, ABCD, EFGH, Plat. XXI. fig. 3, they appear elliptical, if their planes are oblique to the eye: but if the eye was in the planes of them, they would appear like the right lines AC, EG, Plat. XXV. fig. 3; where AC may repreſent the plane of the ecliptic, EG the plane of the moons orbit inclined to the plane of the ecliptic, and D the node. This is ſuppoſed to be the ſituation of the plane of the ecliptic and plane of the moons orbit in reſpect of the readers eye, when RNR, Plat. XXV. fig. 7, is to repreſent to him a part of the ecliptic, ONO a part of the moons orbit, and N the node.

The center of the ſun, and the center of the earth, and conſequently C, fig. 7, the center of the earths ſhadow are always in the plane of the ecliptic. Now if the earths ſhadow at the time of the full moon is at A, the moon F muſt be even with the ſhadow: for the earths ſhadow is always oppoſite to the ſun; and the moon at this time muſt be oppoſite to the ſun, becauſe it is at the full: therefore the longitude of the ſhadow and of the moon will be the ſame, or they will be even with one another, ſince they are both in oppoſition to the ſun and conſequently the longitude of each differs from that of the ſun 180°. But ſince C the center of the ſhadow is in the plane of the ecliptic, only half the ſhadow will be on the ſame ſide of the ecliptic with the moon, and the other half will be on the

the contrary ſide. The ſemidiameter of the ſhadow is about 45′, by propoſition 173. Now *c* the center of the moon F is always in the plane of its orbit; and conſequently half the moons breadth is on the ſame ſide of its orbit with the earths ſhadow, and the other half is on the contrary ſide. The moons apparent ſemidiameter is about 16′. Thus we have ſeen that the ſhadow reaches 45′ above the plane of the ecliptic and the moon reaches 16′ below the plane of its orbit: theſe ſemidiameters added together make 61′. Therefore if the moons center *c* at the time of the full is more than 61′ from C the center of the ſhadow, the lower edge of the moon will be above the upper edge of the ſhadow, and the moon will paſs clear of the ſhadow, ſo as not to be eclipſed. Or if *c* the moons center is juſt 61′ from C the center of the ſhadow, then the lower edge of the moon will, as it paſſes through the oppoſition, juſt touch the upper edge of the ſhadow without entering into it, and conſequently at this oppoſition there will be no eclipſe. The numbers here made uſe of are not very exact: but they are not far from the truth, and will ſerve very well to give the reader a general notion of what we have undertaken to explane.

Let GNE, Plat. XXV. fig. 4, be an arc in the moons orbit, ANB an arc in the ecliptic, N the node, *c* the center, and *cl* the ſemidiameter of the moon, C the center, and Ct the ſemidiameter of the earths ſhadow; *c*C an arc in a ſecondary of the ecliptic, which paſſes through the centers of the moon and of the ſhadow at the time of the moons oppoſition. The arc *c*C, or the diſtance of the moons center from the plane of the ecliptic, is the moons latitude. Now C*t* is about 45′, by propoſition 173, and *cl* is about 16′. Therefore C*t*+*cl* is about 61′. But if the moons latitude C*c* was equal to C*t*+*cl*, the ſum of the ſemidiameters of the ſhadow and moon, or to 61′, *l* and *t* would juſt touch one another, or the moons edge would juſt touch the earths ſhadow without entering into it, and conſequently the moon at that oppoſition would not be eclipſed. Much leſs would it be eclipſed, if, as is repreſented in the figure, the moons latitude C*c* is greater than C*t*+*cl*, the ſum of the ſemidiameters of the ſhadow and moon, or than 61′, for then *l* the lower edge of the moon will be above *t* the upper edge of the ſhadow.

178. *The moon, when it is at the full, will be partially eclipſed, if its latitude is leſs than the ſum, but greater than the difference of its own ſemidiameter and the ſemidiameter of the earths ſhadow.*

The moon G, Plat. XXV. fig. 7, may at its oppoſition be ſo near C the center of B the earths ſhadow, that a part of it will come within the ſhadow;

PLATE XXIV.

Fig. 1.

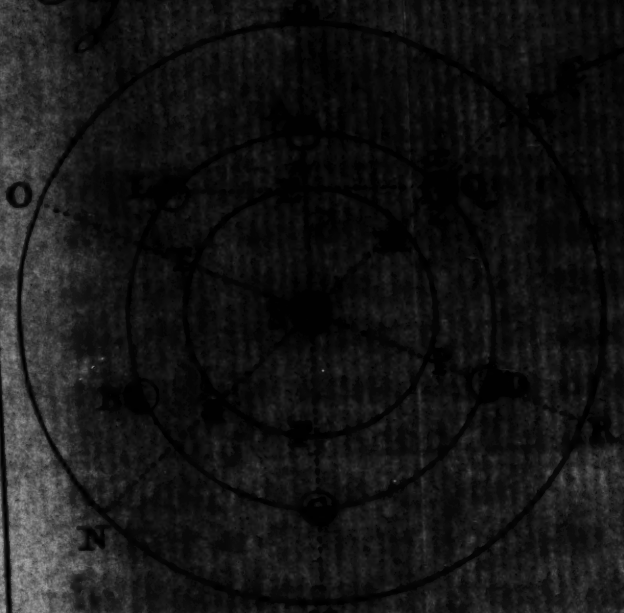


Fig. 2.

Fig. 3.

Fig. 4.

Fig. 6.

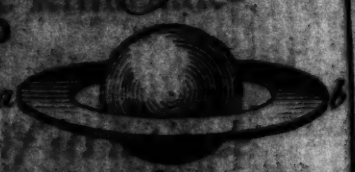
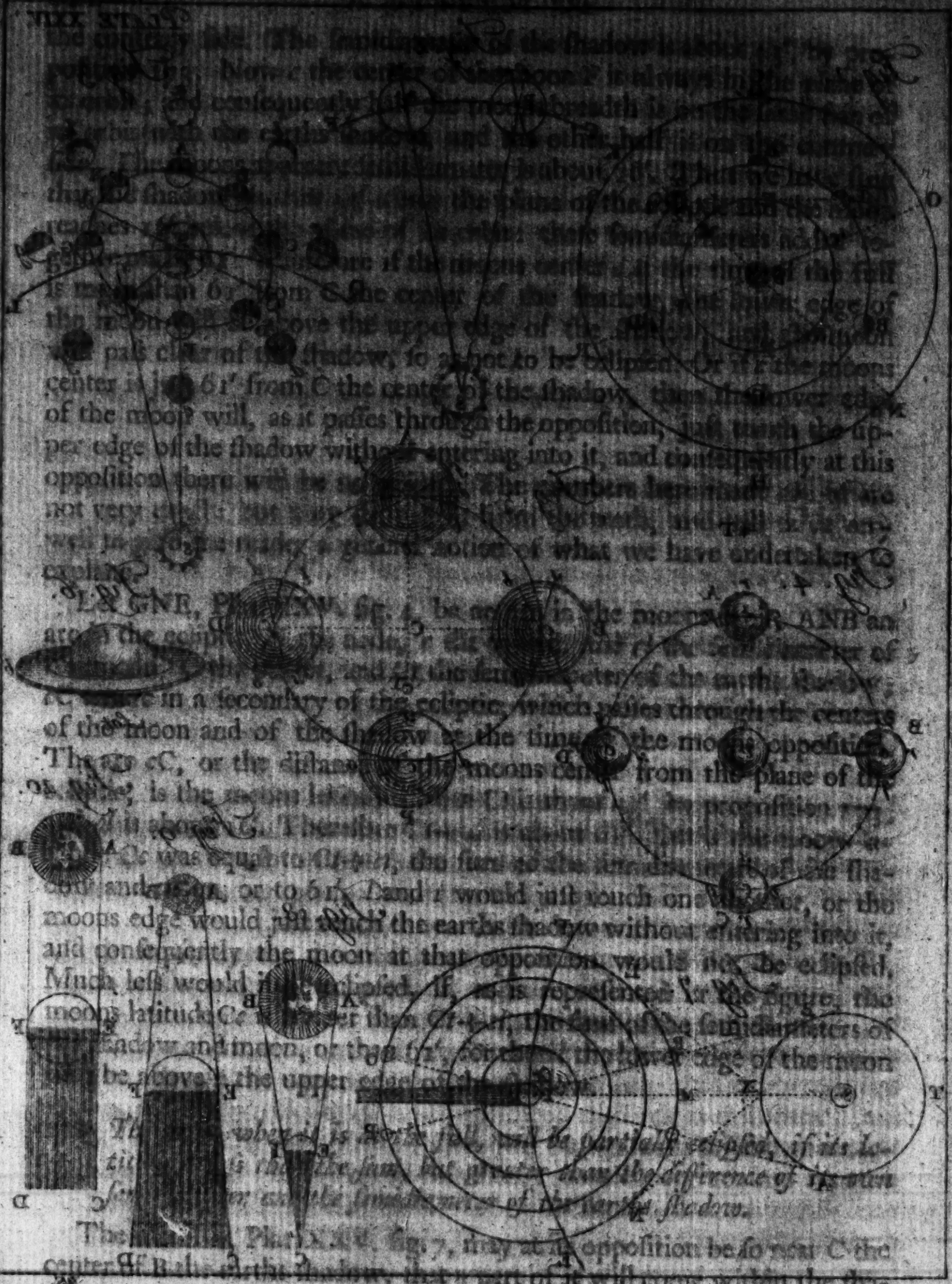


Fig. 10.

Fig. 9.

Fig. 8.

Fig. 7.



dow;

dow; and yet it may be far enough off for a part of it at the ſame time to be without the ſhadow; in this caſe the moon will be partially eclipſed.

Let GE, Plat. XXV. fig. 5, repreſent an arc of the moons orbit, AB an arc of the ecliptic, N the node, C*c* the moons latitude at the time of oppoſition, C*t* the ſemidiameter of the earths ſhadow, *cl* or *cm* the ſemidiameter of the moon. Here the latitude C*c* is planely leſs than C*t*+*cl*, or than the ſum of the ſemidiameters of the ſhadow and the moon, or the moons center *c* is ſo near the plane of the ecliptic that *l* the lower edge of the moon will be below *t* the upper edge of the ſhadow, and conſequently the lower ſide of the moon, or the ſide, which is next the ecliptic, will be eclipſed.

But as C*c*+*cm* is greater than C*t*, or as the moons latitude added to its ſemidiameter is greater than the ſemidiameter of the earths ſhadow, the upper edge of the moon *m* will be ſo far from the plane of the ecliptic, that it will not come within the ſhadow, and conſequently the eclipſe will be but a partial one. Now if C*c*+*cm* is greater than C*t*, it follows that C*c* is greater than C*t*—*cm*, or, when the moon is partially eclipſed, the moons latitude is greater than the difference between the ſemidiameters of the ſhadow and the moon.

179. *The moon, when it is at the full, will be totally eclipſed, if its latitude is leſs than the difference between its own ſemidiameter and the ſemidiameter of the earths ſhadow.*

The earths ſhadow is three times as large as the moon, by propoſition 173. Therefore if the moon at the time of oppoſition is at I, Plat. XXV. fig. 7, near to one of its nodes, the whole moon will be within the ſhadow, and then it is totally eclipſed. It enters the ſhadow at H, is totally eclipſed at I, and comes out of the ſhadow at L.

If C*c*, Plat. XXV. fig. 6, is the moons latitude at the time of the full, we have ſeen, in propoſition 178, that if this latitude was greater than C*t*—*cm*, or greater than the difference between the ſemidiameters of the ſhadow and the moon, then ſome part of the moons edge *m* will be above the ſhadow, and the moon will be only partially eclipſed. Conſequently, if C*c* is leſs than C*t*—*cm*, the upper edge *m* of the moon will be within the ſhadow, and the moon will be totally eclipſed.

180. *The moon is not centrally eclipſed, unleſs it is in one of the nodes at the time of the full.*

The

The center of the earths ſhadow is always in the plane of the ecliptic, by propoſitions 28, 166: therefore the center of the ſhadow cannot fall upon the center of the moon, unleſs the moons center is likewiſe in the plane of the ecliptic. Now the moons center is always in the plane of its orbit: and conſequently it cannot be at the ſame time in the plane of the ecliptic, unleſs it is at one of the points where the plane of its orbit croſſes the plane of the ecliptic. Theſe points of interſection are the nodes. Therefore the center of the ſhadow cannot fall upon the moons center, or the moon cannot be centrally eclipſed, unleſs it is in one of the nodes at the time of the full.

If ONO, Plat. XXV. fig. 7, is a part of the lunar orbit, and RNR is the plane of the ecliptic, then will N be the node. Now the node N, which is the common interſection of the moons orbit and plane of the ecliptic, is both in the orbit ONR, and likewiſe in the plane of the ecliptic RNR. Therefore when the moon is in the node, its center is in the plane of the ecliptic as well as in its own orbit. The center of the earths ſhadow E is always in the plane of the ecliptic: and conſequently, if the moon is at the node at the time of the full, the center of it will be in the center of the ſhadow: the moon, as it moves in its orbit, will enter the ſhadow at M, will be totally and centrally eclipſed at N, and will come out of the ſhadow again at P.

The moon moves from weſt to eaſt in its orbit, and the ſhadow of the earth being always oppoſite to the ſun will move in the ecliptic on one ſide the ſame way that the ſun appears to move on the other ſide, that is, the ſhadow moves in the ecliptic from weſt to eaſt. Thus E the ſhadow and N the moon both move the ſame way, and conſequently, if they moved with equal velocities, the ſhadow and moon would always keep together ſo as to be even with one another, and the moon would never get clear of the ſhadow, till they were got far enough from the node, the moon far enough towards O, and the ſhadow far enough towards R, for the moon by the encreaſe of its latitude to be below the ſhadow. But the ſhadow does not move ſo faſt as the moon: for the ſhadow moves no faſter than the ſun; and the ſun takes up a years time to go round the earth, whereas the moon goes round in a month. Therefore the moon will get clear of the ſhadow at P by the difference between its own velocity and the velocity of the ſhadow. Or otherwiſe: the moon by moving faſter in its orbit than the ſun appears to move in the ecliptic will not be always in oppoſition to the ſun. But the earths ſhadow is always oppoſite to the ſun. Therefore the moon, as it gets from the oppoſition, will get clear of the ſhadow.

181. *When*

181. *When the ſemiangle of the cone of the earths ſhadow is known, the height of the ſhadow may be determined.*

The ſemiangle CMT, Plat. XXV. fig. 2, of the cone of the earths ſhadow is equal to the ſuns apparent ſemidiameter, or is about 16′ by propoſition 170: and the ray CM, which terminates the ſhadow on one ſide is a tangent to the earth at C, by propoſition 169. Now if a ſemidiameter of the earth CT is drawn from the point of contact, it will be perpendicular to the tangent CM: Euc. b. III. prop. 18: and conſequently CT is the ſine of the angle CMT, or of an angle of 16′, in a circle whoſe radius is TM. Therefore as the ſine of an angle of 16′ is to the radius in the tables of ſines, ſo is CT the earths ſemidiameter to TM the height of the cone. But the ſine of 16′ is to the radius about as 1 to 217. Therefore TM is about 217 times as long as CT; or the height of the ſhadow is equal to about 217 ſemidiameters of the earth; that is, the ſhadow reaches from the earth to the diſtance of about 217 of the earths ſemidiameters.

182. *The moon in a total eclipſe receives light enough from the ſun to make it viſible.*

Thoſe, who imagined it impoſſible for any light of the ſun to fall upon the moon, when the earth is between them, have concluded the moon not to be entirely an opake body, but to ſend out a ſmall quantity of light from itſelf, or to be in ſome degree luminous: becauſe otherwiſe they could not conceive why the moon ſhould appear at the time of a total eclipſe. But we have already ſhewn, in propoſition 144, that the moon is an opake body: and we may now ſhew whence that feint ruddy light comes, which makes the moon viſible to us when it is eclipſed.

Let ADCE, Plat. XXV. fig. 8, be the earth, and *rp* the atmoſphere that ſurrounds it. Rays as *d* and *c* which are tangents to the earths ſurface terminate its ſhadow at B, by propoſition 169. Other rays as *b* and *e*, if they were to paſs ſtrait forwards would not come into the ſhadow. But as the rays *b* and *e*, when they are to paſs out of void ſpace into the atmoſphere, fall more obliquely upon the atmoſphere than *c* and *d*, will be more refracted, by propoſition 57 of optics, and conſequently, by propoſitions 40, 48, 49, of optics will converge to ſome point nearer to the earth than B. Therefore they will get into the ſhadow, and paſſing through it will come out at *l* and *h*. Other rays as *a* and *f* fall ſtill more obliquely, and for that reaſon will be more refracted, and will converge more, ſo that coming out of the atmoſphere at *i* and *g* they will converge

to a point ſtill nearer, and will paſs through the ſhadow in the lines *im*, *gn*. The rays, which are thus refracted into the ſhadow by the earths atmoſphere will fall upon the moon, when it is in the ſhadow and will make it viſible to us in a total eclipſe. But as only few rays will be thus refracted, and only thoſe, which have the greateſt moment, as the red rays have, by propoſition 282 of optics, will make their way through the atmoſphere, the light, which the moon receives in this manner, will be feint, and of a muddy red colour.

183. *The ſun is eclipſed, when the moon paſſes directly between the ſun and the earth.*

When the moon is between the ſun and the earth, it will intercept the ſuns light or will hinder us from ſeeing the ſun.

184. *The ſun can never be eclipſed, but at the new moon.*

The moon cannot be directly between the ſun and us, unleſs they are both in the ſame part of the heavens, that is, unleſs they are in conjunction. But an eclipſe of the ſun is cauſed by the moons paſſing directly between the ſun, and the earth, by propoſition 183. Therefore the ſun cannot be eclipſed, unleſs the moon is in conjunction with it. But the conjunction of the ſun and moon is the new moon, by propoſition 135. Therefore the ſun can never be eclipſed but at the new moon.

From hence we may eaſily prove that the darkneſs at our Saviours crucifixion was not owing to an eclipſe of the ſun. For the crucifixion happened at the time of the jewiſh paſſover: and the paſſover by the appointment of the law was to be celebrated at the full moon. Therefore the ſun could not be eclipſed at the time of the paſſover.

185. *The ſun will not be eclipſed at the new moon, unleſs the moon at that time is in or near one of its nodes.*

The ſun is always in the plane of the ecliptic, by propoſition 27. But the moons orbit is inclined to the plane of the ecliptic at an angle of between 5° and 6°. And conſequently the moon does not appear in the ſame plane with the ſun, unleſs it is in one of the nodes, that is, in one of the points where its orbit croſſes the plane of the ecliptic: nor will it appear near the plane where the ſun is, unleſs it is near one of its nodes. But the moon cannot appear to us to move directly acroſs the ſun, ſo as to cover either the whole or a part of it, unleſs it is in or near the plane of the ecliptic where the ſun is. Therefore the moon at the new will not cover either the whole or a part of the ſun, that is, the ſun will not be eclipſed, unleſs the moon at that time is in or near one of its nodes.

The

The ſun, by propoſitions 27, 49, has no latitude, for it is always in the plane of the ecliptic. But the moon at the new may have latitude, becauſe its orbit is inclined to the plane of the ecliptic. Therefore though the moon at the new is even with the ſun, or has the ſame longitude with the ſun, ſo as to be neither to the eaſt nor to the weſt of it; yet it may have latitude enough, it may be enough above or below it, enough to the north or to the ſouth of it, not to cover any part of it: and then at the new there will be no eclipſe of the ſun.

But if the moon at the new is in one of the nodes, it has no latitude, but is in the plane of the ecliptic. Then, becauſe it is in conjunction, it has the ſame longitude with the ſun, or is neither to the eaſt nor to the weſt of it; and becauſe it has no latitude and the ſun has none, it will be neither above nor below, neither to the north nor to the ſouth of the ſun, and conſequently the whole moon will go directly between the ſun and the earth, ſo that it will cover the whole ſun, provided the apparent ſize of the moon is equal to the apparent ſize of the ſun. At the new moon, if it is in this ſituation, there will be a total eclipſe of the ſun.

Or if the moon at the conjunction is near one of its nodes, then it will be near the plane of the ecliptic: the center of the moon is indeed in the plane of its orbit; and the center of the ſun is in the plane of the ecliptic; ſo that unleſs the moon is in one of the nodes its center will not go acroſs the ſuns center. But half the moon is above the plane of its orbit, and half is below it; and ſo likewiſe half the ſun is above the plane of the ecliptic, and half is below it. Therefore if the moon is near the plane of the ecliptic, a part of the moon will paſs directly between a part of the ſun and us, and that part of the ſun will be covered or eclipſed by the moon in its paſſage. At the new moon, if it is in this ſituation, there will be a partial eclipſe of the ſun.

186. *In a total eclipſe of the ſun, the ſhadow of the moon falls upon that part of the earth, where the eclipſe is ſeen.*

Let *ls*, Plat. XXV. fig. 9, be the ſun, *rt* the moon, *mv* a part of the earths ſurface. If a ſpectator is any where between *m* and *v*, he will ſee a total eclipſe of the ſun, for the moon *rt* is interpoſed between the ſun and him, ſo as to hide the whole ſun from him. Now it is plane that in this ſituation the ſun is on one ſide of the moon and *mv*, that part of the earths ſurface where the eclipſe is ſeen, is directly on the oppoſite ſide. But as the moon is an opake body, it caſts a ſhadow oppoſite to the ſun. Therefore the moons ſhadow falls upon *mv* that part of the earths ſurface, where the total eclipſe of the ſun is ſeen.

187. The moons ſhadow is a cone ending in a point.

By what has been ſaid, under propoſition 169, concerning the figure of the earths ſhadow, it appears that the moons ſhadow, like the earths ſhadow, will be a cone ending in a point, if the moon is leſs than the ſun. Now we have ſhewn, in propoſition 169, that the earth is leſs than the ſun, and if we can ſhew that the moon is leſs than the earth, then it will certainly follow that the moon muſt be much leſs than the ſun. We have ſhewn, in propoſition 173, that the diameter of the moon is leſs than the diameter of the earths ſhadow. And the diameter of the earths ſhadow at the moon is leſs than the diameter of the earth, by propoſition 169. Therefore the diameter of the moon is much leſs than the diameter of the earth. Conſequently, by propoſition 169, the moon is much leſs than the ſun, and therefore its ſhadow will be a cone ending in a point.

The cone of the moons ſhadow is terminated by lines that are tangents to the ſun and moon on the ſame ſide. If a tangent *lra* is drawn from any point *l* in the ſun, Plat. XXV. fig. 9, to any point *r* on the ſame ſide of the moon, this tangent will terminate the cone of the ſhadow on one part, and ſuch another tangent *sta* drawn from *s* a point in the ſuns ſurface to *t* a point on the ſame ſide of the moon, this will terminate the ſhadow on the other part. For all the rays of the ſun that paſs without the tangents *la* and *sa* will paſs freely by the moon, and will not be intercepted by it: but all rays that come from the ſun within the ſpace *lsrt* included between thoſe tangents will be ſtopped by the moon, and conſequently there will be no light paſs into the ſpace *rta* that is included between thoſe tangents behind the moon. Therefore this dark ſpace *rta* will be the ſhadow.

188. The ſemiangle of the cone of the moons ſhadow is equal to the apparent ſemidiameter of the ſun.

Let FBGA, Plat. XXV. fig. 1, be the ſun, and HED the moon; then, by what has been ſaid under propoſition 187, the tangents BHM, ADM will terminate the ſhadow, and the conical ſpace HMD behind the moon will be the ſhadow. The angle HMD is the angle of this cone, and if a line is drawn from S the ſuns center through C the moons center to M the vertex of the ſhadow, this line divides the angle HMD into two equal parts, ſo that CMD will be the ſemiangle of the cone. SA is the ſemidiameter of the ſun, and the angle SDA, which this ſemidiameter would ſubtend, if it was to be viewed by an eye placed at D, is called the apparent ſemidiameter of the ſun. The propoſition therefore affirms that CMD is equal to SDA.

The

The proof of this propoſition is the ſame with that of propoſition 170; with this difference only, that in propoſition 170, HED repreſented the earth and here it is put to repreſent the moon.

Indeed when HED repreſents the moon, we ſhall only prove that CMD the ſemiangle of the cone is equal to SDA the angle, which the ſuns ſemidiameter would ſubtend, if the ſpectators eye was at the moon. But as the moon and earth are very nearly at the ſame diſtance from the ſun, the apparent ſemidiameter of the ſun ſeen from the moon may be reckoned equal to the apparent ſemidiameter of the ſun ſeen from the earth, without any ſenſible error.

189. *A ſemidiameter of the moons ſhadow at the place where the earth paſſes through it is equal to the difference between the apparent ſemidiameters of the moon and ſun.*

Let CDE, Plat. XXV. fig. 2, repreſent the moon, CME the cone of its ſhadow, FH the diameter, and FG the ſemidiameter of that part of the moons ſhadow, through which the earth paſſes at the time of a ſolar eclipſe. FMG the ſemiangle of the cone is equal to the ſuns apparent ſemidiameter, by propoſition 188, CT is the ſemidiameter of the moon, and CFT is the angle which this ſemidiameter would ſubtend, if it was ſeen at the diſtance FC, or from the earth, which we have ſuppoſed at F; or CFT is the apparent ſemidiameter of the moon. Now if FG the ſemidiameter of the moons ſhadow was to be ſeen from an equal diſtance TF or from the moon, it would ſubtend the angle FTG. We are therefore to prove that FTG is the difference between CFT the apparent ſemidiameter of the moon and FMG the apparent ſemidiameter of the ſun.

In the triangle TFM the external angle at the baſe is equal to the two internal oppoſite angles taken together, or TFC=FTG+FMG. But ſince TFC=FTG+FMG, it follows that TFC—FMG=FTG, that is, if FMG the ſuns apparent ſemidiameter is taken from TFC the moons apparent ſemidiameter, the difference or remainder is equal to FTG, or to the angle under which an eye at the moon would ſee FG a ſemidiameter of that part of the moons ſhadow, through which the earth paſſes at a ſolar eclipſe.

190. *In a partial eclipſe of the ſun an imperfect ſhadow of the moon falls upon that part of the earth, where the partial eclipſe is ſeen: this imperfect ſhadow we call the moons penumbra.*

If *ls*, Plat. XXV. fig. 9, is the ſun, *rt* the moon, and *np* a part of the earths ſurface; a ſpectator between *m* and *v* would ſee a total eclipſe, for

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the whole ſun would be covered by the moon: but a ſpectator at *p* or *n* would ſee no eclipſe at all: for if the ſpectator was at *p*, he might certainly ſee the edge *s* of the ſun, becauſe no part of the moon is between *s* and *p*; and he might likewiſe ſee the oppoſite edge *l*, becauſe a ſtrait line may be drawn from *l* to *p* without paſſing through the moon, and conſequently a ray of light might paſs from *l* to *p* without being intercepted by the moon: in like manner if he was at *n* he might for the ſame reaſons ſee both the edge *l* of the ſun and the oppoſite edge *s*. Therefore in either of theſe ſituations he might ſee the whole ſun.

But if the ſpectator was any where between *p* and *v* on one ſide, or between *n* and *m* on the other ſide, he would ſee a partial eclipſe of the ſun. Thus particularly ſuppoſe he was at *d*, he might ſee the edge *s* of the ſun indeed, becauſe no part of the moon is between *s* and *d*: but if a ſtrait line is drawn from the ſun ſo as to touch the moon at *t* this line will be *bcgtd*, which divides the ſun in halves; the ſpectator at *d* might ſee the half *bsg*, but the other half *blg* will be behind the moon and will be hidden by it: and conſequently this half of the ſun will be eclipſed. In like manner if he was at *f*, the line *bcqrf*, which is a tangent to the moon, divides the ſun in halves, and he would ſee only the half *blq*, whilſt the other half *bsq* would be eclipſed by the moon. Now it is evident, from what has been ſaid, that but half the ſun can ſhine upon the places *d* and *f*, where it appears half eclipſed, and in the ſame manner we might ſhew that it would appear partially eclipſed in any place between *p* and *v*, or *n* and *m*: and conſequently in all thoſe places, though they are not in total darkneſs, there will be leſs light from the ſun, than if it ſhone full out and was not eclipſed at all. This gloom or dimneſs, which ariſes from a partial eclipſe of the ſun we call the moons penumbra.

191. *The moons penumbra is an encreaſing cone.*

If a ſpectator was at *p*, Plat. XXV. fig. 9, upon the earths ſurface, the moon *rt* would hide no part of the ſun from him. But if he was to walk from *p* towards *v* he would immediately be in the moons penumbra and would loſe ſight of a part of the ſun: the firſt part that he would loſe ſight of would be the edge *l*. Therefore ſince, whilſt he is in the line *ltp*, he can ſee the whole ſun; and as ſoon as he gets beyond *p* towards *v*, he is immediately in the penumbra, it follows that the line *ltp* terminates the penumbra on one part. In like manner we may ſhew that the line *srn* terminates the penumbra on the other part: becauſe a ſpectator, whilſt he is at *n* in the line *srn*, can juſt ſee the whole ſun; but if he was to walk

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from *s* towards *m*, he would immediately get into the penumbra, and would ſee the ſun partially eclipſed, or would loſe ſight of the edge *s*. Theſe lines *lp* and *sm* are tangents to the ſun and moon, but on contrary ſides: the line *lp* touches the ſun at *l*, and the moon on the contrary ſide at *t*; and the line *sm* touches the ſun at *s*, and the moon on the contrary ſide at *r*. By ſuch lines as theſe drawn all round the ſpherical bodies of the ſun and moon the penumbra is terminated all round. But as theſe lines are tangents to the ſun on one ſide and to the moon on the contrary ſide, they croſs each other at the point *i* between the ſun and moon, and diverge from thence as they paſs on: conſequently the farther they go, the farther they will be aſunder. Therefore the cone, which theſe lines include, will grow broader the farther it is from the moon, or will be an encreaſing one. Indeed it is a truncate cone; for the ſun ſhines upon the ſpace *irt* between the vertex and the moon, ſo that in this ſpace there will be no penumbra.

But now we have ſeen what the moons penumbra is owing to, we may obſerve by the way that the earths ſhadow in a lunar eclipſe muſt be encompaſſed with juſt ſuch another penumbra. For as the moon by hiding a part of the ſun from us occaſions a gloom or caſts a penumbra, ſo the earth, by hiding a part of the ſun and preventing that part from ſhining upon the moon in a partial eclipſe of the moon, muſt occaſion a gloom at the moon, or muſt caſt a penumbra upon that part of the diſc, which is not within the ſhadow.

The earths penumbra, like the moons, muſt be an encreaſing cone: and conſequently this penumbra will extend itſelf to an infinite diſtance. Therefore here it may be aſked, whether the earths penumbra does not eclipſe mars: for though we have proved, in propoſition 169, that the earths ſhadow ends before it reaches mars, becauſe it is a decreaſing cone, yet the penumbra is an encreaſing cone; and therefore ought to reach as far as mars. This difficulty ſhall receive its anſwer in a proper place.

192. *The darkneſs of the moons penumbra encreaſes gradually as we are nearer to the ſhadow,*

The gloom occaſioned by a partial eclipſe of the ſun, is leaſt in thoſe places where the edges of the penumbra fall, and encreaſes by degrees as the place is nearer to the ſhadow itſelf. If a ſpectator is at *p*, Plat. XXV. fig. 9, he is juſt at the edge of the penumbra *rtnp*, and may ſee the whole ſun; but if he was to walk from *p* towards *v* he would ſoon loſe ſight of a part of it; and the farther he goes from *p* towards *v* the greater part of the ſun will be hidden from him. Now the greater the part of the ſun

is

is, that the moon hides from him, the greater will be the gloom. Therefore the gloom or darkneſs of the penumbra encreaſes as he is nearer to the ſhadow.

193. *The ſemiangle of the moons penumbra is equal to the ſuns apparent ſemidiameter.*

Let BAD, Plat. XXVI. fig. 1, be the ſun, and GH the moon; then by what has been ſaid, under propoſition 191, it appears that the tangents BHT and DGR terminate the cone of the penumbra. The angle GFH, which theſe tangents make with one another at F, is the angle of this cone, and if a line CI is drawn from the ſuns center to the moons, this line biſects the angle of the penumbra, ſo that IFH is its ſemiangle. BC is a ſemidiameter of the ſun, and BHC is the angle, under which this ſemidiameter would appear, if ſeen from the moon H; this angle BHC we call the ſuns apparent ſemidiameter. We affirm therefore that IFH the ſemiangle of the penumbra is equal to BHC the ſuns apparent ſemidiameter.

In the triangle FCH the external angle at the baſe is IFH: this angle IFH is equal to the two internal oppoſite angles taken together, that is, to FHC+FCH. Euc. b. I. prop. 32. Now the angle FHC is the ſame with BHC; and the angle FCH is the ſame with ICH. Therefore IFH is equal to BHC+ICH. Conſequently we have proved that IFH is bigger than we affirmed it to be; for we affirmed it to be equal to BHC, and we have proved it to be equal to BHC and ICH together. But then ICH is only the angle, which IH the moons ſemidiameter would ſubtend, if it was ſeen from C the ſun, and this is ſo ſmall that we may look upon it as nothing without any ſenſible error: and if we neglect this angle ICH or look upon it as nothing, then IFH the ſemiangle of the cone of the penumbra will be equal to BHC the ſuns apparent ſemidiameter.

194. *The ſemidiameter of the moons penumbra in that part, through which the earth paſſes in an eclipſe of the ſun, is equal to the ſum of the apparent ſemidiameters of the ſun and moon.*

Let CD, Plat. XXVI. fig. 2, be the moon, CDGH its penumbra, and conſequently CMD the angle of the cone of the penumbra: then, if a line as MLT is drawn from the vertex M through the moons center, it will divide the angle CMD into two equal parts. Therefore CML, which is the ſemiangle of the penumbra, is equal to the ſuns apparent ſemidiameter. Let AEBF be the ſection of the penumbra, which falls upon the earth in an eclipſe of the ſun, AB the diameter, and AT the ſemidiameter of this ſection. Now if CL the ſemidiameter of the moon, when ſeen from

from the earth at A, that is, when ſeen from the diſtance AC, ſubtends the angle CAL; this is the moons apparent ſemidiameter. And AT a ſemidiameter of the penumbra, in that part, through which the earth paſſes at an eclipſe of the ſun, when ſeen from an equal diſtance LA, ſubtends the angle ALT. This angle ALT is equal to the ſum of CML and CAL added together. For ſince ALT is the external angle at the baſe of the triangle AML, it is equal to the two internal oppoſite angles taken together, or to CML added to CAL.

195. *By the earths diſc we mean the hemiſphere of the earth, which is towards the moon, appearing like a flat circle, as it would do to a ſpectator at the moon.*

As the hemiſphere of the moon, which we ſee from the earth, appears to be a flat circle and is called the moons diſc; ſo, if there were any inhabitants of the moon, the hemiſphere of the earth, which is towards the moon, would appear to them like a flat circle, and this is what we call the earths diſc.

196. *At the new moon the whole earths diſc is enlightened.*

At the new moon, the moon is between the ſun and the earth, that is, the ſun is on one ſide of the moon and the earth is on the oppoſite ſide. Therefore when the moon appears in conjunction with the ſun to an inhabitant of the earth, the earth will appear in oppoſition to the ſun to an inhabitant of the moon: and conſequently at our new moon the enlightened hemiſphere of the earth is towards the moon; and for that reaſon an inhabitant of the moon would ſee the earth at the full or would ſee the whole diſc enlightened.

197. *The ſemidiameter of the earths diſc is equal to the moons horizontal parallax.*

The earths diſc is a hemiſphere of the earth, viewed from the moon; and conſequently the ſemediameter of the diſc is a ſemediameter of the earth viewed from the moon. But the moons horizontal parallax, by what has been ſaid of it, under propoſition 172, is equal to the apparent ſemediameter of the earth, when viewed from the moon. Therefore the ſemediameter of the diſc is equal to the moons horizontal parallax.

198. *In a partial eclipſe of the ſun the moons penumbra falls upon ſome part of the earths diſc: and in a total eclipſe the moons ſhadow falls upon ſome part of the earths diſc.*

In a partial eclipſe of the ſun, the penumbra of the moon falls upon ſome part of that hemiſphere of the earth, which is towards the moon, by propoſition 190. But this hemiſphere is the earths diſc. Therefore in a partial eclipſe of the ſun the moons penumbra falls upon ſome part of the earths diſc.

In a total eclipſe of the ſun the ſhadow of the moon falls upon ſome part of that hemiſphere of the earth, which is towards the moon, by propoſition 186. But this hemiſphere is the earths diſc. Therefore in a total eclipſe of the ſun the ſhadow of the moon falls upon ſome part of the earths diſc.

199. *If the latitude of the moon at the new is equal to the ſum of the ſemidiameter of the penumbra added to the moons horizontal parallax, or if its latitude is greater than that ſum, there will be no eclipſe of the ſun.*

Let *fng*, Plat. XXVI. fig. 3, repreſent a part of the ecliptic, and let *rno* be in the plane of the moons orbit: let *n* be the node, *dc* the earths diſc, and *pb* a ſection of the moons penumbra, which at the time of the new moon is even with the earth. The center of the moons penumbra is always oppoſite to the moons center, and conſequently the diſtance of the penumbras center *s* from the plane of the ecliptic is always equal to the diſtance of the moons center from the plane of the ecliptic or to the moons latitude. The earths center, or center of the earths diſc *t* is always in the plane of the ecliptic. Therefore *st* the diſtance between the center of the moons penumbra and center of the earths diſc is always equal to the moons latitude. Now if *st* or the moons latitude is equal to *sl* the ſemidiameter of the moons penumbra added to *et* the ſemidiameter of the earths diſc, then *l* one edge of the penumbra will touch *e* one edge of the diſc. But no part of the penumbra will fall upon the diſc, and conſequently, by propoſition 190, there will be no eclipſe of the ſun.

Or if *st*, which is equal to the moons latitude, as has been already proved is greater than *lt* the ſemidiameter of the earths diſc added to *sl* the ſemidiameter of the moons penumbra, then the penumbra will not touch the diſc, there will be the ſpace *le* between them. Therefore in this ſituation again there will be no eclipſe.

Thus we have ſhewn that there will be no eclipſe of the ſun at the new moon, if the moons latitude is equal to the ſum of the ſemidiameter of the moons penumbra added to the ſemidiameter of the earths diſc, or if the moons latitude is greater than this. But the ſemidiameter of the diſc is equal to the moons horizontal parallax, by propoſition 197. Therefore

fore there will be no eclipſe of the ſun at the new moon, if the moons latitude is equal to the ſum of the ſemidiameter of the moons penumbra added to its horizontal parallax, or if the moons latitude is greater than this.

The ſemidiameter of the moons penumbra is equal to the ſum of the apparent ſemidiameters of the ſun and moon, by propoſition 194: the diſtances of the ſun and moon from the earth are variable, and conſequently their apparent ſemidiameters will not always be the ſame, by propoſition 126 of optics. However to apply this propoſition we will ſuppoſe the ſuns apparent ſemidiameter to be 16′ 23″ and the moons to be 15′ 37″ then the ſum of them or the ſemidiameter of the penumbra will be 32′. The moons horizontal parallax is the angle which the ſemidiameter of the earth would ſubtend ſeen from the moon, this likewiſe will be variable according as the moon is nearer to the earth or farther from it, by propoſition 126 of optics. We will ſuppoſe it in a round number to be 60′. Then if the moons latitude at the new is either equal to or greater than 32′+60′=92′, there will be no eclipſe of the ſun.

200. *If the moons latitude at the new is leſs than the ſum of the ſemidiameter of its penumbra added to its horizontal parallax, but greater than the ſum of the ſemidiameter of its ſhadow added to its horizontal parallax, there will be a partial eclipſe of the ſun.*

Let *onr*, Plat. XXVI. fig. 4, be an arc in the plane of the lunar orbit, *fng* an arc of the ecliptic, and *n* one of the moons nodes: let *dc* be the earths diſc, and *pb* the ſection of the moons penumbra, which is even with the earth at the time of the new moon: and laſtly let the little circle *s* about the center of the penumbra be the ſection of the moons ſhadow, which is likewiſe even with the earth at the ſame time. We have proved, under propoſition 199, that *st* the diſtance between the centers of the moons penumbra and of the earths diſc at the time of the new moon is equal to the moons latitude. Now if *st* or the moons latitude is leſs than *sl* the ſemidiameter of the penumbra added to *et* the ſemidiameter of the diſc; then *l* the edge of the penumbra will be nearer to the ecliptic than *e* the edge of the diſc; conſequently ſome part of the penumbra will fall upon ſome part of the diſc; and in that part of the earth where this penumbra falls upon its diſc, there will be a partial eclipſe of the ſun, by propoſition 190. But the ſemidiameter of the diſc is equal to the moons horizontal parallax. Therefore if the moons latitude at the new is leſs than the ſemidiameter of the penumbra added to the moons horizontal parallax, there will be a partial eclipſe of the ſun ſeen in ſome parts of the earth.

But if the moons ſhadow does not fall upon the diſc there will be no total eclipſe of the ſun ſeen in any part of the earth, by propoſition 186. And if the moons latitude or *st*, which is equal to it, at the time of the new, is greater than the ſemidiameter of the ſmall circle *s* added to *et* the ſemidiameter of the diſc, then no part of this ſmall circle can paſs over the diſc *dc*. But the ſemidiameter of the ſmall circle *s* is, by the conſtruction, the ſemidiameter of the moons ſhadow and *et* the ſemidiameter of the diſc is equal to the moons horizontal parallax, by propoſition 197. Therefore, if at the new moon the latitude of the moon is greater than the ſemidiameter of its ſhadow added to its horizontal parallax, there will be no total eclipſe of the ſun.

201. *If the moons latitude at the new is leſs than the ſemidiameter of its ſhadow added to its horizontal parallax, there will be a total eclipſe of the ſun ſeen in ſome part of the earth.*

Let *onr*, Plat. XXVI. fig. 5, be an arc in the plane of the lunar orbit, *fng*, an arc in the ecliptic, and *n* the moons node: let *dc* be the earths diſc, *bp* the moons penumbra, and the little circle *s* its ſhadow. The moons latitude at the time of the new has been proved, under propoſition 199, to be equal to *st* the diſtance between the centers of its penumbra and of the earths diſc. If this diſtance is leſs than the ſemidiameter of the ſmall circle *s* and *et* the ſemidiameter of the earths diſc, then ſome part or the whole of the ſmall circle *s* will paſs acroſs the diſc. But, by the conſtruction the ſemidiameter of the ſmall circle *s* is the ſemidiameter of the moons ſhadow, and *et* the ſemidiameter of the diſc is equal to the moons horizontal parallax, by propoſition 197. Therefore if the moons latitude at the new is leſs than the ſemidiameter of its ſhadow added to its horizontal parallax, the moons ſhadow will paſs over ſome part of the earths diſc; and then, by propoſition 186, in thoſe parts of the earth, where the ſhadow of the moon croſſes the diſc, there will be a total eclipſe of the ſun.

Indeed the ſemidiameter of the moons ſhadow is little or nothing. For, by propoſition 189, it is equal to the exceſs of the apparent ſemidiameter of the moon above that of the ſun. But the moons apparent ſemidiameter is never much greater than the ſuns. Therefore the ſemidiameter of the moons ſhadow in that part where it croſſes the earths diſc is at all times very ſmall. Conſequently the ſum of the ſemidiameter of the ſhadow added to the moons horizontal parallax will be very nearly equal to the moons horizontal parallax alone. So that unleſs the moons latitude at the new is leſs than its horizontal parallax there will ſeldom be a total eclipſe of the ſun.

Nay

Nay ſometimes the moons apparent ſemidiameter of the earth may be leſs than that of the ſun; and in that caſe, as we ſhall ſhew preſently, there can be no total eclipſe of the ſun.

202. *If the moon at the time of the new is in one of the nodes, the eclipſe of the ſun will be central.*

The ſuns center is always in the plane of the ecliptic, by propoſition 27. And if the moon is in one of the nodes, its center is likewiſe in the plane of the ecliptic. Therefore if the moon at the time of the new is in one of its nodes, its center will paſs between the ſuns center and the earth: and ſuch a ſolar eclipſe as this is called a central one.

203. *The penumbra of the moon in a central eclipſe of the ſun will not cover the whole diſc of the earth.*

The ſemidiameter of the penumbra, as has been ſhewn, under propoſition 199, is about 32′, and conſequently its diameter is about 64′. The ſemidiameter of the earths diſc, as has been ſhewn in the ſame place, is about 60′, and conſequently its diameter is about 120′. Therefore ſince the diameter of the moons penumbra is leſs than that of the earths diſc, the penumbra will not be large enough to cover the whole diſc.

204. *From the proportion of the earths ſemidiameter to the moons ſemidiameter, and from the height of the earths ſhadow being known, we may determine the height of the moons ſhadow.*

The ſemiangle of the earths ſhadow, and likewiſe the ſemiangle of the moons ſhadow are either of them equal to the ſuns apparent ſemidiameter, by propoſitions 170, 188. But ſince as the ſemiangles of thoſe cones are equal, the cones will be ſimilar. Therefore as the ſemidiameter of the baſe of the earths ſhadow is to the ſemidiameter of the baſe of the moons ſhadow, ſo is the height of the earths ſhadow to the height of the moons ſhadow. Now the ſemidiameter of the earth is the ſemidiameter of the baſe of the earths ſhadow, and the ſemidiameter of the baſe of the moons ſhadow is the ſemidiameter of the moon. Therefore as the ſemidiameter of the earth is to the ſemidiameter of the moon, ſo is the ſemidiameter of the earths ſhadow to the ſemidiameter of the moons ſhadow.

The ſemidiameter of the earth is to that of the moon about as 100 to 28, and the height of the earths ſhadow is about 217 ſemidiameters of the earth, by propoſition 18. But as 100 to 28, ſo is 217 to 60 $\frac{1}{2}$ nearly. Therefore the height of the moons ſhadow is equal to about 60 $\frac{1}{2}$ ſemidi-

a-

ameters of the earth, that is, the moons ſhadow extends itſelf from the moon to the diſtance of about 60 $\frac{1}{2}$ ſemidiameters of the earth.

205. *An eclipſe of the ſun is called an annular one, when at the time of the eclipſe a ring of the ſun appears round the edges of the moon.*

The appearance of ſuch an eclipſe is repreſented, Plat. XXVI. fig. 6, where the dark circle is the opake body of the moon, and the white circle which ſurrounds it, is the lucid annular part or ring of the ſun, which is ſeen round the edges of the moon.

206. *A central eclipſe of the ſun will be an annular one, if the moons diſtance from the earth at the time of the eclipſe is greater than its middle diſtance.*

Let ABD, Plat. XXVI. fig. 1, be the ſun, GH the moon, and GPH the moons ſhadow. IP the height of this ſhadow is about 60 $\frac{1}{2}$ ſemidiameters of the earth, by propoſition 204. Now if at the time of a central eclipſe, KLMN is a part of the earths ſurface, the parts KL and MN are in the moons penumbra, and thoſe, who live there, will ſee a partial eclipſe of the ſun, by propoſition 190. LM is within the moons ſhadow. Therefore thoſe, who live between L and M, will ſee a total eclipſe of the ſun, by propoſition 186.

But if the diſtance of the earth from the moon at the time of a central eclipſe is equal to about 60 $\frac{1}{2}$ ſemidiameters of the earth or to the height of the moons ſhadow IP, that is, if OVPQ is a part of the earths ſurface; then P the vertex of the ſhadow will juſt reach to the earths ſurface. In this caſe a ſpectator at P will ſee a total eclipſe of the ſun, becauſe he will be in the ſhadow. But one, who is at V, will ſee only a partial eclipſe: for the ſhadow of the moon has no breadth at the earth, and conſequently a ſpectator ever ſo little either to one ſide or the other of P will be out of the ſhadow, though he is in the penumbra. Now, by propoſition 189, the ſemidiameter of the moons ſhadow at the earth is equal to the exceſs of the apparent ſemidiameter of the moon above that of the ſun. But when a ſpectator is at the vertex of the moons ſhadow, the moons apparent ſemidiameter GPI is equal to the ſuns apparent ſemidiameter BPC. Therefore as the moons apparent ſemidiameter does not exceed the ſuns, the ſemidiameter of the moons ſhadow at the earth will be nothing.

Laſtly if the diſtance of the moon from the earth is greater than its middle diſtance, which is about 60 $\frac{1}{2}$ ſemidiameters of the earth, that is, if IS the diſtance of RST a part of the earths ſurface from the moon is greater than 60 $\frac{1}{2}$ ſemidiameters of the earth, the ſhadow of the moon

GPH

GPH will not reach to the earth: for, by propoſition 204, IP the length of the ſhadow is only equal to about 60½ ſemidiameters of the earth. Conſequently, though a ſpectator at S would ſee a central eclipſe of the ſun, becauſe I the center of the moon and C the center of the ſun are in the ſame line SIC with his eye, yet this eclipſe would not be total, by propoſition 186, becauſe he is not in the ſhadow of the moon, which does not reach ſo far as his eye. Thus particularly, if a tangent to the moon at G was drawn from his eye at S to the ſun, this tangent would fall upon the ſun at *b*. Therefore at this edge of the ſun he would ſee the part B*b* beyond the edge of the moon. He might likewiſe ſee an equal part of the ſun at the oppoſite edge D. And for the ſame reaſon he might ſee an equal part of the ſun all round the moon. Therefore as he ſees a part of the ſun all round the edges of the moon, this part will be a lucid ring encompaſſing the moon, and the eclipſe will appear as is repreſented in Plat. XXVI. fig. 6.

It is plane that a central eclipſe muſt be an annular one, if at the time of the eclipſe the apparent ſemidiameter of the moon is leſs than the apparent ſemidiameter of the ſun. For in that caſe the apparent ſize of the moon will be leſs than that of the ſun. And therefore, if the center of the moon comes between the center of the ſun and our eye, the moon will not be apparently big enough to cover the whole ſun from our ſight; but the edges of the ſun will be ſeen all round the moon.

207. *In an annular eclipſe of the ſun, it is the penumbra of the moon, which falls upon the earth.*

An annular eclipſe is a partial one, by propoſition 206; for as there is in ſuch an eclipſe a lucid ring of the ſun, which appears round the edges of the moon, it is plane that only a part of the ſun is covered. Therefore, by propoſition 190, in an annular eclipſe of the ſun it is not the ſhadow, but the penumbra of the moon, which falls upon the earth.

The ſhadow of the moon GPH, Plat. XXVI. fig. 1, does not reach to the earth, if the earth is in the line RST, by propoſitions 204, 206: but as the moons penumbra GHRT, by propoſition 191, is an encreaſing cone, it will reach to this and to much greater diſtances. Therefore a ſpectator at S, who ſees a central and annular eclipſe of the ſun, is not in the moons ſhadow but in its penumbra.

208. *The penumbra of the earth reaches to the planet mars; and the penumbra of venus reaches to the earth.*

We have already obſerved, in propoſition 191, that the earth has a penumbra, as well as the moon: and in conſequence of this we propoſed

ſed a difficulty, which was whether this penumbra, as it is an encreaſing cone, ought not to reach as far as the planet mars. We now anſwer, that it does reach ſo far. For in propoſition 207, we have ſeen, if the moon covers a part in the ſuns diſc, though the ſun appears all round it, as in an annular eclipſe, that then the earth is in the moons penumbra. In like manner, when venus at its inferior conjunction appears, as by propoſition 103 it may appear, in the ſuns diſc, then though venus does not cover the whole diſc, yet the earth is in the penumbra of venus. Indeed the moon covers a much greater part of the ſun in a central eclipſe than venus covers at the inferior conjunction; and conſequently the gloom of the moons penumbra will be greater than that of venus's, as it ought to be, by propoſition 192; becauſe we are nearer to the moons ſhadow in an annular eclipſe, than we are to venus's ſhadow in its inferior conjunction. Nay the part of the ſun, which venus covers is ſo ſmall, or the diſtance of the earth from its ſhadow is ſo great, that the gloom of its penumbra is not ſenſible.

Now as venus is an inferior planet in reſpect of the earth, ſo the earth is an inferior planet in reſpect of mars: and conſequently as we ſee venus ſometimes croſſing the ſuns diſc, ſo an inhabitant of mars, if there are any inhabitants there, may ſometimes ſee the earth going acroſs the diſc of the ſun. But ſince the earth, when viewed from mars, covers part of the ſun, mars muſt then be in the earths penumbra, by propoſitions 190, 207. But then as the part of the ſun, which the earth covers, is ſo ſmall, or as mars is ſo far from the earths ſhadow, the gloom of this penumbra will inſenſible to an inhabitant of mars, that is, the brightneſs of the ſun at mars will, as to ſenſe, be the ſame, when ſuch a ſort of eclipſe happens, as when the earth is not between the ſun and mars. And conſequently, ſince the ſun ſhines as bright upon mars at this time as at any other, mars will reflect as much light to us, when this penumbra of the earth falls upon it, as it does at any time. Therefore though the penumbra of the earth does ſometimes fall upon mars, yet as it makes no alteration in the light of that planet, we ſhall not ſee any thing like an eclipſe of mars ariſing from this penumbra.

209. *The lunar ecliptic limit is the leaſt diſtance that the moon can be at from one of its nodes without being eclipſed at the time of oppoſition.*

210. *The ſolar ecliptic limit is the leaſt diſtance that the moon can be at from one of its nodes without eclipſing the ſun at the time of conjunction.*

212. *The ſolar ecliptic limit is greater than the lunar.*

We have already ſhewn, in propoſition 201, that if the moons latitude is about 92′, at the time of the new, the edge of the moons penumbra will touch the edge of the earths diſc: and conſequently this is the leaſt latitude that the moon can have at the time of the new without eclipſing the ſun. For if the latitude is leſs than this, then, by propoſition 202, ſome part of the penumbra will fall upon the diſc, and the ſun will be eclipſed. From hence we know that *ns*, Plat. XXVI. fig. 3. or the diſtance of the moons ſhadow, and conſequently of the moon itſelf, from the node at the time of the new muſt be ſuch that its latitude *st* may be equal to 92′. Now if we can find out what is the length of *ns* when the moons latitude or *st* is 92′, the length of *ns* ſo found is the ſolar ecliptic limit: for if *ns* is leſs than this, the moons latitude or *ts* will at the new be leſs than 92′, and conſequently it will eclipſe the ſun. In this triangle *nts*, we have the angle at *t* a right one, becauſe *st* is perpendicular to *fg* the plane of the ecliptic; we have likewiſe the angle at *n* about 5° 30′, for it is the inclination of the moons orbit to the plane of the ecliptic; and laſtly we have the ſide *st* of 92′. From theſe particulars, which are known, trigonometry will teach us to determine the length of the ſolar ecliptic limit or of the ſide *ns*, and it is found to be about 16°.

We have ſhewn, in propoſition 177, that if *c*C, Plat. XXV. fig. 4. or the moons latitude at the oppoſition is 61′, the edge of the earths ſhadow will juſt touch the moon, and there will be no eclipſe. But, by propoſition 178, if the latitude of the moon is leſs than this, ſome part of the earths ſhadow will fall upon ſome part of the moon, and the moon will be eclipſed. Therefore in order to determine the lunar ecliptic limit, we muſt find out what is the length of N*c* or the diſtance of the moon from its node N, when its latitude *c*C is 61′: for if the moon is at a leſs diſtance from the node than this at the oppoſition, its latitude will be leſs than 61′, and conſequently it will be eclipſed. Here we have in the triangle *c*CN the angle at C a right one, and the angle at N about 5° 30′ as before; and we have the ſide *c*C about 61′. From theſe particulars trigonometry will teach us to determine the length of the ſide N*c* or the lunar ecliptic limit, and it is found to be about 12°.

From hence then we ſee that the ſolar ecliptic limit is greater than the lunar: for the former is about 16° and the latter about 12°. I have here taken round numbers pretty near the truth, without endeavouring to fix the preciſe numbers; for I labour rather to be intelligible than to be exact.

212. *There are more eclipſes of the ſun in a courſe of years, than there are of the moon.*

If the moon at the new is leſs than 16° from one of its nodes, on either ſide of the node, it will eclipſe the ſun, by propoſition 211. Therefore there are 32° of the lunar orbit at each node, 16° on one ſide of it and 16° on the other ſide, and conſequently 64° in all, in any of which if the moon is at the time of the new, the ſun will be eclipſed. But unleſs the moon at the full is leſs than 12° degrees from its nodes on either ſide of the node, it will not be eclipſed, by propoſition 211. Therefore there are only 12° on each ſide of either node or 24° at each node, and conſequently only 48° in the lunar orbit, where the moon muſt be at the time of the full in order to be eclipſed. But ſince there are more degrees in the lunar orbit where the moon at the new will eclipſe the ſun, than there are where at the full it will be eclipſed itſelf; it follows that the ſun will be more frequently eclipſed than the moon in a courſe of years.

213. *Every eclipſe of the moon will be viſible, if the moon is above the horizon at the time of the eclipſe.*

For if the ſuns rays are intercepted by the earth, that is, if the earths ſhadow falls upon the moon, the part of the moon where the ſhadow falls will loſe its light, and conſequently the eclipſe muſt be ſeen at all places where the moon can be ſeen.

214. *The ſun may be eclipſed, and yet the eclipſe may be inviſible in places where the ſun is above the horizon.*

All places where the ſun is above the horizon are in that hemiſphere of the earth, which is towards the ſun, or which, by propoſition 196, makes the earths diſc. But neither the ſhadow nor the penumbra of the moon are large enough to cover the whole diſc, by propoſition 203. Therefore there are places upon the diſc, or upon that hemiſphere of the earth, which is towards the ſun, or places where the ſun is above the horizon, on which the penumbra of the moon will not fall at the time of a ſolar eclipſe; and conſequently, by propoſition 190, in thoſe places the eclipſe will not be ſeen.

215. *In any given place, more eclipſes of the moon than of the ſun will be ſeen in a courſe of years.*

For though more eclipſes of the ſun than of the moon happen in a courſe of years; yet many eclipſes of the ſun may be inviſible in any

PLATE XXV.

Fig. 1.

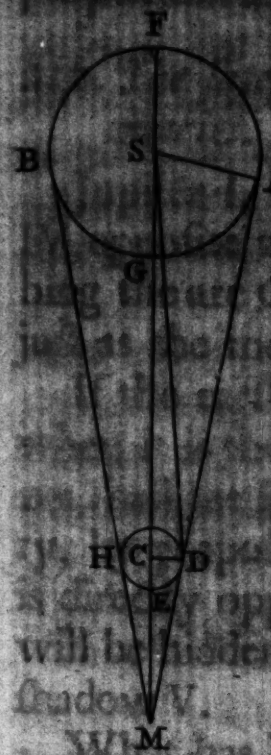


Fig. 2.

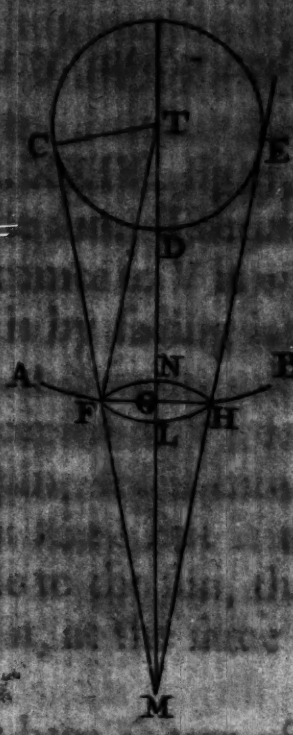


Fig. 3.

Fig. 4.

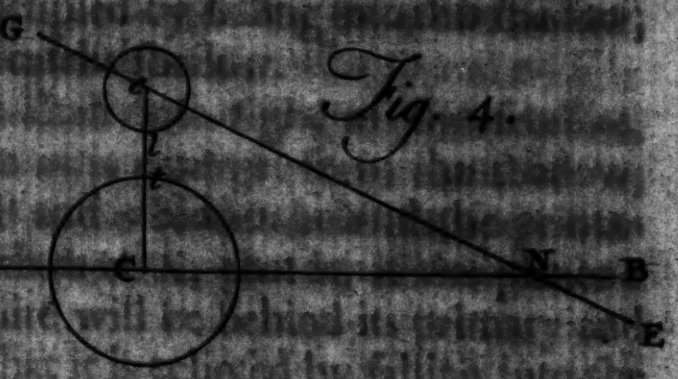


Fig. 5.

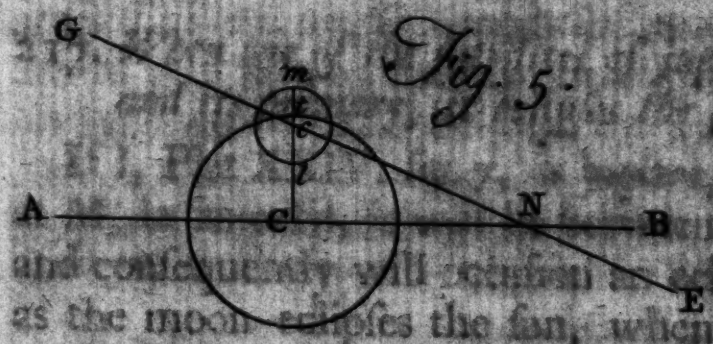


Fig. 6.

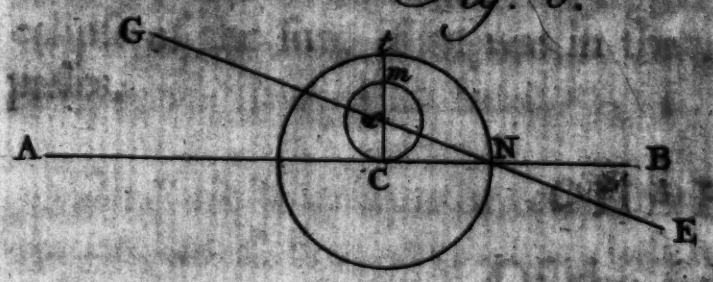


Fig. 7.

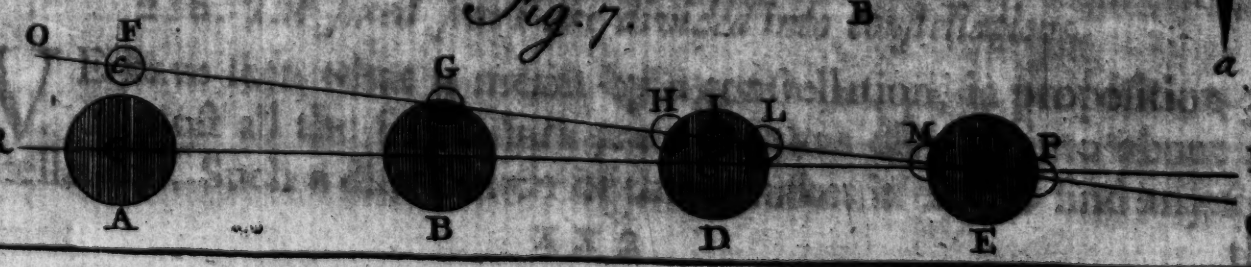
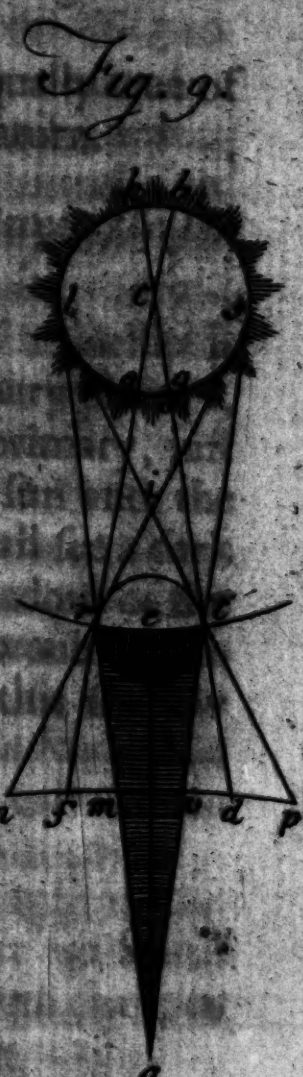
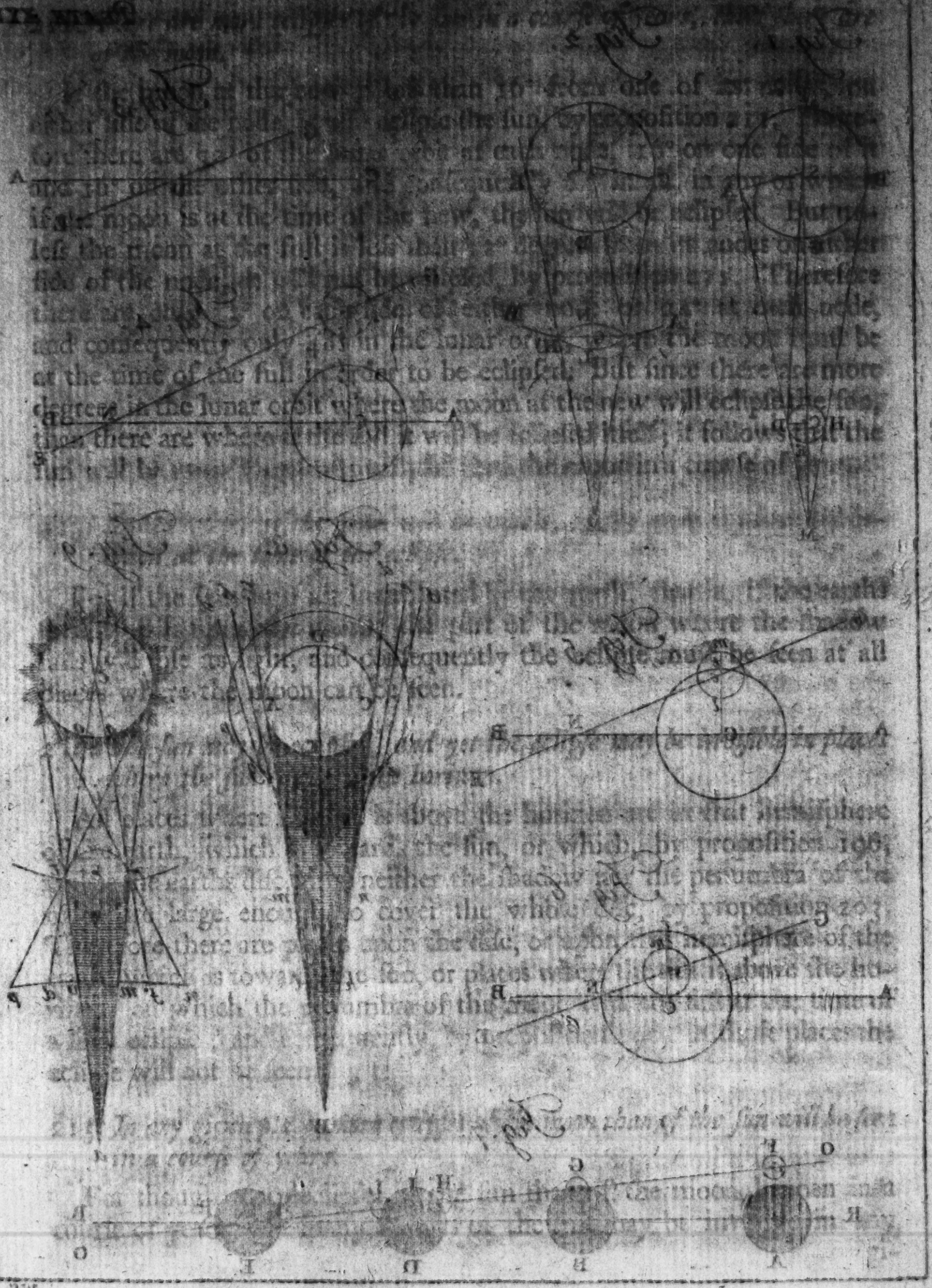


Fig. 8.

Fig. 9.



WS.



given place, though the ſun is above the horizon at the time of the eclipſe, by propoſition 214. Whereas all the eclipſes of the moon will be viſible, if the moon at the time of the eclipſe is above the horizon, by propoſition 213.

216. *The ſatellites of jupiter and ſaturn are eclipſed by their reſpective primaries.*

Jupiter I, Plat. XXIV. fig. 7, as it is an opake body, caſts a ſhadow IV oppoſite to the ſun. Therefore, when one of its ſatellites in deſcribing the arc OP comes to V it will be eclipſed by falling into this ſhadow, juſt as the moon is by falling into the earths ſhadow.

If the earth is at B in its orbit, we ſhall loſe ſight of the ſatellite, when it is thus eclipſed at V; and then as it emerges from the ſhadow, we ſhall ſee it again, till it comes to P and is hidden behind the primary, by propoſition 162. But if the earth is at X, that is, if the planet I is directly oppoſite to the ſun, the ſatellite will be behind its primary and will be hidden by it, at the ſame time that it is eclipſed by falling into the ſhadow V.

What has here been ſhewn of one ſatellite of jupiter is equally true of them all, and may be applyed likewiſe to the ſatellites of ſaturn.

217. *When one of the ſatellites of jupiter or ſaturn paſſes between the ſun and its primary, it eclipſes the ſun.*

If I, Plat XXIV. fig. 7, is jupiter, and S the ſun, then if a ſatellite is at M, in its orbit, it will be between the ſun S and the primary planet I, and conſequently will occaſion an eclipſe of the ſun at the primary, juſt as the moon eclipſes the ſun, when it paſſes between the ſun and the earth. If the earth in this caſe is at B or A in its orbit, we ſhall ſee a dark ſpot moving acroſs the diſc of jupiter; which ſpot is the ſhadow of the ſatellite, and an inhabitant of jupiter, if there is any ſuch, would ſee an eclipſe of the ſun, if he was in that part of jupiter where this ſhadow paſſes.

CHAP. VII.

Of the fixed ſtars.

218. *The fixed ſtars are divided into conſtellations.*

WE have ſeen what is meant by a conſtellation, in propoſition 30. Almoſt all the fixed ſtars are now thus divided into about 74 conſtellations. Such a diſtribution of them makes it eaſy to find any ſtar,

that we want to look for upon the artifical globe; and then by feeing its ſituation there in reſpect of the other ſtars about it, we may find it in the heavens. By this diſtribution of the fixed ſtars any perſon is enabled dily to deſcribe any part of the heavens, ſo that another will underſtand what part he means: if he has diſcovered a comet and can tell in what conſtellation it is, aſtronomers will know at once whereabouts to look for the comet: if he has diſcovered a new fixed ſtar, he need only ſay in what conſtellation it is, and in what part of the conſtellation, and this deſcription will ſufficiently direct others to the place where he has made the diſcovery.

The reader will not be able to remember the names of the ſeveral conſtellations, if they were to be reckoned up, ſo as to make any uſe of them: in order to know them to any purpoſe, he had better acquaint himſelf with their names and places together by ſtudying the artificial repreſentation of the heavens upon a celeſtial globe.

219. *There are ſome ſtars, which are not within any conſtellation; theſe are called unformed ſtars.*

In the ancient ſphere there were fewer conſtellations than we have at preſent; for many ſtars, which were left unformed by the ancients have been formed by Hevelius into new conſtellations. There are ſtill ſome ſtars that are unformed, but when we want to ſpeak of that part of the heavens where any of theſe ſtars are, we deſcribe them by the conſtellation that is neareſt to them.

220. *The milky way conſiſts of innumerable fixed ſtars, which are too ſmall to be ſeen diſtinctly by the naked eye.*

The peculiar brightneſs of that part of the heavens, which we call the milky way, had never been accounted for, till the invention of teleſcopes. But we now find that this part of the heavens is brighter than the reſt on account of the great number of ſtars, which are placed there, and which are ſo ſmall that the naked eye cannot diſtinguiſh them from one another.

There are indeed ſmall bright ſpots in ſome other parts of the heavens, which appear like the milky way to the naked eye. And theſe ſpots are found by the help of teleſcopes to owe their brightneſs to the number of ſmall fixed ſtars that are placed there.

221. *It is impoſſible to determine the number of fixed ſtars.*

Improvements in teleſcopes may diſcover many to us, which are now inviſible, and many more there may be, which are too remote to be ſeen

 through

through telescopes, after they are improved as far as they can be. Flamsteeds catalogue of fixed stars consists of no less than 3000. In a catalogue published by Bayer there are only 1160; but this catalogue contains only such stars as were known to and had been taken notice of by Ptolemy and Tycho. The catalogue is a very useful one, because every star has a mark given to it, which will distinguish it as readily, as if it had a proper name given to it: for each star in every constellation is marked with some letter; the biggest star in the constellation is marked with the first letter in the greek alphabet; the next in apparent size is marked with the second letter, and so on: if there are more stars in a constellation than there are letters in the greek alphabet, he marks the remaining stars with the letters of the roman alphabet. Thus if any one speaks of the star in leo marked α by Bayer, we may know by looking upon Senex's celestial globe, where these marks are used, what star he means, as well as if he had called it by its proper name cor leonis: if he speaks of the star in auriga marked ϑ by Bayer, we cannot fail of knowing by a globe, where these marks are used, which star is meant, as well as we should if the star had a proper name to call it by.

222. *The stars are sometimes divided into six different magnitudes.*

Some stars appear much brighter and larger than others. Perhaps those, which appear the brightest, may be either larger in themselves or nearer to us, than those, which appear less bright and seem smaller. However in consequence of this different appearance, astronomers have divided them into six different classes. Those, which seem the largest and brightest, are called stars of the first magnitude; the smallest, that we can see with the naked eye, are called stars of the sixth magnitude; and the intermediate ones according to their different apparent size are of the second, third, fourth, or fifth magnitude. Such stars as we can only see through telescopes are not reckoned in any of these classes, but are called telescopic stars.

223. *Some discoveries have lately been made of new stars, which for a while appear very bright, and then decay gradually, till they quite vanish.*

We shall shew hereafter, that a comet, in approaching to the sun, will be retarded by the suns atmosphere, and that this retardation will make it after several revolutions fall into the sun. If the fixed stars are suns, and have comets moving round them, such an addition of matter, and the shock, which they must receive by a comets falling into them, may make them blaze out with an unusual splendor, so as to exceed the brightness of any of the other stars: and then, though they were invisible to us before, this would make them visible. But then a splendor arising from

ſuch a ſudden ſhock will decay gradually, till at laſt we ſhall loſe ſight of them.

Indeed all the new ſtars do not blaze out thus at once; for ſome of them appear and diſappear periodically; their light encreaſes and decreaſes by ſlow degrees; and when they are the brighteſt, they do not appear bigger than ſtars of the third magnitude. Now if theſe fixed ſtars have ſpots upon them, ſuch as the ſun has, but larger, and if they turn round ſlowly upon their axes; ſometimes that ſide, where the ſpots are, and ſometimes the other ſide will be towards us; and, according as one or the other ſide is towards us, the ſtar will appear or diſappear.

224. *The longitude of the fixed ſtars encreaſes; but their latitude is not altered.*

The longitude of a ſtar is the arc of the ecliptic, which is intercepted between the firſt of aries or vernal equinoctial point and a ſecondary of the ecliptic that paſſes through the ſtar. Thus if I ſay that the ſtar in the conſtellation aries marked β by Bayer has 29° longitude, I mean that if a ſecondary of the ecliptic, or circle perpendicular to the ecliptic is drawn through this ſtar, there will be 29° of the ecliptic between the vernal equinoctial point and ſuch a circle, if we reckon thoſe degrees from the vernal equinoctial point towards the eaſt. Now the longitude of this or any other ſtar will encreaſe, as its diſtance from the vernal equinoctial point towards the eaſt encreaſes, or as that arc, by which its longitude is meaſured, encreaſes. But, by propoſition 64, the vernal equinoctial point moves weſtward. Therefore the diſtance between the ſtar and the vernal equinoctial point will encreaſe, or the ſtar will be farther to the eaſt of that point, not by any motion of its own from weſt to eaſt, but by a motion of the vernal equinoctial point or firſt of aries from eaſt to weſt.

But, ſince this change of longitude in the fixed ſtars is owing to the preceſſion of the equinoxes, and ſince the motion, from whence the preceſſion of the equinoxes ariſes, is performed round the axis of the ecliptic, by propoſition 66; ſuch a motion will make no change in the diſtance of the fixed ſtars from the poles of the ecliptic, and conſequently it will make no change in their diſtances from the plane of the ecliptic. But the diſtance of a ſtar from the plane of the ecliptic is its latitude. Therefore the ſame motion, which encreaſes their longitude, will not alter their latitude. If the latitude of the fixed ſtars is variable at all, the alterations made in it are too ſmall to have been yet exactly determined, and they muſt be owing to ſome other cauſe.

This motion, which changes the longitude of the fixed ſtars will not alter their ſituation in reſpect of one another. Becauſe the motion is not

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in the ſtars themſelves but in that point, from whence we begin to reckon their longitude.

From hence we may ſee the reaſon why the ſigns or conſtellations of the zodiac are to the eaſt of thoſe ſigns or arcs of the ecliptic, which are called by the ſame names. The firſt part of the conſtellation aries in the zodiac is about 30° from the arc of the ecliptic, which is called aries. For if the conſtellation aries, when the ſphere was firſt invented, was in the ſame part of the heavens with the vernal equinoctial point, yet by the preceſſion of the equinoxes, the conſtellation would in time get as far to the eaſt of that point as we now find it is. In like manner if the conſtellation libra was originally at the other equinoctial point, yet, as the longitude of libra encreaſed by the preceſſion of the equinoxes, the conſtellation libra would be, as it now is, to the eaſt of the ſign or arc libra in the ecliptic, which begins from the autumnal equinoctial point. The ſolſtitial points muſt be always 90° from the equinoctial points, and conſequently the ſame cauſe that makes a preceſſion of the equinoxes will make a preceſſion of the ſolſtices. Therefore though originally one ſolſtitial point, where the arc cancer begins in the ecliptic, might be in the conſtellation cancer, yet now the conſtellation cancer muſt be to the eaſt of it: and though originally the other ſolſtitial point, or point where the ecliptic touches the tropic of capricorn, might be in the conſtellation capricorn, yet the longitude of the conſtellation has encreaſed, and conſequently it muſt now be more to the eaſt than the ſolſtitial point.

The colure of the equinoxes is a ſecondary of the equator which paſſes through the equinoctial points. And the colure of the ſolſtices is another ſecondary of the equator, which paſſes through the two ſolſtices. Sir Iſaac Newton tells us, in his chronology, that Chiron and Muſaeus made a ſphere for the uſe of the argonauts, and that they drew the colure of the equinoxes through the middle or 15th degrees of the conſtellations aries and libra: and the colure of the ſolſtices through the middle or 15th degrees of the conſtellations cancer and capricorn. What I have here called libra, he calls chelae or the claws in conformity to the antient ſphere, in which the ſcorpion was drawn with long claws, that took up that part of the heavens where we now place libra.

Having ſettled, as he thought, the place of Chirons colures he reaſons in the following manner. In the end of the year 1689, a ſecondary of the equator drawn through the middle of the conſtellations aries and libra would cut the ecliptic 1 ſign, 6°. 29′. to the eaſt of the equinoctial points. But ſuch a ſecondary of the equator was the equinoctial colure, as it was drawn by Chiron. Therefore from the time of firſt drawing

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the colure, or ſince the time of the argonautic expedition, the ſolſtitial points have gone backwards in the ecliptic 1 ſign, 6°. 29′. In like manner a ſecondary of the equator paſſing through the middle of the conſtellations cancer and capricorn was the colure of the ſolſtices in the time of Chiron or the time of the argonautic expedition. But at the end of the year 1689 a ſecondary of the equator paſſing through the middle of theſe conſtellations cancer and capricorn cut the ecliptic 1 ſign, 6°. 29′. to the eaſt of the ſolſtitial points. Therefore between the time of the argonautic expedition and the end of the year 1689, the longitude of the ſtars in cancer and capricorn had encreaſed 1 ſign, 6°. 29′. or the ſolſtitial points had gone backwards ſo much. Since the cardinal points move at the rate of 1° in 72 years, by propoſition 64, they will take up 2627 years to move 1 ſign, 6°. 29′. Therefore the argonautic expedition was 2627 years before the end of the year 1689. Count theſe years backwards from the end of the year 1689, and the reckoning will place the argonautic expedition 43 years after the death of Solomon, which is about 300 years later than the greek chronologers place it.

The whole of this argument for ſettling the time of the argonautic expedition depends upon theſe principles. Firſt, that Chiron and his contemporary Muſaeus made a ſphere for the uſe of the argonauts. Secondly, that they drew the colures of the equinoxes through the 15th degrees of the conſtellations aries and libra, and the colure of the ſolſtices through the 15th degrees of the conſtellations cancer and capricorn. And thirdly, that they knew how to draw the colures exactly.

The firſt of theſe principles is not ſufficiently eſtabliſhed. Sir Iſaac in proof of it alledges the authority of an old poet quoted by Clemens Alexandrinus, who ſays, that Chiron drew σχήματα ὁλύμπου the figures of the heaven: to this he adds the teſtimony of Diogenes Laertius in order to ſhew that Muſaeus made a ſphere: and laſtly he concludes from the names of the conſtellations that the ſphere was delineated in the time of the argonauts; becauſe the expedition is delineated in the conſtellations, together with ſeveral other more antient hiſtories of the Greeks, and without any thing later. There is the golden ram, the bull with brazen hoofs tamed by Jaſon, and the twins Caſtor and Pollux two of the argonauts, with the ſwan of Leda their mother. There is the ſhip argo, and hydrus the watchful dragon, &c. Let us examine each of the three particulars.

The words quoted by Clemens, Strom. I. 306. concerning Chiron are

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Εἰς τε δικαιοσύνην θνητῶν γένος ἤγαγε δείξας
ἄρμον, καὶ θυσίας ἱλαρὰς, καὶ σχήματ' ὀλύμπου.

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He firſt led men to the practice of juſtice by introducing the obligation of oaths, propitiatory ſacrifices, and the figures of the heaven. The ſenſe planely requires that theſe figures of the heaven ſhould have ſome relation to religion: for delineating conſtellations for the uſe of ſailors would be of no ſervice in leading men to the practice of juſtice. Clemens adds that Chirons daughter Hippo ſtudyed nature in the ſame manner or to the ſame purpoſes that her father had done, and cites Euripides to prove it.

Ἡ πρῶτα μὲν τὰ θεῖα προμαντεύσατο
Χρησμοῖσιν, ἢ δὲ ἀστέρων ἐπαναστολάς.

She was one of the firſt who ſignifyed the will of the gods by oracular anſwers or from the riſing of the ſtars. The uſe, which Hippo made of her knowledge of nature, was to teach the will of the gods from the motion of the ſtars; ſo that her aſtronomy was applyed to the purpoſes of augury. And as ſhe ſtudyed nature in the ſame manner that her father had done, we may conclude that his aſtronomy was applyed to the ſame purpoſes, and that, if he drew a ſphere, it was for this uſe, and not for the uſe of ſailors. Though perhaps, as he was an augur, to draw σχήματα οὐλύμπου may not mean to form conſtellations, but to divide the heaven into regions in the ſame manner that the Roman augurs uſed to do, as we learn from Ciceros firſt book concerning divination. Diogenes Laertius, prooem. ſegm. 3. tells us that Muſaeus did ποιῆσαι θεογονίαν καὶ σφαῖραν πρῶτον firſt make a poem concerning the birth of the gods, and concerning the ſphere. The word ποιῆσαι manifeſtly ſignifyes here, as it often does, to make a poem. Muſaeus was a poet: one of his works was called, as one of Heſiods is, a theogony, which treated of the birth of the gods; and another, like Aratus's phaenomena and Manilius's aſtronomica, treated of the ſphere. From this account of Muſaeus there is no ſort of reaſon to conclude, that he was a practical aſtronomer, and aſſiſted Chiron in delineating the conſtellations for the uſe of the argonautic ſailors. The argonautic expedition was probably an embaſſy to ſeveral princes upon the coaſts of the Euxine and Mediterranean ſeas. The accounts of it, that we meet with in the poets, are only its fabulous hiſtory. Now the names of the conſtellations, as Sir Iſaac has explaned them, are taken from this fabulous hiſtory, which doubtleſs had its origin much later than the expedition itſelf: therefore the preſent names of the conſtellations, ſuppoſing them to relate to the argonautic expedition, are no proof that the ſphere was firſt delineated for the uſe of the argonauts; ſince the names are taken from the fabulous account of their expedition, and conſequently were names that could not be given to thoſe conſtellations by Chiron at the time of the expedition, for he knew only the true and

was unacquainted with the fabulous hiſtory of it. Beſides, Pagaſæ from whence the argonauts ſailed is in about 39° and Colchi, to which they were ſailing, is in about 45° north latitude; the ſtar Canobus of the firſt magnitude marked α by Bayer in the conſtellation argo is only 37° from the ſouth pole; and great part of this conſtellation is ſtill nearer to the ſouth pole. Therefore this principal ſtar and great part of the conſtellation argo could not be ſeen either in the place that the argonauts ſet out from, or in the place to which they were ſailing. Now the ſhip was the firſt of its kind and was a principal thing in the expedition: which makes it very unlikely that Chiron would chuſe to call a ſet of ſtars by the name argo, moſt of which were inviſible to the argonauts. If he had delineated the ſphere for their uſe, he would have choſen to call ſome other conſtellation by this name; moſt likely he would have given the name argo to ſome conſtellation in the zodiac; however, certainly to one that was viſible to the Argonauts, and not to one, which was ſo far to the ſouth that the principal ſtar in it could not be ſeen by them, either when they ſet out, or when they came to the end of their voyage.

But though there are no arguments alledged ſufficient to prove that Chiron delineated the conſtellations for the uſe of the argonauts, and though the names of the conſtellations, and the ſituation of thoſe ſtars, to which the name argo is given, ſeem to prove the contrary; yet let us ſuppoſe that he was the inventor of the ſphere, and that he invented it at the time Sir Iſaac Newton imagines he did. Upon this ſuppoſition in order to ſettle the time of the argonautic expedition from the preceſſion of the cardinal points, it would be neceſſary to make good Sir Iſaacs ſecond principle; which is, that Chirons colure of the equinoxes paſſed through the 15th degrees of the conſtellations aries and libra, and his colure of the ſolſtices through the 15th degrees of the conſtellations cancer and capricorn. Eudoxus, ſays Sir Iſaac, placed the ſolſtices and equinoxes in the middle of the conſtellations aries, cancer, chelae and capricorn. For Hipparchus, ad phaenom. Eudox. et Arat. l. 1. tells us that Eudoxus drew the colure of the ſolſtices through the middle of the great bear, and the middles of cancer, and the neck of hydrus, and the ſtar between the poop and maſt of argo, and the tail of the ſouth fiſh, and through the middle of capricorn, and of ſagitta, and through the neck and right wing of the ſwan and the left hand of cepheus. Here Sir Iſaac leaves off in his deſcription of Eudoxus's colure, and omits a very material part, which Hipparchus adds; that this colure goes on from the left hand of cepheus through the flexure of the dragon, and by the tail of the little bear. Hipparchus's account of the colure of the equinoxes, as drawn by Eudoxus,

is, that it paſſed through the left hand of arctophylax, and along the middle of his body, and croſs the middle of chelae, or libra, and through the right hand and fore knee of the centaur, and through the flexure of eridanus and head of caetus and the back of aries acroſs, and through the head and right hand of perſeus. It ſeems impoſſible to conclude from this deſcription of the cardinal points that Chiron placed them in the middle of the conſtellations aries, cancer, libra, and capricorn. Eudoxus is deſcribing the courſe of the colures in his own time, and not the ſituation, in which Chiron drew them. This is plane from the circumſtance which Sir Iſaac has omitted. Eudoxus ſays that the colure of the ſolſtices paſſed by the tail of the little bear. Therefore he was deſcribing a ſphere, in which there was ſuch a conſtellation as the little bear. But if Chiron did make a ſphere, as Sir Iſaac ſuppoſes, yet he left the ſtars, which compoſe this conſtellation unformed: in Greece they were reduced to a conſtellation long after his time by Thales, as Diogenes Laertius informs us from Callimachus. Thal. ſegm. 23. L. I. And conſequently when Eudoxus deſcribes the colures in a ſphere that has the little bear in it, he cannot be deſcribing Chirons ſphere. The ſum of the argument is this Eudoxus draws the colures through the middle of the conſtellations aries, cancer, libra, and capricorn: this Sir Iſaac urges to prove that the colures were ſo placed in the time of Chiron; whereas no ſuch thing can be proved from it, becauſe from the mention of the little bear it appears that Eudoxus muſt be deſcribing the ſphere as it ſtood after the time of Thales.

Thirdly, let us enquire whether Chiron knew how to draw the colures exactly. Eudoxus was probably endeavouring to deſcribe the ſphere as it ſtood in his own time. If he was, we are ſure that he placed the cardinal points wrong: for we can prove that the cardinal points were not in the middle of the conſtellations aries, cancer, libra, and capricorn, at the time when he lived, from the ſituation in which they are at preſent. But whoever delineated the ſphere, which Eudoxus deſcribes, he was very capable of making a miſtake about the true place of the cardinal points: for Hipparchus has ſhewn, in the place referred to above, that no ſecondary of the equator could be ſo drawn as to paſs through the middle of the great bear, the middle of cancer, the neck of hydrus, the ſtar between the poop and maſt of argo, the tail of the ſouth fiſh, the middle of capricorn and of ſagitta, the neck and right wing of the ſwan, the left hand of cepheus, the flexure of the dragon, and the tail of the little bear: and that no other ſecondary of the equator could be ſo drawn as to paſs through the left hand of arctophylax, and along the middle of his body, then croſs the middle of chelae, through the right hand and

fore knee of the centaur, through the flexure of eridanus, the head of cætus, the back of aries acroſs, and the head and right hand of perſeus. The former is the courſe of the ſolſtitial and the latter of the equinoctial colure, according to Eudoxus. Therefore Eudoxus did not know the true courſe of two ſecondaries of the equator, one of which cuts the ecliptic in the middle of the conſtellations aries and libra, and the other cuts it in the middle of the conſtellations cancer and capricorn. Now if his ſkill in aſtronomy was not ſufficient to determine the courſe of two ſecondaries of the equator, which any one, who has a ſphere rightly delineated, may eaſily determine, we may venture to ſay that he was not able to ſettle the true places of the cardinal points with any exactneſs: he might place them in the middle of the conſtellations aries, cancer, libra, and capricorn, though that was not their true place: and conſequently though we are ſure that the colures in the time of Eudoxus were not in the ſituation, in which he has deſcribed them, yet he might deſign in his deſcription to give an account of the ſphere as it ſtood in his time. If he did, we are ſupplyed with a double argument againſt Sir Iſaac Newton. Firſt, the place of the colures, as deſcribed by Eudoxus, will not determine what their place was in the time of Chiron; becauſe Eudoxus was endeavouring in that deſcription to give an account of the ſphere, not as it ſtood in the time of Chiron, but as it ſtood in his own time. And ſecondly, aſtronomy was ſo imperfect in the days of Eudoxus that he was not able to tell the courſe of a ſecondary of the equator among the fixed ſtars, and much leſs was he able to determine the true place of the cardinal points. Therefore Chiron, who lived much earlier than he, and in the very infancy of aſtronomy, cannot be ſuppoſed to draw the colures through their true places. So that though we know certainly the ſituation of the cardinal points at preſent, and though we grant that Chiron placed them where Sir Iſaac ſuppoſes; yet it is impoſſible to determine from the preceſſion of the equinoxes how many years have paſſed between the time of the argonautic expedition and the preſent time, becauſe we have great reaſon to believe that Chiron was not able to fix the true place of the cardinal points.

Nay though we were to grant, what I have ſhewn to be falſe, that Eudoxus deſcribes the ſphere as it was delineated by Chiron, yet, ſince we have ſeen that the author of this ſphere, whoever he was, had not ſkill enough in aſtronomy to determine the true place of the cardinal points, no uſe can be made of this conceſſion. Chiron, as we will now ſuppoſe, fixed the cardinal points in the middle of the conſtellations aries, cancer, libra, and capricorn: if theſe were their true places at the time

time of the argonautic expedition, then we might conclude from their places at the end of the year 1689, that 2627 years and no more had paſſed between the time of the argonautic expedition and the end of that year. But Chirons aſtronomy was ſo imperfect that we may be ſure he did not know how to determine the true places of the cardinal points. And though we are ſuppoſed to know where he placed them, yet as in all probability he placed them wrong, we cannot tell how far they have gone back ſince his time, nor ſhew by their preceſſion how many years have paſſed ſince the argonautic expedition.

CHAP. VIII.

Of the Copernican ſyſtem.

WE have hitherto ſuppoſed the truth of the Copernican ſyſtem, and upon that ſuppoſition have explaned the ſeveral apparent motions of the heavenly bodies. We will now from ſome of thoſe apparent motions, prove that the Copernican ſyſtem is true, by ſhewing that ſuch apparent motions cannot be explaned either by the Ptolemaic or the Tychonic ſyſtem.

225. *All the planets are ſpherical.*

We have proved already, in propoſition 11, that the earth is ſpherical. We find from obſervations, that venus, mars, and jupiter turn round their reſpective axiſes, and by that means turn different parts towards the earth at different times. But their diſcs are round at all times: and all the different ſides of a body cannot appear round unleſs the body is ſpherical. Therefore theſe three planets are ſpherical. Indeed the whole diſc either of venus or mars is not always enlightened, by propoſitions 104, 131: but the part, which is enlightened, is a part of a circle, and from thence we may be ſure that, if the whole diſc was enlightened and we could ſee it all, it muſt be circular. Beſides, when theſe two planets have their whole diſcs enlightened, the ſame ſide of each of them is not always towards the earth, and yet their diſcs appear round. Therefore they appear round, when they have different ſides towards the earth, and conſequently they are ſpherical. We find that mercury changes in the ſame manner as the moon does, and its diſc is round, when it is all enlightened. Now becauſe the diſc is round when it is all enlightened, mercury is either a plane flat circle with one of its ſides towards the earth, or elſe it is a ſphere. The changes of its diſc, like thoſe of the moon, will prove that it is not a flat circle. For if mercury was a plane flat circle, the ſun muſt be either on the ſame ſide of the plane of mercury

cury with the earth, or the ſun muſt be on one ſide of this plane and the earth on the other ſide, or laſtly the ſun muſt be in the plane of this flat planet. If the ſide of mercury, which is towards the earth, was quite flat, then when the ſun is on the ſame ſide of this plane with the earth, it would ſhine upon the whole diſc at once, and we ſhould ſee the whole diſc enlightened; but when the ſun is on the contrary ſide, it could not ſhine at all upon the ſide, that is towards us, and mercury would become inviſible; and when the ſun is in the plane of the flat planet, it would ſhine only upon the edge of the plane and not upon either ſide of it, ſo that in this ſituation likewiſe mercury would be inviſible. Therefore if mercury was a plane flat circle, it muſt either appear round to us or be inviſible. But mercury is ſometimes horned, ſometimes halved, and ſometimes gibbous. Therefore mercury is not a plane flat circle, and conſequently it is a ſphere. We cannot make uſe of the ſame argument to prove that mercury is a ſphere, that we made uſe of to prove that venus, mars, and jupiter are, becauſe no obſervations have yet been made, that will determine whether it has a rotation round its axis. This argument will fail us likewiſe in the caſe of ſaturn and for the ſame reaſon. Nor can we prove that ſaturn is ſpherical by the argument juſt now made uſe of to prove that mercury is ſpherical: becauſe there are no ſuch changes in the phaſe of ſaturn as there are in that of mercury. But ſince we have ſhewn that all the other primary planets are of this ſhape, we may conclude by analogy that this is the ſhape of ſaturn.

If the moon always turned exactly the ſame ſide towards the earth, we could then prove that it is a ſphere and not a flat circle only by the argument before made uſe of in the caſe of mercury. For the moon, if it was a flat circle, would always be either at the full or elſe inviſible: it would be at the full, whenever the ſun is on the ſame ſide of this plane circle with the earth, and would be inviſible, whenever the ſun is either on the oppoſite ſide, or in the plane of it, ſo as to ſhine only upon its edge. Therefore the changes made in the phaſe of the moon prove that it is a ſphere.

But in the moons librations an alteration is made in the ſide that is towards us, by propoſitions 147, 148. And in theſe librations, though exactly the ſame ſide of the moon is not always towards us, the diſc appears round. But a body muſt be ſpherical, if in this change of ſides it appears always round. Therefore the moon is ſpherical.

The ſatellites of jupiter are ſpherical: for they always appear round, though, by propoſition 162, they turn different ſides towards us. And the ſatellites of ſaturn are proved to be ſpherical by analogy, becauſe this is the ſhape of all the other bodies in the planetary ſyſtem.

227. *All*

226. *All the planets are opake bodies.*

Opake is frequently oppoſed to *tranſparent*, and then by opake bodies we mean ſuch as tranſmit no light. But here opake is oppoſed to luminous, and when we call the planets opake bodies, we mean, that they are not luminous, that they have no light of their own, as the ſun has, but are made viſible and ſhine only becauſe the ſun ſhines upon them, and they reflect the ſuns rays.

Mercury, venus, and mars are opake bodies, becauſe, by propoſitions 104. 131, thoſe parts of their diſc, upon which the ſun does not ſhine, are inviſible. Jupiter and ſaturn are opake bodies, becauſe when the ſhadow of a ſatellite falls upon either of them, that is, when the ſuns light is intercepted by a ſatellite, that part of the diſc, where the ſhadow falls, becomes dark, or does not ſhine, by propoſition 217. The moon is an opake body, becauſe thoſe parts of the diſc, which neither the ſun nor the earth ſhines upon, are inviſible, by propoſitions 142, 144. The ſatellites of jupiter and ſaturn are opake bodies, becauſe when a ſatellite falls into the ſhadow of its primary, that is, when the primary intercepts the ſuns light, the ſatellite becomes inviſible, by propoſition 216.

227. *The orbits of the inferior planets encompaſs the ſun; but the earth is placed on the outſide of thoſe orbits: and conſequently the Ptolemaic ſyſtem is falſe.*

We have already ſhewn, by one argument, in propoſition 110, that the Ptolemaic ſyſtem is falſe. But there are other arguments drawn from the apparent motions of the inferior planets, by which we may prove the ſame thing. Venus, for inſtance, is ſometimes ſeen on the eaſt ſide and ſometimes on the weſt ſide of the ſun; it is ſometimes between the ſun and the earth or in its inferior conjunction, and ſometimes it is beyond the ſun or in its ſuperior conjunction. But ſince venus is thus at different times on all ſides of the ſun, the orbit that venus moves in muſt go round the ſun. Now in the Ptolemaic ſyſtem the orbit of venus is ſuppoſed to be within the ſuns orbit, or venus is ſuppoſed to move in an orbit, which does not encompaſs the ſun. Therefore the Ptolemaic ſyſtem is not true.

If the earth was within the orbit of venus, as the Ptolemaic ſyſtem ſuppoſes, venus might ſometimes be on one ſide of the earth, whilſt the ſun is on the oppoſite ſide, or venus might be ſometimes in oppoſition to the ſun. But venus is never ſeen in oppoſition. Therefore the earth is on the outſide of the orbit of venus, and conſequently the Ptolemaic ſyſtem is not true.

What has here been ſaid of venus, may be applyed to mercury the other inferior planet.

228. *The earth is not at reſt within the orbits of the ſuperior planets: and conſequently the Tychonic as well as the Ptolemaic ſyſtem is falſe, and the Copernican ſyſtem is true.*

If the earth was at reſt either in the center or in any other point within the orbit of mars, the apparent motion of mars ſeen from the earth would always be direct. Let ABCD, Plat. XXVI. fig. 7, be the orbit of mars, and let mars move from weſt to eaſt according to the order of the letters A, B, C, D: then ſuppoſe the earth to be at reſt either at O or in any other point within this orbit, a ſpectator upon the earth could never ſee mars move in any other direction than according to the order of the letters A, B, C, D, or from weſt to eaſt. Therefore upon this ſuppoſition mars's apparent motion would always be direct. Indeed if the earth was at O and not at the center of the orbit, the apparent velocity of mars would be unequal: for if it was to deſcribe the equal ſpaces AB and CD in equal times, when it is in theſe different parts of its orbit; theſe equal ſpaces would appear of unequal lengths, the ſpace AB ſubtends the angle AOB, and the equal ſpace CD ſubtends the greater angle COD, or the apparent length of thoſe equal ſpaces would be inverſely as their reſpective diſtances from O, by propoſition 126 of optics. Now the apparent velocity of mars, by propoſition 10, of mechanics, will be as the apparent length of the ſpaces deſcribed in equal times; and conſequently mars would appear to move ſwifter in the arc CD than in the arc AB. But ſtill, though the apparent velocity of the planet would be different at different times, yet it would always appear to move the ſame way, or its apparent motion would always be in conſequentia; the planet could never be ſtationary, and its apparent motion could never be retrograde. But mars is ſometimes ſtationary and ſometimes retrograde. Therefore the earth is not at reſt in any point within the orbit of mars.

What we have here ſaid of mars is applicable to jupiter and ſaturn the other two ſuperior planets.

Now the Ptolemaic ſyſtem ſuppoſes the earth to be at reſt in the common center of the orbits of the ſuperior planets; and the Tychonic ſyſtem, though it does not place the earth in the center, yet it ſuppoſes the earth to be at reſt within their orbits. But we have proved from the apparent ſtations and regreſſes of theſe planets that the earth is not at reſt within their orbits. Therefore neither the Ptolemaic nor Tychonic ſyſtem is true.

We

We are ſure indeed that the earth is within the orbits of the ſuperior planets, becauſe they are ſometimes ſeen in oppoſition to the ſun. And we have proved that it is not at reſt, but moves within their orbits, as the Copernican ſyſtem ſuppoſes. Therefore the Copernican ſyſtem is true.

CHAP. IX.

Of the ſphere.

THE artificial globes will be very uſeful in helping the reader to underſtand many of the following propoſitions, and particularly the definitions. I muſt deſire him to remember, that when I repreſent a circle by a right line in any of the figures referred to in this chapter, it is becauſe the figure is deſigned to give him ſuch a perſpective view of a circle, as he would have if his eye was in the plane of the circle: and in ſuch a ſituation of his eye the circle would appear like a right line.

229. *If innumerable circles are ſuppoſed to be drawn through the zenith and nadir of any place, ſo as to cut the horizon at right angles, theſe are called vertical circles.*

230. *A vertical circle, which paſſes through the two poles, is called the meridian.*

Let T, Plat. XXVI. fig. 8, be the earths center, R a place upon its ſurface, Z the zenith of the place, N the nadir: *ps* the earths axis, *p* and *s* the two poles of the earth: then PS is the axis of the earth continued, and upon this axis, by the earths daily rotation, the whole concave ſphere of the heavens will appear to revolve, by propoſition 23; therefore PS will be the axis of the world, P will be one pole, and S will be the other. The vertical circle ZPNS, paſſing through the zenith and nadir Z and N, and through the two poles P and S, is the meridian of the place R.

231. *The meridian of the place cuts the horizon in the cardinal north and ſouth points.*

ZPNS, Plat. XXVI. fig. 8, is the meridian of the place R, and HO is the horizon. The meridian cuts the horizon at O on one ſide directly under P the north pole, and at H on the other ſide directly over S the ſouth pole. Therefore O is the cardinal north point and H is the cardinal ſouth point.

232. *A meridian line is the common interſection of the plane of the meridian and plane of the horizon.*

HO, Plat. XXVI, fig. 8, in a perſpective view will repreſent the plane of the horizon. Or, if we conſider HO as a right line, it is then the interſection of the plane of the horizon with the plane of ZPNS the meridian. Such a line as this is the meridian line; it runs along upon the plane of the horizon from H the ſouth point to O the north point. So that any line, which is drawn due north and ſouth upon a horizontal plane, may be looked upon as part of the meridian line.

233. *The altitude of a ſtar or of any other point in the heavens above the horizon, and the depreſſion of a ſtar or of any other point in the heavens below the horizon, is the arc of a vertical circle intercepted between the horizon and that ſtar or that point of the heavens.*

If HO, Plat. XXVI. fig. 8, is the horizon, the altitude of a ſtar at P is the arc PO, which is an arc in the vertical circle ZHNO intercepted between the horizon O and the ſtar P. The depreſſion of a ſtar at S is the diſtance of the ſtar S below the horizon, which is meaſured by the arc HS intercepted between the horizon H and the ſtar S.

Now ſince the arc PO ſubtends the angle PTO, which PT a line drawn from the ſtar to the earths center makes with the horizon, and ſince this angle is proportional to the arc, which ſubtends it, we may call PTO the altitude of the ſtar P. In like manner we may call STH the depreſſion of the ſtar S.

234. *The altitude of a ſtar may be determined by the help of a quadrant.*

AED, Plat. XXVI. fig. 9, is a quadrant or quarter of a circle made of a piece of wood or braſs. The arc ED is divided into 90°, and thoſe degrees are again ſubdivided into 60′ each; and thoſe minutes are again ſubdivided into 60″ each, if the quadrant is large enough to admit of ſuch ſmall diviſions. Two pieces of braſs, each of which has a ſmall hole in it, are placed at A and B; theſe are called the ſights. And laſtly a plummet P hangs by a ſtring from C the center of the circle, of which the quadrant is a part.

If S is the ſun, and we move the quadrant backwards and forwards, till we have got it into ſuch a ſituation that we can ſee the ſun through the two ſights A and B, then the angle contained between the ſtring of the plummet CP and the ſide CE, on which the ſights are not placed, that is, the angle ECP, is the ſuns altitude. ECD is a right angle, by the form of the quadrant: HC is a line parallel to the horizon, and Z is the zenith, the plummet P, by propoſition of 42 mechanics, hangs directly down from Z the zenith towards the nadir, and conſequently ZP is perpendi-

cular

cular to the horizon, and the angle ZCH is a right angle. A ray of light SAD coming ſtrait from the ſun through the two ſights makes the angle SCZ with the line ZC, and the angle PCD with the plummet. But as the ray of light moves in a right line, by propoſition 34 of optics, and as ZP has juſt been proved to be a right line, it follows that SCZ and PCD are equal to one another, becauſe they are vertical angles. Euc. b. I. prop. 15. Since theſe angles are equal to one another, the reſpective complements of each of them to a right angle will likewiſe be equal. But SCH is the complement of SCZ, and ECP is the complement of PCD. Therefore SCH the ſuns altitude is equal to ECP.

234. *The prime vertical is that, which croſſes the meridian at right angles in the zenith and nadir, and cuts the horizon in the cardinal eaſt and weſt points.*

All the vertical circles croſs one another in the zenith and nadir, by propoſition 229, and the meridian is one of the vertical circles, by propoſition 230. Amongſt the other innumerable vertical circles there muſt be one, which croſſes the meridian at right angles, this is called the prime vertical. Now as the prime vertical makes right angles with the meridian, the points where the prime vertical cuts the plane of the horizon muſt be 90° diſtant from thoſe where the meridian cuts the ſame plane. But, by propoſition 23, the meridian cuts the plane of the horizon in the cardinal ſouth and north points: and the cardinal eaſt and weſt points are 90° diſtant from thoſe. Therefore the prime vertical cuts the plane of the horizon in the cardinal eaſt and weſt points.

235. *The azimuth of a heavenly body is an arc of the horizon intercepted between the meridian and a vertical circle paſſing through that body: the azimuth is either eaſtern or weſtern.*

Any heavenly body has ſome azimuth, unleſs it is in the meridian. The ſun has ſome azimuth before it comes to the meridian, and then its azimuth is eaſtern, becauſe then it is on the eaſt ſide of the meridian; it has likewiſe ſome azimuth after it has paſſed the meridian, but then its azimuth is weſtern, becauſe then it is on the weſtern ſide of the meridian. Wherever the ſun is, if we conceive a vertical circle to be drawn through it, the diſtance between the point where this circle cuts the horizon and either of the points where the meridian cuts the ſame horizon is called its azimuth. Now the meridian cuts the horizon in the cardinal north and ſouth points. Therefore if a vertical circle is conceived to

be drawn through the ſun or any other heavenly body, the diſtance of this vertical circle from the cardinal north or ſouth points meaſured by an arc of the horizon is the azimuth of the ſun, or of that other body.

236. *The amplitude of a heavenly body at riſing is an arc of the horizon intercepted between the point where the body riſes and the cardinal eaſt point; its amplitude at ſetting is an arc of the horizon intercepted between the point where the body ſets and the cardinal weſt point: the amplitude is either northern or ſouthern.*

If a ſtar riſes exactly in the eaſt and ſets exactly in the weſt, it has no amplitude either at riſing or ſetting: this is the caſe of the ſun at the equinoxes, as ſhall be ſhewn hereafter. If a ſtar riſes in the north-eaſt and ſets in the north-weſt, which in our latitude is nearly the caſe of the ſun at the ſummer ſolſtice; then it has 45° northern amplitude at riſing, and likewiſe 45° northern amplitude at ſetting: for it riſes in the middle between the cardinal eaſt and north points or 45° to the north of the cardinal eaſt point; and it ſets in the middle between the cardinal weſt and north points or 45° to the north of the cardinal weſt point. If a ſtar riſes in the ſouth-eaſt and ſets in the ſouth-weſt, which is nearly the caſe of the ſun at the winter ſolſtice, it has 45° ſouthern amplitude at riſing, and likewiſe 45° ſouthern amplitude at ſetting.

237. *Any heavenly body is ſaid to riſe or ſet coſmically, when it riſes or ſets at the ſame time that the ſun riſes.*

The moon, for inſtance, riſes coſmically, when it riſes in a morning with the ſun, that is, at the conjunction: it ſets coſmically, when it ſets in a morning at the ſame time that the ſun riſes; conſequently when the moon ſets coſmically it muſt be on the weſtern ſide of the horizon when the ſun is on the eaſtern ſide, that is, it muſt be in oppoſition.

Sirius or the dog-ſtar ariſes with the ſun, or riſes coſmically in our latitude when the ſuns place, not in the zodiac, but in the ecliptic is about the 15th degree of leo. At this time the dog-days, as they are called, begin. But then it muſt be obſerved that the beginning of the dog-days will be variable: for as the longitude of the dog-ſtar encreaſes, by propoſition 224, it will get farther to the eaſt in reſpect of any point in the ecliptic, at the rate of 1° in 72 years. Thus Cicero tells us, in his ſecond book concerning divination, that ſirius roſe coſmically not many days after the ſummer ſolſtice. Therefore 1791 years ago, which is about the time when he wrote this book, the dog-days began, when the ſuns place in the ecliptic was in cancer or more to the weſt than it is when they begin

by the conſtant encreaſe of ſirius's longitude ariſing from the preceſſion of the equinoxes, in proceſs of time ſirius will have got ſo much further to the eaſt than it is at preſent, that it will not riſe coſmically till the ſuns place in the ecliptic is in capricorn, and then the dog-days will begin in december. From hence the reader may judge how little the heat of the weather at that time of the year, when the dog-days happen at preſent, depends upon the coſmical riſing of the dog-ſtar.

238. *Any heavenly body is ſaid to riſe or ſet achronically, if it riſes or ſets at the ſame time that the ſun ſets.*

The moon, for inſtance, when it is at the full, riſes when the ſun ſets, that is, it riſes achronically. But it does not ſet achronically, when it is at the full, becauſe it ſets not when the ſun ſets, but when the ſun riſes. At the new it ſets achronically, becauſe at the new it is in conjunction with the ſun, and conſequently ſets, when the ſun ſets.

239. *Any planet or fixed ſtar is ſaid to ſet heliacally, when it gets ſo near to the ſun as to be hidden by the ſuns ſtronger light, and is ſaid to riſe heliacally, when it gets far enough from the ſun to emerge out of that light and to be again viſible to us.*

Soon after the moon has ſet achronically, that is, by propoſition 239, ſoon after the new, it riſes heliacally; for it gets far enough from the ſun to be ſeen in an evening after the ſun is down. Not long before the moon ſets achronically, it ſets heliacally; for juſt before the new it gets ſo near to the ſun as not to be ſeen.

240. *The equator, and all its parallels and ſecondaries, as we have already deſcribed them in the concave ſphere of the heavens, are conceived to be drawn likewiſe upon the ſurface of the earth.*

Let *peſt*, Plat. XXVII. fig. 1, repreſent the earth, *p* and *s* the two poles, and *ee* the equator, as deſcribed in propoſition 14: we muſt imagine innumerable circles encompaſſing the earth parallel to the equator, ſuch as *bq*, *in*, *tc*, &c. ſuch circles as theſe, when drawn upon the concave ſphere of the heavens are called parallels of the equator, but when they are imagined to be drawn upon the earths ſurface, they are called circles or parallels of latitude, for a reaſon that will appear preſently. We muſt likewiſe imagine innumerable circles to be drawn through the poles

poles *p* and *s* ſo as to be perpendicular to the equator, ſuch as *pefs*, *pigs*, &c. Only half of all theſe circles, except *pefs*, is viſible in the figure, becauſe, as the figure repreſents a ſphere, the other half is hidden behind the ſphere. This ſort of circles, when conceived to be drawn on the concave ſphere of the heavens, are called ſecondaries of the equator, in propoſition 114; but then each of them is likewiſe called the meridian of that place, through the zenith of which it paſſes, in propoſition 230. Hence the ſame ſort of ſecondaries of the earths equator, when conceived to be drawn upon the earths ſurface are likewiſe called the meridians of thoſe places, through which they paſs; but theſe to diſtinguiſh them are called geographical meridians. Thus a ſecondary of the earths equator conceived to be drawn through the city of London is called the geographical meridian of London.

241. *Two parallels of latitude, each at the diſtance of 23 ½ degrees from the equator, one on the north ſide and the other on the ſouth ſide, are called the tropics; that on the north ſide is the tropic of cancer, and that on the ſouth ſide is the tropic of capricorn.*

If *ee*, Plat. XXVII. fig. 1, is the equator, then *tc* is the tropic of cancer, and *tn* is the tropic of capricorn: theſe two tropics, have their names from ſimilar circles in the heavens, by propoſitions 39, 240.

242. *Two parallels of latitude 23 ½ degrees from each pole of the earth are called polar circles; that round the north pole is the arctic circle, and that round the ſouth pole is the antarctic circle.*

If *p*, Plat. XXVII. fig. 1, is the north pole of the earth, then a circle *la*, parallel to the earths equator *ee* and 23 ½ degrees diſtant from *p* is the arctic polar circle. If *s* is the ſouth pole ſuch another circle *rk* is the antarctic polar circle. Theſe circles upon the earths ſurface have their names from two ſimilar circles in the heavens, by propoſitions 46, 240.

243. *The latitude of a place upon the earths ſurface is its diſtance from the earths equator: and this diſtance is meaſured by the arc of a geographical meridian, which is intercepted between the place and the equator.*

Suppoſe that London is at *i*, Plat. XXVII. fig. 1, by the latitude of London we mean the diſtance of *i* from the earths equator *ee*; and when we ſay that the latitude of London is 51°. 31′, we mean that, in *pifms* the geographical meridian of London, the arc *if*, which lies between London and the equator, contains 51°. 31′.

From hence it is obvious that all places, which lie in the ſame parallel of latitude, have equal latitudes: for the ſame parallel, and conſequent-

quently all places in it, muſt be every where at the ſame diſtance from the equator.

All places, that lie round the earths ſurface in the equator *ee*, have no latitude, for they are at no diſtance from the equator.

The two poles have each of them 90° latitude; for they are 90° diſtant from the equator.

All places that lie round the earths ſurface in either of the tropics have 23 ½ latitude, by propoſition 241.

All places, that lie round the earth in either of the polar circles, have 66 ½ latitude: for, the poles are 90° from the equator, and, by propoſition 242, the polar circles are 23 ½ degrees from the poles, that is, they are 23 ½ degrees nearer to the equator: therefore their diſtance from the equator is 66 ½ degrees.

244. *The latitude of places upon the earths ſurface is either north or ſouth.*

Thoſe places are in north latitude, which are on the north ſide of the equator; and thoſe are in ſouth latitude, which are on the ſouth ſide.

245. *The longitude of a place is the diſtance between the geographical meridian of that place and ſome other geographical meridian, which is taken at pleaſure and called the firſt meridian; this diſtance is meaſured by an arc in the equator, that is intercepted between thoſe two meridians.*

It is uſual for geographers to look upon the meridian of their own principal city as the firſt meridian, and to begin their reckoning from thence. Thus the Engliſh geographers reckon the longitude from the meridian of London, the French from that of Paris, the Dutch from that of Amſterdam. If *pifms*, Plat. XXVII, fig. 1, is the geographical meridian of London, all places lying under the ſame ſemicircle *pifms*, have the ſame longitude with London: but all places lying under any other meridian have a different longitude from London; and this difference of longitude is either eaſtern or weſtern. A place at *d*, for inſtance lies under the miridian *pdgs*, to the eaſt of the meridian of London; and the arc *fg* in the equator, which lies between the meridian of London *pifms*, and the meridian of the place *pdgs* meaſures the difference of their longitude: this is an arc of 10°: therefore we ſhould ſay that place differs in longitude from London 10° eaſtward. A place at *y* lies in the meridian *poys* to the eaſt of *pifms*, the arc of the equator intercepted between *pifms* the meridian of London and *poys* the meridian of the place is 100°: therefore we ſhould ſay that the place *y* differs in lon-

longitude from London 100° eaſtward. A place at *x* is in the meridian *pxs* to the weſt of *pfms* the meridian of London, and the arc of the equator intercepted between thoſe two meridians is 20°: therefore we ſhould ſay that the place *x* differs in longitude from London 20° weſtward.

Two places that are in oppoſite ſemicircles of the ſame geographical meridian differ from each other in longitude 180°: for as the two ſemicircles of the ſame meridian cut the equator in oppoſite points, there will be an arc in the equator of 180° between the two oppoſite ſemicircles of the ſame meridian.

246. *The altitude or elevation of one pole and the depreſſion of the other in any place upon the earths ſurface is equal to the latitude of the place.*

Let R, Plat. XXVI. fig. 8, be a place upon the earths ſurface, HO the horizon, PS the axis of the world, P one pole and S the other, EE the celeſtial equator, and *ee* the geographical equator, Z the zenith and N the nadir. Then the latitude of the place is either *e*R the diſtance of the place from the earths equator, or EZ the diſtance of its zenith from the celeſtial equator: for ſince either *e*R or EZ ſubtend the ſame angle ZTE, they are ſimilar arcs, that is, they contain an equal number of degrees. So that if R is London, *e*R its latitude is 51°. 31′, and EZ the diſtance of the zenith of London from E the celeſtial equator is likewiſe 51°. 31′. The arc PO is the altitude of one pole and the arc SH is the depreſſion of the other. Now we affirm that either PO or SH is equal to EZ. ZO is an arc of 90° becauſe it is the diſtance of the zenith from the horizon: and EP is an arc of 90°, becauſe it is the diſtance of the pole from the equator: conſequently ZO is equal to EP. Therefore, if from each of theſe two equal quantities, ZO and PE, we take the ſame common arc ZP, the remainder PO on one ſide will be equal to the remainder EZ on the other ſide, or PO the elevation of the pole is equal to EZ the latitude of the place. Now PS the axis of the world is a right line, which croſſes the plane of the horizon in T the earths center: conſequently the vertical angles PTO and STH are equal, and ſo are the arcs PO and SH. But PO the elevation of one pole is equal to the latitude of the place. Therefore SH the depreſſion of the other pole is likewiſe equal to the latitude. Thus at London the altitude of one pole and the depreſſion of the other is 51° 31′.

From hence we have a method of meaſuring the circumference of the earth. If, for inſtance, in the place where a perſon makes his firſt obſervation, the north pole is elevated 52°, then he is in 52° north latitude.

Let

Let him go on northward till he finds that the same pole is elevated 53°, then he is in 53° north latitude. Consequently the place of his second observation is 1° from the place of his first observation. Therefore, if he measures this one degree and determines how many miles one of these places is due north of the other, he may then find how many miles are contained in the whole circumference. For in the whole circumference of the earth there are 360°: and consequently, if he multiplys the number of miles in one degree by 360, the product will be the number of miles contained in the whole circumference.

A degree of latitude contains 69½ English miles: and if this is multiplyed by 360, the product 24930 is the number of miles contained in the earths circumference. The semidiameter is about a fixth part of the circumference. Therefore we may reckon the earths semidiameter about 4000 miles long.

247. *The elevation of the equator in any place upon the earths surface is equal to the complement of the places latitude.*

EZ, Plat. XXVI. fig. 8, has been shewn, in proposition 246, to be equal to the latitude of the place R. Consequently, since PE is an arc of 90°, ZP is the complement of EZ or the complement of latitude to an arc of 90°. EE is the equator; and EH is the elevation of the equator. This arc EH is equal to ZP. For ZH is an arc of 90°, and PE is likewise an arc of 90°, as in proposition 246. Therefore ZH is equal to PE. Now if from these equal arcs, ZH and PE, we take the same common arc ZE, the remainder EH on one side will be equal to the remainder ZP on the other side, or EH the elevation of the equator is equal to ZP the complement of latitude.

The latitude of London is 51°. 31'. north: the complement of this to an arc of 90° is 38°. 29': therefore in the latitude of London the equator EE makes an angle of 38°. 29'. with the horizon.

248. *The axis of the world is perpendicular to the planes of all the circles, which the heavenly bodies describe in their apparent diurnal motions.*

If PS, Plat. XXVII. fig. 2, is the axis of the world, the rotation of the earth round its own axis will make all the heavenly bodies, the sun, planets, and fixed stars appear to revolve in circles from east to west round this axis PS, by proposition 23. But all bodies, that revolve round an axis, move in planes perpendicular to this axis. Therefore the axis of the world is perpendicular to all the circles of daily motion. Thus in the apparent daily motion of the heavens, a star at *c* will revolve in a circle

cc, a ſtar at *b* in a circle *bb*, a ſtar at *a* in a circle *aa*, a ſtar at E in a circle EQ, a ſtar at *x* in a circle *xx*, a ſtar at *y* in a circle *yy*, and a ſtar at *z* in a circle *zz*: and the axis PS will be perpendicular to the planes of all theſe or any other intermediate circles, that any ſtar or planet can deſcribe in the apparent daily motion of the heavens.

249. *The planes of all the circles of daily motion are parallel to the equator.*

The planes of all the circles of daily motion, *cc*, *bb*, *aa*, *xx*, *yy*, *zz*, Plat. XXVII. fig. 2, are parallel to the plane of the equator EQ. For the axis of the world PS is perpendicular to the planes of all the circles of daily motion, by propoſition 248; and the ſame axis is perpendicular to the plane of the equator, by propoſitions 14, 18. But all places, to which the ſame right line is perpendicular, are parallel to one another. Euc. b. XI. prop. 15. Therefore all the planes of all the circles of daily motion are parallel to the equator.

250. *The axis of the world paſſes through the centers of all the circles of daily motion.*

The axis PS, Plat. XXVII. fig. 2, paſſes through the centers of the circles *cc*, *bb*, *aa*, EQ, *xx*, *yy*, *zz*. For ſince all the heavenly bodies appear every day to move in circles round the axis of the world, by propoſition 23, they will, in every part of this apparent motion, be at the ſame diſtance from the axis; and conſequently the axis PS is in the center of every circle, that a heavenly body can deſcribe by this daily apparent motion.

251. *In all places, that are in the earths equator or have no latitude, the two poles of the world lie in the horizon.*

Places at the earths equator have no latitude, by propoſition 243. But the elevation of one pole above the horizon of any place and the depreſſion of the other below the horizon is equal to the places latitude, by propoſition 246. Therefore in all places that are in the earths equator, one pole will not be elevated above the horizon, and the other will not be depreſſed below it, that is, the two poles will be in the horizon. Thus if T, Plat. XXVII. fig. 3, is the earth, and HO the horizon of a place that is at the earths equator or has no latitude, the two poles of the world will be in the horizon, one at P the other at S.

252. *In*

PLATE XXVI.

Fig. 1.

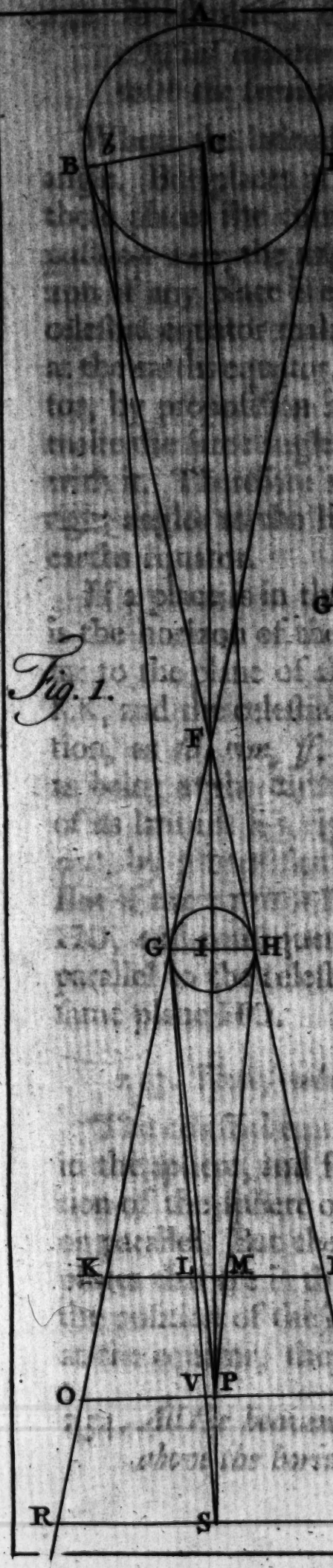


Fig. 2.

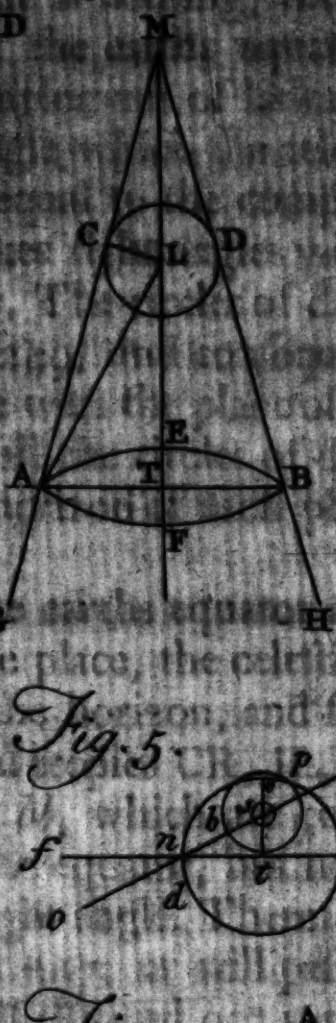


Fig. 3.

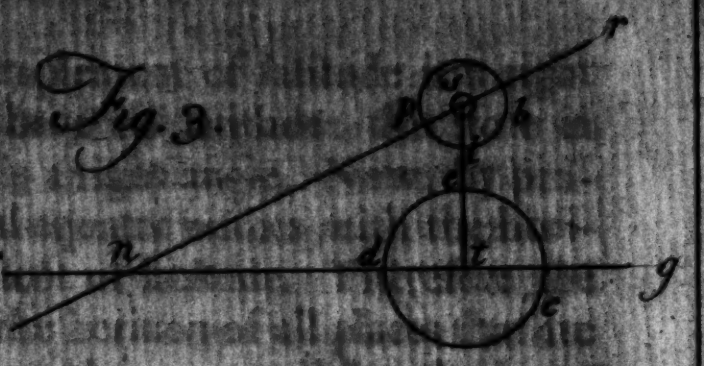


Fig. 4.

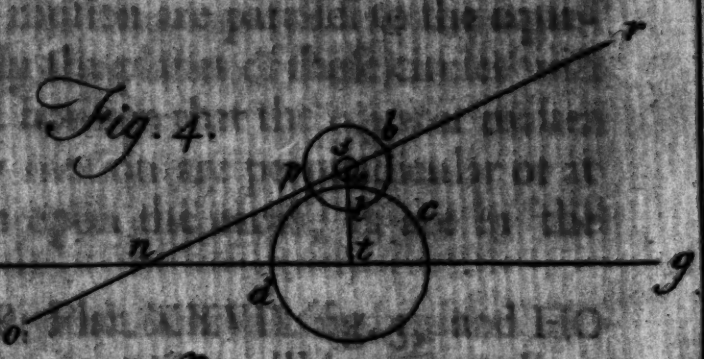


Fig. 5.

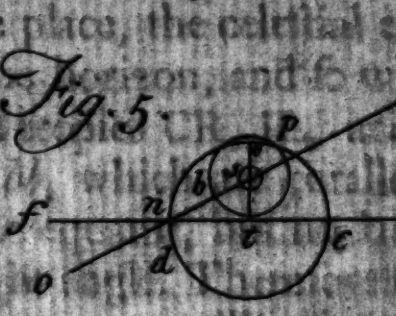


Fig. 6.

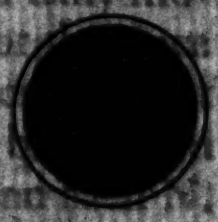


Fig. 7.

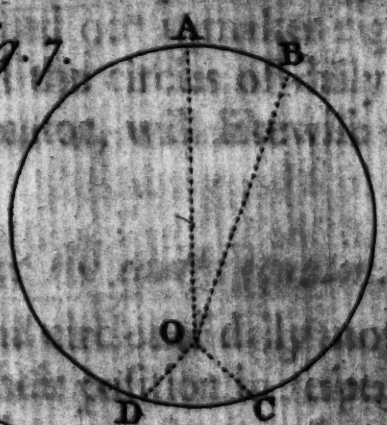


Fig. 8.

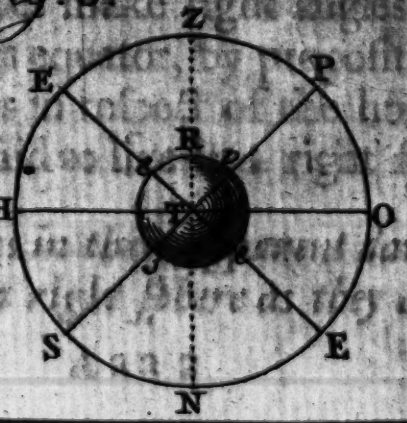
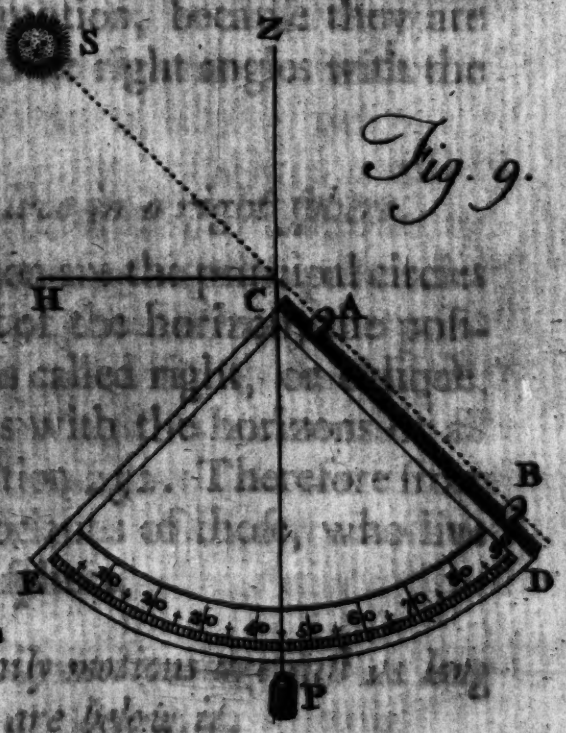
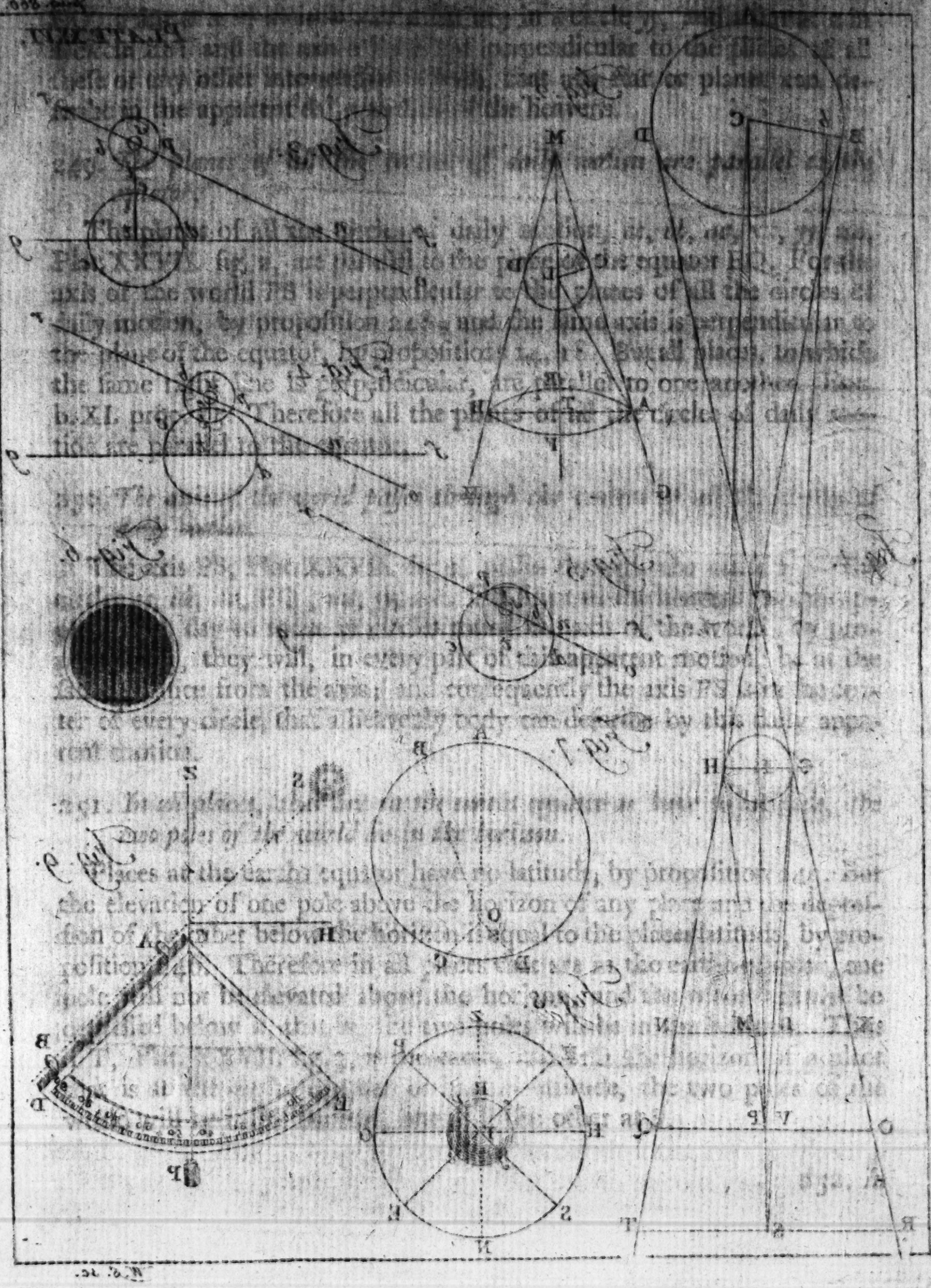


Fig. 9.



W. S. sc.

PLATE XXIV



252. *In all places, that are in the earths equator or have no latitude, the celeſtial equator and all the circles of daily motion make right angles with the horizon.*

Where the latitude is nothing, the complement of latitude is a right angle. But places at the earths equator have no latitude: therefore in thoſe places the complement of latitude is a right angle. Now, by propoſition 247, the angle which the celeſtial equator makes with the horizon of any place is equal to the complement of latitude. Therefore the celeſtial equator makes right angles with the horizon of all places that are at the earths equator. The circles of daily motion are parallel to the equator, by propoſition 249, and conſequently the planes of theſe circles will make the ſame angle with the plane of the horizon that the equator makes with it. Therefore all the circles of daily motion are perpendicular or at right angles to the horizon of ſuch places upon the earth, as are in the earths equator.

If a place is in the earths equator at T, Plat. XXVII. fig. 3, and HO is the horizon of the place, the celeſtial equator EQ will be perpendicular to the plane of the horizon, and ſo will the celeſtial polar circles AA, KK, and the celeſtial tropics CR, IL, and any other circles of daily motion, as *dd*, *mm*, *ff*, *bb*, which are parallel to the equator. For the place as being at the earths equator, has no latitude, ſo that, the complement of its latitude is a right angle. Therefore the equator EQ will be elevated 90°, by propoſition 247, or will paſs through the zenith Z and nadir N. But if the equator is elevated 90° it makes right angles with the horizon HO, and conſequently all the circles of daily motion, becauſe they are parallel to the celeſtial equator, will likewiſe make right angles with the ſame plane HO.

253. *Thoſe, who live at the earths equator, live in a right ſphere.*

The celeſtial equator and circles of daily motion are the principal circles in the ſphere, and from their poſition in reſpect of the horizon, the poſition of the ſphere of the heavens in any place is called right, or oblique, or parallel. But theſe circles make right angles with the horizons of all places that are in the earths equator, by propoſition 252. Therefore from the poſition of theſe circles in reſpect of the horizons of thoſe, who live at the equator, they are ſaid to live in a right ſphere.

254. *All the heavenly bodies in their apparent daily motions are juſt as long above the horizon of a right ſphere as they are below it.*

The poles of the world P and S, Plat. XXVII. fig. 3, are in the horizon HO of a right ſphere, by propoſitions 251, 253. Therefore PTS the axis of the world is in the plane of the horizon. Now the axis of the world paſſes through the centers of all the circles of daily motion. Therefore the plane of the horizon of a right ſphere paſſes likewiſe through all their centers: and conſequently HO the horizon of a right ſphere divides all the circles of daily motion into two equal parts; ſo that one half of any of theſe circles will be above the horizon, and the other half will be below it. But all the heavenly bodies in their apparent daily motion deſcribe one of theſe circles. And ſince any heavenly body will be juſt as long in deſcribing one half of the circle as it is in deſcribing the other half, it will be juſt as long above the horizon of a right ſphere as it is below it.

255. *The days and nights are equal to one another at all times of the year in all places, that are at the earths equator or have no latitude.*

The ſun by its annual motion appears to deſcribe the ecliptic LC, Plat. XXVII. fig. 3, which is inclined to the equator EQ at an angle of 23½ degrees, by propoſitions 27, 31. Now whatever part of the ecliptic the ſun is in, it will appear every day by the earths rotation to revolve from eaſt to weſt in ſome circle of daily motion, by propoſition 23. Thus if the ſun is at L its utmoſt ſouthern limit in the ecliptic, where the ecliptic, by propoſition 34, touches the tropic of capricorn LI, then by the daily motion it will apppear to revolve round the earth in the tropic Ld. If the ſun in the ecliptic is at *b*, it will by the daily motion appear to revolve in the circle *dbd*. If it is at T in the ecliptic or at one of the points where the ecliptic croſſes the equator, its apparent daily motion at that time will be in the celeſtial equator EQ. If it is at *g* in the ecliptic, its apparent daily motion will be in the circle *mgm*. And if it is at C its utmoſt northern limit in the ecliptic where the ecliptic, by propoſition 34, touches the tropic of cancer CR, it will then by the daily motion appear to revolve in the tropic of cancer CR. Therefore ſince the ſun at all times of the year deſcribes ſome circle of daily motion every 24 hours, the ſun, like any other of the heavenly bodies, will, by propoſition 254, be juſt as long above the horizon of a right ſphere as below it: and conſequently to thoſe, who live in a right ſphere or at the earths equator, the days and nights will each of them be 12 hours long at all times of the year.

256. *At the poles of the earth, one pole of the world is in the zenith and the other is in the nadir.*

The

The latitude of either pole of the earth is 90°. Therefore by propoſition 246, one pole of the world will be elevated, and the other will be depreſſed 90°, that is, one pole P, Plat. XXVII. fig. 5. will be in the zenith Z, and the other will be in the nadir N.

257. *At the poles of the earth, the celeſtial equator coincides with the horizon, and all the circles of daily motion are parallel to the horizon.*

At either pole of the earth the latitude is 90°, and the complement of latitude is nothing. But the elevation of the celeſtial equator above the horizon of a place is equal to the complement of the places latitude, by propoſition 247. Therefore, at either pole of the earth, the celeſtial equator EQ, Plat. XXVII. fig. 5, will not be elevated above the horizon H at all, but will coincide with it. Now all the circles of daily motion, ſuch as AA, KK, the two polar circles in the heavens, CR, IL, the two tropics, *bb*, *mm*, *dd*, *ff*, &c. are parallel to the equator, and conſequently, by propoſition 249, where the equator coincides with the horizon, all the circles of daily motion are parallel to the horizon.

258. *A perſon, who is at either pole, is in a parallel ſphere.*

The poſition of the principal circles of the ſphere, in reſpect of his horizon, is the reaſon why he is ſaid to live in a parallel ſphere; for all the circles of daily motion are parallel to his horizon.

259. *In a parallel ſphere all the heavenly bodies are carryed round by the apparent daily motion in circles parallel to the horizon, ſo that thoſe which are above the horizon will never ſet by this motion, and thoſe which are below it will never riſe.*

All the circles of daily motion are parallel to the horizon of a parallel ſphere, by propoſitions 257, 258. Therefore all the heavenly bodies, by the apparent daily motion, muſt deſcribe circles parallel to the horizon. Now a motion, which is parallel to the horizon, cannot make any heavenly body either deſcend below the horizon or aſcend above it: and conſequently the daily motion in a parallel ſphere will not make the heavenly bodies either ſet or riſe. If HO, Plat. XXVII. fig. 5, is the horizon, AA, *bb*, CR, *mm*, *dd*, IL, *ff*, KK, circles of daily motion, theſe circles are all of them parallel to HO, and it is evident, that a motion in theſe circles cannot alter the ſituation of the body in reſpect of HO, it can neither make thoſe bodies riſe, which move in the circles below HO, nor make thoſe ſet, which move in the circles above it.

261. *The*

260. *The day and the night are each of them half a year long at either pole of the earth.*

CL, Plat. XXVII. fig. 5, is the ecliptic: the center of the earth T is the center of the ecliptic, and the center of the earth is likewiſe the center of the horizon HO. But ſince theſe circles have the ſame center T, they divide each other into two equal parts. Therefore half the ecliptic is above the horizon, and half is below it. Now though at either pole of the earth the ſun does not riſe or ſet by the daily motion, as was proved in propoſition 259, yet by the annual motion in the ecliptic it will riſe above the horizon and ſet below it. The celeſtial equator is in the horizon of a parallel ſphere, and conſequently at that time of the year when the ſun is in either point where the ecliptic croſſes the equator, that is, by propoſition 34, at either equinox, the ſun will be in the horizon EQ or HO of a parallel ſphere, and the daily motion will juſt at that time make the ſun revolve in the plane of the horizon. But for one half of the year, the ſun is in CT, that half of the ecliptic, which is above the horizon, and the other half of the year it is in LT, that half of the ecliptic, which is below the horizon. Therefore in a parallel ſphere, the ſun for one half of the year will, by the daily motion, revolve in ſuch circles as are above the horizon, and for the other half year it will revolve in ſuch circles as are below the horizon, that is, in a parallel ſphere, or to a perſon, who lives at either pole of the earth, the ſun will be ſeen for half a year, and then it will be day, and will be hidden for half a year, and then it will be night.

261. *In any place upon the earths ſurface, that is between either of the poles and the equator, one pole of the world will be elevated, and the other depreſſed at angles that are leſs than right ones.*

The place ſuppoſed is not at either pole, and conſequently its latitude is not 90°. Therefore, by propoſitions 246, 256, the elevated pole will not be in the zenith, nor the depreſſed one in the nadir, that is, the elevation of one pole and the depreſſion of the other will not be equal to a right angle. The place ſuppoſed is not at the equator, and conſequently it has ſome latitude. Therefore, by propoſitions 246, 251, the two poles of the world do not lie in the horizon, but one of them will be elevated, and the other will be depreſſed: and we have already ſhewn, becauſe the place is not at either pole of the earth, that the elevation of one pole of the world and the depreſſion of the other will be leſs than a right angle.

The

The latitude of London is 51°. 31′. Let EQ, Plat. XXVII. fig. 4, be the celeſtial equator, and EZ an arc of 51°. 31′. then, by propoſitions 21, 22, Z will be the zenith of London, and HO the horizon: and, by propoſition 246, PO the elevation of one pole of the world, and SH the depreſſion of the other will be 51°. 31′. The place is between the equator and one pole of the earth, or it has ſome latitude but leſs than 90°: therefore one pole is elevated and the other depreſſed at angles which are leſs than right ones.

262. *In all places on the earths ſurface, that are between the equator and either pole, the celeſtial equator will make a leſs angle than a right one with the horizon.*

The place ſuppoſed is not at either pole of the earth, or the latitude of it is leſs than 90°, therefore the complement of latitude is ſomething, and by propoſitions 247, 257, the celeſtial equator will not coincide with the horizon, but will be inclined to it. But then the place ſuppoſed is not at the earths equator, but has ſome latitude: therefore the complement of latitude will be leſs than 90°, and, by propoſitions 247, 252, the celeſtial equator will not make right angles with the horizon.

The latitude of London is 51°. 31′: the complement of this is 38°. 29′. Therefore by propoſition 247, if HO, Plat. XXVII. fig. 4, is the horizon of London, the celeſtial equator EQ will make an angle of 38°. 29′, with the horizon HO.

263. *Thoſe, who live in any place on the earths ſurface between the equator and either pole, live in an oblique ſphere.*

The poſition of the ſphere of the heavens is oblique where the circles of daily motion are oblique to the horizon, that is, where their planes are neither parallel to the horizon nor perpendicular to it. Now, by propoſition 262, in all places on the earths ſurface between the equator and either pole the celeſtial equator is oblique to the horizon, and conſequently, ſince the circles of daily motion, by propoſition 249, are parallel to the equator, they muſt likewiſe in all ſuch places be oblique to the horizon. Therefore in all ſuch places the poſition of the ſphere will be oblique.

If HO, Plat. XXVII. fig. 4, is the horizon of a place that is ſomewhere between the earths equator and either of its poles, the celeſtial equator EQ, and all the other circles of daily motion, as AA, Z*b*, *d*O, *ff*, CR, *mm*, *xx*, IL, *yy*, H*v*, *w*N, KK, will be oblique to the horizon. Therefore in all ſuch places the heavenly bodies from the daily rotation of the

earth

earth will appear to move in circles, that are oblique to the horizon, by propoſition 23.

264. *In an oblique ſphere thoſe heavenly bodies, which have no declination, will be juſt as long above the horizon as they are below it.*

Thoſe heavenly bodies which have no declination are in the celeſtial equator EQ, Plat. XXVII. fig. 4, by propoſition 47. Now I the center of the ſphere is the center both of the horizon and of the celeſtial equator. Therefore the plane of the horizon HO paſſes through the center of the equator, and conſequently divides the equator into two equal parts, of which one half ET is above the horizon, and the other half QT is below it. But ſince a ſtar or planet, which has no declination, is in the equator, it will appear in the daily revolution of the heavens to deſcribe the equator, and therefore for juſt half the time of ſuch a revolution it will be above the horizon, and during the other half of its revolution it will be below the horizon.

265. *When the ſun is in the celeſtial equator, the day will be equal to the night in all places upon the earths ſurface, that are in an oblique ſphere.*

As the ſun deſcribes the ecliptic by its apparent annual motion, it croſſes the celeſtial equator twice every year, by propoſition 34, and conſequently at thoſe times it is in the equator or has no declination. But, by propoſition 264, it will be juſt as long above the horizon of any oblique ſphere as it is below it. Therefore the day will be juſt as long as the night. For this reaſon the two points where the ecliptic croſſes the equator, are called the equinoctial points.

Propoſition 264, which we have made uſe of to prove this, relates only to the horizon of an oblique ſphere. But then, by propoſitions 253 258, 263, all places upon the earths ſurface are in an oblique ſphere, except thoſe which are at the equator or at one of the poles. Now in thoſe places, which are at the earths equator, the day is equal to the night at all times of the year, by propoſition 255, and not only when the ſun is in the equator. And at the two poles, by propoſition 260, the day and night, which are each of them half a year long are not cauſed, by the ſuns apparent daily motion.

266. *In an oblique ſphere, thoſe heavenly bodies, which have declination towards the elevated pole, are longer above the horizon than they are below it; and thoſe, which have declination towards the depreſſed pole, are longer below the horizon than they are above it.*

Let

Let HO, Plat. XXVII. fig. 4, be the horizon, and PS the axis of the world, in this poſition the ſphere is oblique, by propoſitions 262, 263. It is evident that T the middle of the axis is in the plane of the horizon, becauſe T the center of the ſphere is the middle of the axis, and T the center of the ſphere is likewiſe the center of the horizon. Therefore half the axis TP is above the horizon, and the other half TS is below it. If a ſtar or planet has no declination or is in the equator EQ, we have proved, in propoſition 264, that it will be juſt as long above the horizon, as below it. But ſuppoſe the ſtar or planet was not in the equator, but had declination towards the elevated pole P, and let its declination or diſtance from the celeſtial equator be equal to E*m*. Such a ſtar or planet in the apparent daily revolution of the heavens will appear to revolve in the circle of daily motion *mm*. Now the center of this circle is in PT, or in that half of the axis which is above the horizon: and conſequently the horizon HO does not paſs through the center of this circle. But if a plane cuts a circle and does not paſs through its center, the circle will be divided into two unequal parts, and the greater part of the circle will be on the ſame ſide of the plane with its center. Therefore the horizon HO divides this circle *mm* into two unequal parts, the greater part is above the horizon, and the leſſer part is below it: and conſequently a ſtar or planet, which deſcribes this circle *mm* in the apparent daily revolution of the heavens, will be longer in deſcribing the greater part, which is above the horizon, than in deſcribing the leſſer part, which is below it.

If the ſtar or planet had declination towards the depreſſed pole S, and its declination was equal to E*x*, then *xx* would be its circle of daily motion. The center of this circle is in TS the depreſſed half of the axis, and conſequently is below the horizon. Therefore the greater part of the circle *xx* will be below the horizon, and the leſſer part above the horizon: ſo that a ſtar or planet, which in the apparent daily motion deſcribes the circle *xx*, will be longer below the horizon than above it.

267. *Where the latitude of a place in an oblique ſphere is given, the greater a heavenly bodys declination is towards the elevated pole, the longer will be its ſtay above the horizon, and the ſhorter its ſtay below the horizon in its apparent daily motion: and the greater its declination is towards the depreſſed pole, the ſhorter will be its ſtay above the horizon, and the longer its ſtay below it.*

If the latitude of the place is given, the elevation of one pole and the depreſſion of the other is given, by propoſition 246. Let PTO, Plat.

XXVII. fig. 4, be the elevation of the pole P, and STH the depreſſion of the other pole S. We have ſhewn, in propoſition 266, that if EQ is the equator, and the declination of a ſtar or planet is equal to E*m* it will in the apparent daily revolution of the heavens deſcribe the circle *mm*, and will be longer above the horizon than below it, becauſe the center of this circle is in PT that half of the axis of the world, which is above the horizon. But if the declination is EC, the ſtar or planet will ſtay a longer time above and a ſhorter time below the horizon, than it will if its declination is only E*m*. For when the ſtar or planets declination is EC its circle of daily motion is CR. But CR is nearer to the elevated pole than *mm*, becauſe the ſtar or planet has a greater declination, or is farther from the equator EQ, when in the daily motion it deſcribes CR, than when it deſcribes *mm*. But the nearer any circle of daily motion is to the pole P, the farther the center of that circle will be from T, which is the middle of the axis. Now T is in the horizon. Therefore the nearer the circle of daily motion is to the elevated pole the farther the center of that circle will be above the horizon; the center of the circle CR, for inſtance, is farther above the horizon HO, than the center of the circle *mm*. Conſequently the parts, into which the horizon divides a circle of daily motion, will be the more unequal, or the part above the horizon will be greater in proportion to the part below it as that circle is nearer to the elevated pole. Therefore, ſince the greater the declination of a ſtar or planet is towards the elevated pole, the nearer that ſtar or planet will be to the elevated pole, it follows that the ſtar or planet will deſcribe a circle in its apparent daily motion, of which ſo much the greater part will be above the horizon, and ſo much the leſſer part will be below the horizon; and conſequently the greater the declination of a ſtar or planet is towards the elevated pole, ſo much the longer in its daily motion will be its ſtay above the horizon, and ſo much ſhorter its ſtay below the horizon.

For a like reaſon a ſtar or planet will make ſo much a ſhorter ſtay above the horizon and ſo much a longer below it, as its declination towards the depreſſed pole is greater. A ſtar or planet, whoſe declination towards the depreſſed pole S, is equal to EI, will make a ſhorter ſtay above the horizon HO and a longer below it, than it would do if its declination had been only E*x*. For if the ſtars or planets declination is EI, then IL will be its circle of daily motion, and if its declination is E*x*, then *xx* will be its circle of daily motion. The centers of both theſe circles are in the depreſſed half of the axis TS. But IL is farther from the equator and conſequently is nearer to the pole than *xx*. Therefore the center of IL will be farther below the horizon than the center of *xx*, and conſequently the

two

two parts, into which the horizon divides IL, will be more unequal to one another, than the two parts, into which the ſame horizon divides *xx*; or in the circle IL the part, which is above the horizon, will be leſs in proportion to the part, which is below it, than in the circle *xx*. Therefore a ſtar or planet, which in the daily motion deſcribes IL, will make a ſhorter ſtay above the horizon and a longer below it, than a ſtar or planet, which has a leſs declination and deſcribes *xx*.

268. *As the ſuns declination towards the elevated pole encreaſes, the days in an oblique ſphere grow longer, and the nights ſhorter: and as its declination towards the depreſſed pole encreaſes, the days grow ſhorter and the nights longer.*

This follows from propoſition 267. For, by that propoſition, as the ſuns declination towards the elevated pole encreaſes, its ſtay above the horizon, or the day, will be longer, and its ſtay below the horizon, or the night, will be ſhorter. And on the contrary, as its declination towards the depreſſed pole encreaſes, its ſtay above the horizon, or the day, will be ſhorter, and its ſtay below the horizon, or the night, will be longer.

Thus to us who live in north latitude the north pole is elevated, and as the ſun moves in the ecliptic from the vernal equinox to the tropic of cancer, it gets to the north of the equator, or its declination towards our elevated pole encreaſes. Therefore from the vernal equinox, when the days and nights are equal, till the ſun comes to the tropic of cancer, our days lengthen and our nights ſhorten. But when the ſun comes to the tropic of cancer, it is then at its utmoſt northern limit, by propoſition 34, and returns in the ecliptic to the equator again. During this return of the ſun, its declination towards our elevated pole decreaſes, and conſequently the days decreaſe and the nights encreaſe; till the ſun is arrived at the equator again, and is in the autumnal equinoctial point, when the days and nights will be equal again. As the ſun moves from thence towards the tropic of capricorn, it gets to the ſouth of the equator, or its declination towards the depreſſed pole encreaſes. Therefore at that time of the year our days ſhorten and our nights lengthen, till the ſun arrives at the tropic of capricorn. But when the ſun is arrived there, it is then in its utmoſt ſouthern limit, and returns in the ecliptic to the equator again. During this return, its declination towards our depreſſed pole decreaſes, and conſequently the days will lengthen and the nights will ſhorten; till they become equal, when the ſun is come round to the vernal equinoctial point.

269. *When the ſun is at either of the tropics, the length of the days for ſome little time ſcarce changes at all.*

If CL, Plat. XXVII. fig. 4, is the ecliptic, EQ the celeſtial equator, P the north and S the ſouth pole of the world; then the ſun at C will be in that part of the ecliptic, which touches the tropic of cancer CR: at that time its declination EC is $23^{\circ}\frac{1}{2}$ towards the north. But though the circle CL touches the tropic CR only in a point, yet theſe circles for ſome ſpace about the point of contact will be ſo near to one another, that apparently, or as to ſenſe, they will touch one another. Now whilſt the ſun is moving through that part of the ecliptic, which as to ſenſe touches the tropic, its declination as to ſenſe will continue the ſame. But whilſt the ſuns declination continues the ſame, the length of the days will not change, by propoſition 268. Therefore, when the ſun is at the tropic of cancer, the length of the days for ſome little time will ſcarce change at all.

The ſame may be ſhewn in the ſame manner, when the ſun is at L, that point of the ecliptic which touches the tropic of capricorn IL.

From the ſuns thus ſeeming *ſtare* or to ſtand ſtill, without ſenſibly changing its declination, when it is in either of the points, where the ecliptic touches the tropics, have been called the ſolſtitial points.

271. *Any heavenly body, that is in the celeſtial equator, has no amplitude either at riſing or ſetting.*

Any ſtar or planet that is in the celeſtial equator deſcribes the equator in its daily motion, and conſequently riſes in that point where the equator cuts the horizon on one ſide, and ſets in that point where the equator cuts the horizon on the other ſide. Now the equator is at right angles to the meridian, by propoſition 230. Therefore the two points where the equator cuts the horizon are 90° diſtant from the two points where the meridian cuts it. But the meridian cuts the horizon in the cardinal ſouth and north points, by propoſition 23. Therefore the equator cuts the horizon in the cardinal eaſt and weſt points. And conſequently a ſtar or planet, that riſes in one point of interſection and ſets in the other, riſes due eaſt and ſets due weſt; or has no amplitude either at riſing or ſetting. From hence it follows, by propoſitions 34, 265, that the ſun at the equinoxes riſes due eaſt and ſets due weſt.

272. *Any heavenly body is at its greateſt altitude, when it is in one ſemicircle of the meridian, and is the moſt depreſſed, when it is in the other ſemicircle.*

The

The meridian cuts the horizon into two parts, one lies on the eaſt ſide of it and the other on the weſt ſide. Any ſtar or planet, by propoſition 23, riſes on the eaſt ſide and ſets on the weſt ſide of the meridian. When the ſtar or planet is riſing, it has no altitude; and again, when it is ſetting, it has no altitude. Therefore whilſt it is up, it is at its greateſt altitude, when it is in the middle between riſing and ſetting, or in the middle between the eaſt and weſt ſide of the meridian. But the meridian itſelf is in the middle between the eaſt and weſt ſides of it. Therefore the ſtar is at its greateſt altitude, when it comes to the meridian.

A ſtar or planet ſets on the weſt ſide of the horizon, and riſes on the eaſt ſide. When it is ſetting it has no depreſſion, and when it is riſing it has no depreſſion. Therefore, whilſt it is below the horizon, it will be depreſſed the moſt, when it is in the middle between ſetting and riſing, or when it is in the middle between the weſt and eaſt ſides of the horizon. But as the meridian is in the middle between theſe two ſides above the horizon, ſo it is in the middle between them below it. Therefore as the ſtar or planet is at its greateſt altitude, when it is on one ſide of the meridian above the horizon, ſo it is at its greateſt depreſſion, when it is on the other ſide of the meridian below it.

If Z, Plat. XXVII. fig. 4, is the zenith of a place, N the nadir, P and S the two poles of the world, then ZPNS is the meridian. Any ſtar or planet, which deſcribes CR in its daily motion, will be at its greateſt altitude when it is at C in the ſemicircle PCS of the meridian, and will be at its greateſt depreſſion, when it is at R in the oppoſite ſemicircle PRS. The reader will here obſerve, that, when we conſider the meridian as divided into two ſemicircles PCS and PRS, it is the axis of the world PS, which makes this diviſion.

273. *The meridian altitude of any heavenly body is its altitude when it comes to the meridian.*

274. *At equal diſtances of time before noon and after it the ſun is at equal altitudes.*

It is noon, when the ſun is in the middle between the eaſtern and weſtern ſides of the horizon, that is, when it is in the meridian. Therefore it takes up juſt the ſame time to riſe from the eaſtern horizon to the meridian, as it does to deſcend from the meridian to the weſtern horizon; and conſequently in any given time it will aſcend juſt as far on one ſide, as in an equal time it will deſcend on the other ſide of the meridian. Therefore whatever is the ſuns diſtance from the meridian on the eaſtern ſide at any time before noon, it will be at the ſame diſtance from

from it on the weſtern ſide at an equal time after noon. But ſince the meridian altitude of the ſun is its greateſt altitude, by propoſition 272, it follows, that at equal diſtances from the meridian its altitudes will be equal. Therefore, whatever is the ſuns altitude at any given diſtance of time before noon, its altitude will be the ſame again at an equal diſtance of time after noon.

275. *The ſhadow of a ſtyle or gnomon, which is perpendicular to the horizon, falls due north from the ſtyle at noon: and conſequently, a line drawn from the ſtyle along its ſhadow at that time is a meridian line.*

Let EDMC, Plat. XXVII. fig. 6, be an horizontal plane, upon which a ſtyle or gnomon is fixed perpendicular to the plane. When the ſun ſhines upon this ſtyle, the ſhadow of it will fall oppoſite to the ſun. And ſince the ſun is in the meridian or due ſouth at noon, the ſhadow of the ſtyle will at that time fall due north. The difficulty then will be to determine the inſtant of noon, in order to find out which way the ſhadow falls at that inſtant. For this purpoſe let an obſervation be made at any time before noon; and ſuppoſe the length and direction of the ſhadow at the time of obſervation to be AD; then making AD the ſemidiameter and the foot of the ſtyle the center ſtrike the circle DMCE. Obſerve again in the afternoon, when the ſhadow AC is a ſemidiameter of the ſame circle, for then the ſhadow at the time of the ſecond obſervation will be juſt of the ſame length that it was at the time of the firſt obſervation. Now as the ſhadow is of the ſame length at the time of both obſervations, it follows that the ſun was juſt at the ſame height at the time when they were both made: therefore, by propoſition 274, the time of the firſt obſervation was juſt as long before noon as the time of the ſecond is after noon. But if at any given time before noon the ſhadow of the ſtyle fell in the line AD, and at an equal diſtance of time after noon it fell in the line AC, then at the inſtant of noon it muſt fall in the middle between AD and AC. Therefore if we divide the angle DAC or the arc DC into two equal parts, and draw a line AM which does thus divide them, the ſhadow of the ſtyle at noon muſt fall upon this line, and conſequently the line AM is drawn from ſouth to north upon a horizontal plane, and for that reaſon it is a meridian line.

276. *If the ſun has declination towards the elevated pole, its meridian altitude is equal to its declination added to the elevation of the celeſtial equator: and if it has declination towards the depreſſed pole, its meridian altitude is equal to its declination ſubtracted from the elevation of the equator.*

Let

Let HO, Plat. XXVII. fig. 7, be the horizon, T the earth, P and S the two poles of the world, Z the zenith, N the nadir, EE the equator. Then if the ſun has declination towards the elevated pole P, and is at C; when it comes to the meridian PZSN, its meridian altitude CH is equal to CE its declination added to EH the elevation of the equator. If it has declination towards the depreſſed pole S, and is at D; when it comes to the meridian, its meridian altitude DH is equal to EH the elevation of the equator after DE the declination is ſubtracted from it.

From hence we may tell what is the ſuns greateſt and leaſt meridian altitudes in the latitude of London. The ſuns greateſt declination towards the elevated pole at the ſummer ſolſtice, and towards the depreſſed pole at the winter ſolſtice is 23°. 30′, by propoſition 34. In the latitude of London the elevation of the celeſtial equator is 38°. 29′. Therefore the ſuns greateſt meridian altitude at the ſummer ſolſtice is 38°. 29′ + 23°. 30′ = 61°. 59′: and its leaſt meridian altitude at the winter ſolſtice is 38°. 29′ — 23°. 30′ = 14°. 59′.

277. *If the declination of a heavenly body towards the elevated pole is equal to the latitude of a place, the body will paſs through the zenith of that place; and if its declination towards the depreſſed pole is equal to the latitude, it will paſs through the nadir.*

If HO, Plat. XXVII. fig. 4, is the horizon of any place, PS the axis of the world, EQ the celeſtial equator, Z the zenith, and N the nadir: then if a ſtar or planet paſſes through Z its declination from EQ the equator towards P the elevated pole muſt be equal to the places latitude. For any ſtar or planet, which paſſes through Z in the daily rotation of the heavens muſt deſcribe the circle Z*b*, ſo that its diſtance from the equator or its declination muſt be EZ. But EZ is likewiſe the diſtance of the places zenith from the equator, and this, by what has been ſhewn under propoſition 246, is equal to the places latitude. Therefore if a ſtar or planet paſſes through the zenith Z of any place in the daily motion of the heavens, its declination EZ towards the elevated pole P is equal to the latitude of the place.

In like manner any ſtar or planet, which in the daily motion deſcribes the circle *w*N, will paſs through the places nadir. But if a ſtar or planet in the daily motion deſcribes the circle *w*N, its declination from the equator EQ towards the depreſſed pole S, muſt be NQ. Now NQ is equal to the diſtance of the places nadir from the equator EQ, and this diſtance is equal to the places latitude. Therefore if a ſtar or planet paſſes through

through the nadir of a place, its declination towards the depreſſed pole is equal to the places latitude.

278. *Any heavenly body ſeen from a place upon the earths ſurface will never ſet in the daily motion of the heavens, if the complement of its declination towards the elevated pole is equal to the latitude or leſs than the latitude of the place. And any heavenly body will never riſe, if the complent of its declination towards the depreſſed pole is equal to the latitude or leſs than the latitude.*

If in the daily motion of the heavens any ſtar or planet deſcribes the circle *d*O, Plat. XXVII. fig. 4, it will never in any part of its daily revolution be below the horizon HO: for when it is at its meridian altitude *d* in one ſemicircle of the meridian it is above the horizon, and when it is at its leaſt altitude O in the oppoſite ſemicircle, it only touches the horizon without going below it. But if HO is the horizon and PS the axis of the world, the elevation of the pole is PO: and it is plane, if P*d* or PO the ſtars or planets diſtance from P the elevated pole is equal to PO the elevation of the pole, that then the ſtar or planet and the horizon will each of them be at the ſame diſtance PO from the elevated pole: conſequently when the ſtar or planet is at its leaſt altitude, it will be no farther from the pole than the horizon is: therefore at its leaſt altitude it will not be below the horizon. Now ſince PE the diſtance of the pole P from the equator EQ is 90°, if any ſtar or planet is any where between the pole and the equator, as at *d*, its diſtance from the pole *d*P is the complement of its declination E*d*. And the elevation of the pole is equal to the latitude, by propoſition 246. Therefore ſince a ſtar or planet will never ſet if its diſtance from the elevated pole is equal to the elevation; we may ſay it will never ſet if its complement of declination towards the elevated pole is equal to the latitude; for this is only ſaying the ſame thing in different words.

Now if a ſtar or planet will never ſet, when the complement of its declination towards the elevated pole is equal to the places latitude; it is evident that it could never ſet, if the complement of its declination was leſs than the latitude. The complement of declination is the diſtance of the ſtar or planet from the pole; and the latitude of a place is equal to the poles elevation. Therefore if the complement of declination towards the elevated pole is leſs than the latitude; the diſtance of the ſtar or planet from the elevated pole is leſs than the poles elevation, that is, the ſtar or planet is nearer to the elevated pole than the horizon is. Conſequently the ſtar or planet will deſcribe ſome ſuch circle in the daily motion round

round the elevated pole as Zb or AA, the loweſt part of which does not reach to the horizon HO: and in deſcribing ſuch a circle as this it can never ſet or go below the horizon.

In like manner, if the complement of declination of a ſtar or planet towards the depreſſed pole S is equal to the latitude of a place upon the earths ſurface, ſuch a ſtar or planet will never riſe, or will never be ſeen above the horizon of that place. The complement of declination towards the depreſſed pole is the diſtance from that pole, and the latitude of the place is equal to the depreſſion of the pole. Therefore, if the complement of declination is equal to the latitude, the ſtars or planets diſtance from the depreſſed pole is equal to the depreſſion of the pole. Now if HO is the horizon and S the depreſſed pole, HS will be the poles depreſſion: and if a ſtars or planets diſtance from the depreſſed pole is HS equal to this depreſſion ſuch a ſtar or planet in the daily motion will deſcribe a circle Hv round the pole. But ſince H in this circle is as far from the pole S as the horizon is; when the ſtar or planet in its daily revolution comes to H, it will only touch the horizon without riſing above it.

If the complement of declination towards the depreſſed pole is leſs than the latitude, then the ſtar or planet will be ſtill nearer to the pole S, and in the daily motion will deſcribe ſome circle as wN, KK, which is nearer to the depreſſed pole than the horizon H is; and conſequently a ſtar or planet in ſuch a ſituation can never riſe above the horizon.

In the latitude of London 51°. 31′ north, thoſe fixed ſtars about the north pole, whoſe diſtance from the pole or whoſe complement of declination towards the north pole is equal to 51°. 31′. or leſs than this will never ſet. And ſuch ſtars as are at the ſame diſtance from the ſouth pole will never riſe or will never be ſeen in our latitude.

279. *The latitude of a place upon the earths ſurface may be determined, by finding the greateſt and leaſt altitude of a fixed ſtar that never ſets.*

The latitude of a place is equal to the elevation of the pole, by propoſition 246, and conſequently, if the altitude of the pole is determined, the latitude is determined too. Now if there was any fixed ſtar exactly in the pole, it would be eaſy to determine the poles altitude: for let HO, Plat. XXVII. fig. 7, be the horizon, and P the elevated pole; if there was any fixed ſtar in the heavens exactly at P, we need only take the altitude of that ſtar, by propoſition 234, and the altitude PO thus found would be the elevation of the pole. But there is no ſtar that is exactly either in the north pole or in the ſouth pole. That, which we call the north polar ſtar, is not at P exactly in the pole, but at p a little on one ſide of

the pole. And there is no ſtar ſo near to the ſouth pole as this is to the north pole. Conſequently as there is no ſtar at either pole, there is nothing that can mark out to our ſight the very point in the heavens where the pole is; and for want of ſuch a viſible mark or object we cannot make uſe of this ready method to determine the poles elevation.

But there are ſome ſtars near each pole, of which, by propoſition 278, thoſe about the north pole will be always viſible in north latitude, and thoſe about the ſouth pole will be always viſible in ſouth latitude. Let A be one of theſe ſtars, which in the daily motion deſcribes the circle AB round the pole P without ſetting, ſo that it may be ſeen both when it is at A its greateſt, and when it is at B its leaſt altitude. Now by propoſition 272, when it is either at A its greateſt, or at B its leaſt altitude, it is in the plane of the meridian. Therefore if a quadrant is placed along a meridian line, ſo as to be in the plane of the meridian, and the ſtar, when at A, appears through the ſights of that quadrant, and, when at B, may be ſeen again through the ſights of the ſame quadrant, then its greateſt altitude AO and its leaſt altitude BO is determined. The difference between AO and BO is the difference between the ſtars greateſt and leaſt altitudes. This difference is AB. Now ſince the ſtar in the daily motion has deſcribed a circle round the pole, by propoſition 23, we are ſure that the ſtar both at its greateſt and leaſt altitudes, or at A and B, was at the ſame diſtance from the pole, which is in the middle of that circle: and conſequently if AB is divided into two equal parts PA and PB, P, which ſo divides AB, is the pole. But we want to find how far this point P is from O the horizon; for this diſtance PO is the poles elevation. Now we know the length of the arc BO, for it is the ſtars leaſt altitude; and we know the length of the arc AO, for it is the ſtars greateſt altitude; and thence, we know the length of the arc AB, for it is the difference between the ſtars greateſt and leaſt altitudes; and conſequently we know the length of the arcs PB and PA, for either of them are half that difference. But PO is equal to PB added to BO. Therefore if PB half the difference of the ſtars greateſt and leaſt altitudes is added to BO its leaſt altitude, the ſum is PO the poles elevation. Or otherwiſe PO is equal to AO when PA is taken from it. Therefore if from AO the ſtars greateſt altitude we take PA half the difference between its greateſt and leaſt altitudes, the remainder PO will be the poles elevation.

280. *The latitude of a place upon the earths ſurface may be determined from the ſuns meridian altitude and declination.*

If the elevation of the celeſtial equator at any place upon the earths ſurface can be found, this will determine the places latitude: for, by pro-

po-

poſition 247, the elevation of the equator is the complement of latitude, and conſequently the latitude is the complement of the elevation of the equator. If HO is the horizon, Plat. XXVII. fig. 7, and EE the equator; then by determining the elevation of the equator above the horizon, or by determining the arc EH, we know the places latitude, for the latitude is the complement of this. Let the ſuns meridian altitude be found by a quadrant; then if the ſun has declination towards the elevated pole, and its meridian altitude is CH, this altitude CH is equal to CE its declination added to EH the elevation of the equator, by propoſition 276. Therefore if from CH the meridian altitude we take CE the declination, when the ſun has declination towards the elevated pole P, the remainder EH will be the elevation of the equator. But if the ſun has declination towards the depreſſed pole S and its meridian altitude is DH, then DH, by propoſition 276, is equal to EH the elevation of the equator after ED the declination is taken from it, and conſequently if to DH the meridian altitude of the ſun we add ED its declination, the ſum will be EH the elevation of the equator.

Now the ſuns declination is different at different times of the year, by propoſition 34. But then as the ſuns motion, by which its declination varys, is regular, the declination may be determined at all times. And there are tables of its declination calculated for every day of the year. Therefore by the help of a quadrant for finding the ſuns meridian altitude, and of ſuch tables for knowing its declination, the elevation of the equator at any place, and conſequently the latitude of the place, which is the complement of this elevation, may be determined.

281. *The two tropics and the two polar circles upon the earths ſurface divide it into five parts, which are called zones.*

As much of the earths ſurface as is included between the two tropics is called the torrid zone. As much as is included between the tropic of cancer and the arctic polar circle is one of the temperate zones, and as much as is included between the tropic of capricorn and the antarctic polar circle is the other temperate zone. One frigid zone is included within the arctic, and the other within the antarctic polar circle.

282. *In any place, that is within the two tropics upon the earths ſurface, the ſun is vertical, or paſſes through the zenith, twice every year.*

As the ſun paſſes in the ecliptic from the celeſtial equator to the tropic of cancer, its north declination encreaſes from nothing to 23 $\frac{1}{2}$ degrees, by propoſition 34, ſo that, whilſt it is in this part of the ecliptic, it has

every intermediate declination towards the north pole, between nothing and 23 ½ degrees. Now as the tropic of cancer upon the earths ſurface is 23 ½ degrees from the earths equator towards the north, all places that lie between the equator and this tropic, have ſome intermediate north latitude between nothing and 23 ½ degrees, and in all thoſe places the north pole will be elevated, by propoſition 261. Conſequently, whilſt the ſun is paſſing from the equator to the tropic of cancer, its declination towards the elevated pole will be equal to the latitude of all places on the earths ſurface between the equator and the tropic. Therefore, by propoſition 277, at ſome time or other during this part of its motion it will paſs through the zeniths of all theſe places. But whilſt the ſun is moving back again in the ecliptic, from the tropic of cancer to the celeſtial equator, its declination towards the north pole decreaſes from 23 ½ degrees to nothing. Therefore in its return, its declination towards the elevated pole will again be equal to the latitude of all ſuch places, as lie upon the earths ſurface between the tropic of cancer and the earths equator, and conſequently, by propoſition 277, it will paſs a ſecond time through the zeniths of all thoſe places.

In like manner we might ſhew, that whilſt the ſun is paſſing in the ecliptic from the celeſtial equator to the tropic of capricorn, its declination towards the ſouth pole will be equal to the ſouth latitude of all ſuch places upon the earths ſurface, as lie between the earths equator and the tropic of capricorn; and that, in its return from the tropic of capricorn to the celeſtial equator, its ſouth declination will again be equal to their latitude. Conſequently, by propoſition 277, it will paſs through all their zeniths once in going and once in returning.

283. *The ſun is vertical once every year in ſuch places upon the earths ſurface as are in either of the tropics.*

The ſuns declination towards the north pole once every year is 23 ½ degrees, by propoſition 34. The latitude of ſuch places as are in the tropic of cancer upon the earths ſurface is 23 ½ degrees north, by propoſition 243. Therefore the ſuns declination towards the north or elevated pole will once every year be equal to their latitude, and conſequently, by propoſition 277, the ſun at that time of the year, and at no other, will paſs through the zenith or vertex of thoſe places.

The ſuns declination towards the ſouth pole once every year is 23° ½, by propoſition 34. The latitude of ſuch places upon the earths ſurface as are in the tropic of capricorn is 23° ½ ſouth, by propoſition 243. Therefore the ſuns declination towards the ſouth or elevated pole will once every

every year be equal to their latitude; and consequently, by proposition 277, the sun at that time of the year, and at no other, will be vertical to those places.

284. *The longest day in summer and the longest night in winter is 24 hours, in such places on the earth as are in either polar circle.*

The latitude of each polar circle is 66 ½ degrees, by proposition 243. When the sun is in the tropic of cancer, its north declination is 23 ½ degrees, and its complement of declination 66 ½ degrees. Therefore at the day of the summer solstice, when the sun is in the tropic of cancer, the complement of its declination towards the pole, that is elevated in such places as are in the arctic polar circle, will be equal to the latitude of those places; and consequently, by proposition 278, the sun in those places will not set during the summer solstice, but that day will be 24 hours long. When the sun is in the tropic of capricorn, its south declination is 23 ½ degrees, and the complement of declination 66 ½ degrees. Therefore at the night of the winter solstice, when the sun is in the tropic of capricorn, the complement of its declination towards the pole, that is depressed in such places as are in the arctic polar circle, will be equal to the latitude of those places, and consequently the sun, by proposition 278, will not rise there during the time of the winter solstice, but that night will be 24 hours long.

In the same manner we might shew, that in the antarctic polar circle the day will be 24 hours long, when the sun is in the tropic of capricorn, and the night 24 hours long, when the sun is in the tropic of cancer.

285. *The longest day and the longest night are either of them more than 24 hours in all places that are in the frigid zones.*

From the time that the suns complement of declination towards the elevated pole is equal to the places latitude, till the sun in returning from the tropic has a complement of declination greater than the latitude, it will be continual day: for the sun during that time will not set below the horizon, by proposition 278. In like manner from the time, when the suns complement of declination towards the depressed pole becomes equal to the latitude of any place in the frigid zone, till the sun returns from the tropic, and the complement of declination becomes greater than the latitude, the sun will not rise, but it will be continual night, by proposition 278. Now the latitude of all places in the frigid zones is greater than 66 ½ degrees, by proposition 243. And the suns declination,

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which it has in the tropic is $23\frac{1}{2}$, and its complement of declination is $66\frac{1}{2}$. Therefore before the ſun comes to the tropic, or before its declination is $23\frac{1}{2}$ degrees, or whilſt its complement of declination is greater than $66\frac{1}{2}$ degrees, this complement will be equal to the latitude of ſuch places as are in the frigid zones. At that time, when the ſun has declination towards the elevated pole, the day will begin, and when it has declination towards the depreſſed pole, the night will begin. And the day in one caſe and the night in the other will continue from that time till the ſun comes to the tropic, and returns far enough back from thence towards the equator to make the complement of its declination greater than the latitude of the place.

We may apply this to a place ſituated at one of the poles, the north pole for inſtance, and ſhew that the longeſt day and likewiſe the longeſt night there will laſt for half a year, as in propoſition 260. Whilſt the ſun is moving towards the north pole, the day will begin, and the ſun will appear above the horizon, as ſoon as the complement of its declination is equal to the places latitude. But the latitude of a place at the north pole of the earth is 90°. Therefore the day will begin as ſoon as the ſuns complement of declination is 90°. Now the complement of declination is 90°, when the declination is nothing, that is, when the ſun is in the celeſtial equator. Conſequently, the day will begin at the vernal equinox. From this time, till the ſun gets to the tropic of cancer, its declination towards the elevated pole encreaſes, and conſequently its complement of declination decreaſes. Therefore, from the vernal equinox till the ſummer ſolſtice, the complement of declination will be leſs than the latitude of a place at the north pole of the earth. From the ſummer ſolſtice to the autumnal equinox the ſuns northern declination decreaſes, and the complement of its declination encreaſes; but, till the autumnal equinox, when it comes to the equator, and its declination is nothing, the complement of declination will be leſs than 90°, and at the autumnal equinox it will be equal to 90°. Therefore from the vernal equinox to the autumnal, the complement of the ſuns declination towards the north pole is either equal to or leſs than 90°, which is the latitude of a place at the north pole of the earth. From hence it follows, by propoſition 278, that during this half year the ſun will have appeared conſtantly above the horizon, or for this half year it will have been day at that place.

In like manner we may ſhew, that from the autumnal equinox to the vernal, the complement of declination towards the ſouth pole will be equal to or leſs than 90°. But at the north pole of the earth the ſouth pole is depreſſed, and the latitude of the north pole is 90°. Therefore

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for this half year the complement of declination towards the depreſſed pole will be equal to or leſs than the latitude; from whence it follows, by propoſition 278, that for this half year the ſun will not have appeared above the horizon, or it will have been night at a place ſituated upon the earths ſurface in the north pole.

286. *The ſun is never vertical to any place in either of the temperate zones.*

The temperate zones are on the outſide or beyond the tropics, by propoſition 281, and conſequently the latitude of all places in either of the temperate zones is greater than 23 ½ degrees, by propoſition 243. Now, by propoſition 277, the ſun is never vertical to any place unleſs its declination is equal to the places latitude, but the ſuns declination either to the north or to the ſouth is never more than 23 ½ degrees, by propoſition 34. Therefore its declination is never equal to the latitude of any place within either of the temperate zones.

287. *The longeſt day in ſummer, and the longeſt night in winter, in either of the temperate zones, is leſs than 24 hours.*

That is, the ſun never continues either above or below the horizon of any place in the temperate zones for 24 hours, or for the time of one apparent daily rotation. The ſun never continues above the horizon without ſetting in the time of a daily rotation, unleſs its complement of declination towards the elevated pole is equal to or leſs than the latitude of the place: and it never continues below the horizon without riſing for the time of a daily rotation, unleſs its complement of declination towards the depreſſed pole is equal to the latitude, by prop. 278. But the ſuns declination towards either pole is never greater than 23 ½ degrees, by propoſition 34, and conſequently its complement of declination towards either pole is never leſs than 66 ½ degrees. Now as all places in the temperate zones are on the outſide of the polar circles, and the latitude of the polar circles is 66 ½ degrees, it follows that the latitude of all places in the temperate zones is leſs than 66 ½ degrees: and conſequently the ſuns complement of declination is never equal to the latitude of any places in the temperate zones.

288. *The longeſt day in ſummer and the longeſt night in winter, at any place upon the earths ſurface, will be longer as the place is nearer to either of the poles.*

The longeſt day at one time of the year, and the longeſt night at the other time of the year, is only 12 hours at all places that are in the earths

earths equator, by propoſition 255: they are either of them equal to 24 hours at the polar circles; and are either of them equal to half a year at the earths poles. Thus as the place is farther from the equator and nearer to the poles the longeſt day at one time of the year and the longeſt night at the other time of the year encreaſes.

289. *The ſun in a circle of daily motion moves at the rate of* 15°, *in one hour.*

The ſun apparently revolves in a circle of daily motion upon account of the earths rotation round its axis, by propoſition 23. But upon account of the progreſs, which the ſun makes in the ecliptic every day, it does not complete an apparent revolution in reſpect of any place upon the earths ſurface in the time of one rotation of the earth, by propoſition 24, for the time of the earths rotation is 23 hours 56 minutes, and if the ſun ſets out from the meridian of any place it takes up 24 hours to return to the ſame meridian again. Thus in the daily motion the ſun apparently deſcribes a circle in 24 hours. That we may ſpeak the more intelligibly of this matter, we will neglect the reaſon why the ſun thus moves at all, and the reaſon why 24 hours is the time of a complete revolution, and will conſider this whole apparent motion of the ſun as if it was real, that is, we will ſpeak of this matter as if the ſun by a real motion of its own, which I here call the daily motion, deſcribed a circle every day in 24 hours. For the appearance will be exactly the ſame, whether it is owing to ſuch a real motion of the ſun, or to the cauſes already aſſigned for it, in propoſitions 23, 24.

Now as the ſun every 24 hours deſcribes a complete circle of its daily motion, in one hour it will deſcribe the 24th part of ſuch a circle. But every circle conſiſts of 360°. and conſequently the 24th part of a circle contains the 24th part of 360° or 15°. Therefore the ſun in a circle of daily motion apparently moves at the rate of 15° in one hour.

290. *The ſun in a circle of daily motion moves at the rate of* 15′ *in one minute of time.*

In a degree of a circle there are 60′; or a minute of a circle is the 60th part of one degree: conſequently 15′ is the 60th part of 15°. But the ſun in a circle of daily motion moves at the rate of 15°. in one hour, by propoſition 289. Therefore it will move at the rate of 15′ in the 60th part of an hour. But a minute of time is the 60th part of an hour. Therefore in a minute of time the ſun moves 15′ in a circle of its daily motion.

291. *The*

291. *The ſun in a circle of daily motion moves at the rate of 15″ in one ſecond of time.*

In a minute of a circle there are 60″; or a ſecond of a circle is the 60th part of a minute of a circle: conſequently 15″ is the 60th part of 15′. But the ſun in a circle of daily motion moves at the rate of 15′ in a minute of time, by propoſition 290. Therefore it will move at the rate of 15″ in the 60th part of a minute of time. But a ſecond of time is the 60th part of a minute of time. Therefore in a ſecond of time the ſun moves 15″ in a circle of daily motion.

292. *In places, which differ in longitude, the hour of the day is different.*

It is noon or 12 o' clock at any place, when the ſun is in the meridian of that place. But the ſun cannot be in the meridians of all places at the ſame time; and conſequently it will not be noon or 12 o'clock in all places at the ſame time. At London, for inſtance, it is 12 o'clock at noon, when the ſun in its daily motion comes to the meridian of London. But the ſuns daily motion is from eaſt to weſt. And conſequently, before it comes to the meridian of London, it muſt have paſſed the meridians of ſuch places as lie to the eaſt of London: ſo that, when it is 12 o'clock at noon in London, it will be paſt 12 o'clock at noon in ſuch places as are under any meridian, which is to the eaſt of the meridian of London. Therefore the hour of the day at London will be different at one time, and conſequently at all other times, from the hour of the day in all ſuch places as are to the eaſt of it. In like manner, when the ſun in its daily motion is come to the meridian of London, it will have ſtill farther to move before it comes to the meridian of any place, that lies to the weſt of London: ſo that, though it is 12 o'clock at noon in London, it will not be 12 o'clock at noon, till ſome time after, in ſuch places as are to the weſt of London. Therefore the hour of the day at London will be different at one time and conſequently at all other times from the hour of the day in all ſuch places as are to the weſt of it. Thus we have ſeen that in all places either to the eaſt or to the weſt of London, that is, by propoſition 245, in all places, which differ from London in longitude, the hour of the day will be different. And the ſame might be ſhewn in the ſame manner of any other place as well as of London.

293. *A difference in longitude of 15° makes a difference of one hour in the time of the day.*

If two places differ 15° in longitude, the meridian of one is 15° diſtant from the meridian of the other, by propoſition 245. Theſe

15° may be reckoned either in the equator or in any circle, that is parallel to the equator, as all the circles of daily motion are, by propoſition 249. Now the ſun is one hour in moving 15°, by propoſition 289. Therefore it will be in the meridian of one of the places an hour before it is in the meridian of the other: and conſequently it will be noon at one of the places an hour before it is noon at the other. But if there is an hours difference in the time of the day at noon, there will be an hours difference likewiſe at any other time of the day.

294. *If two places differ 1 minute in their time of the day, they differ 15′ in longitude.*

A difference of one hour in the time of the day will ariſe from 15° difference in longitude, by propoſition 293. Therefore if two places differ in longitude the 60th part of 15°, they will differ in their account of time the 60th part of an hour, which is 1 minute of time.

295. *If two places differ 1 ſecond in their time of the day, they differ 15″ in longitude.*

A difference of 15′ in longitude makes a difference of 1 minute in time, by propoſition 294. Therefore a difference in longitude of the 60th part of 15′, which is 15″, will make a difference in time of the 60th part of a minute, which is one ſecond of time.

296. *In any place, which is in a meridian to the eaſt of the meridian of London, it is noon ſooner than it is at London.*

For ſince the ſun in its daily motion moves from eaſt to weſt, it muſt neceſſarily be in the meridian of a place, that is to the eaſt of London, ſooner than it is in the meridian of London. But it is noon at any place, when the ſun comes to the meridian of the place. Therefore in any place, which is to the eaſt of London, it will be noon ſooner than it is at London.

A difference of 15° in longitude makes a difference of an hour in time, by propoſition 293. Conſequently if a place differs in longitude from London 15° eaſt, it will be noon at that place an hour before it is noon at London; or it will be 12 o'clock at that place, when it is only 11 at London. If a place differs in longitude from London 30°, or twice 15° eaſt, this will make a difference of two hours in time, or it will be noon, that is, 12 o'clock, at ſuch a place, when it wants two hours of 12 at London, or when at London it is only 10 o'clock in the morning. If a place differs in longitude from London 180° or 12 times 15° eaſt, this will

will make a difference of 12 hours in the account of time, or when it is noon at that place, it will be midnight at London. Now a place, which differs from London in longitude 180°, is under the oppoſite ſemicircle of the ſame meridian with London. And as the reaſon juſt aſſigned for the hours of the day being oppoſite at that place and at London is univerſal, we may ſay, that any two places, which are under oppoſite ſemicircles of the ſame meridian, have oppoſite times of the day, it will be noon at one of the places, when it is midnight at the other.

From hence it appears that if a man was to ſet out from London eaſtwards, and to go round the earth, he would have loſt a day in his reckoning of time, when he came home; or if they, who have ſtayed at London, reckon it to be monday, when he returns, he will reckon it to be tueſday, ſo that he will be a day before them in his account, or in reſpect of their reckoning he will have dropped a day out of the account. For every 15° that he travels he will reckon his noon an hour ſooner than they do at London. Therefore when he has gone round the earth or 360° he will reckon his noon 24 hours ſooner than they do at London, becauſe there are 24 times 15° in 360°: ſo that it will be tueſday at noon with him, when it is only monday at noon with them.

297. *In any place, which is in a meridian to the weſt of the meridian of London, it is noon later than it is at London.*

For as the ſun in its daily motion moves from eaſt to weſt, it will come to the meridian of London, and will make it noon at London, before it comes to the meridian of a place to the weſt of London, and makes it noon at that place.

Every difference in longitude of 15° weſt from London makes a difference of an hour in the time of noon, by propoſition 293. Or when it is noon at London, it will be 11 o'clock in the morning at a place 15° to the weſt, 10 o'clock in the morning at a place 30° to the weſt, 9 o'clock in the morning at a place 45° to the weſt, 12 o'clock at night at a place 180° to the weſt, 11 o'clock at night at a place 195° to the weſt, 10 o'clock at night at a place 210° to the weſt, and ſo on.

From hence it follows, that if a man was to ſet out weſtwards from London and to go round the world, he would have gained a day in his account of time, or if they, who ſtayed at London, reckoned it to be wedneſday, he would have a day more to come in his account of time before it was wedneſday, for in his account it would be only tueſday. He reckoned his noon an hour later than they for every 15° of his voyage. But when he has gone round the earth, he will have travelled 360°. There-

fore he will reckon his noon 24 hours later than they, when he comes home: for in 360° there are 24 times 15°. So that when it is wednesday noon with them it will be only tuesday noon with him.

298. *Obſervations upon any inſtantaneous appearance made at different places will determine how much thoſe places differ in longitude.*

The beginning of a lunar eclipſe, which is the inſtant when the earths ſhadow firſt appears upon the edge of the moon, the end of a lunar eclipſe, which is the inſtant when the ſhadow leaves the moons diſc, the immerſion of a ſatellite of jupiter into the ſhadow of jupiter, or the emerſion of the ſatellite out of the ſhadow; any of theſe and other appearances of the ſame ſort, which happen and are over in an inſtant, are called inſtantaneous appearances. And if ſuch an appearance is obſerved by two perſons at different places, by comparing their obſervations together, the difference in the reckoning of time at thoſe places will be found, from whence by propoſitions 293, 294, 295, 296, 297, the difference of the places in longitude may be determined.

Suppoſe a perſon at London was to obſerve an eclipſe of jupiters innermoſt ſatellite, and was to find that the ſatellite fell into the ſhadow ½ hour after 11 o'clock at night; and ſuppoſe another perſon at another place obſerving the ſame eclipſe was to find that the ſatellite fell into the ſhadow at 12 o'clock at night. The appearance is an inſtantaneous one, and conſequently both obſervers ſaw the ſatellite fall into the ſhadow at one and the ſame inſtant. But this inſtant is ½ hour after 11 at night in London, and is 12 at night in the place of the other obſerver. Therefore London and the other place of obſervation differ ½ an hour in their account of time. Now a difference in longitude of 15° makes a difference in time of a whole hour; and conſequently where the account of time differs half an hour, the difference of longitude is half 15° or 7°. 30′. Therefore London and the other place of obſervation differ in longitude 7°. 30′. The next queſtion might be, is the other place to the eaſt or to the weſt of London? This queſtion may be eaſily anſwered from propoſition 296. For ſince it is 12 o'clock at the other place, when it is only ½ an hour after 11 at London, the other place reckons time ſooner than they do at London, and conſequently, by propoſition 296, the place is to the eaſt of London.

Suppoſe by an obſervation at London it was found that jupiters innermoſt ſatellite fell into the ſhadow or began to be eclipſed at 2 minutes and 3 ſeconds paſt 1 o'clock in the morning, and that in another place, where the ſame eclipſe of the ſatellite was obſerved, it began exactly at

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12 o'clock at night. Since the immerſion or beginning of the eclipſe is an inſtantaneous appearance, it muſt have been obſerved both at London and the other place in one and the ſame inſtant. But this inſtant is reckoned in London 2 minutes and 3 ſeconds paſt 1 in the morning, and in the other place it is reckoned 12 at night. Therefore London and the other place differ 1 hour 2 minutes and 3 ſeconds in their account of time; and by this we may determine how much they differ in longitude. An hours difference in time is owing to a difference in longitude of 15°, by propoſition 293: a minutes difference in time is owing to 15′ difference in longitude, and conſequently 2 minutes difference in time is owing to twice 15′ or 30′ difference in longitude, by propoſition 294: a ſeconds difference in time is owing to 15″ difference in longitude, and conſequently 3 ſeconds difference in time is owing to thrice 15″ or 45″ difference in longitude, by propoſition 295. But the reckoning of time in London and in the other place of obſervation differs 1 hour 2 minutes and 3 ſeconds; therefore London and that other place differ 15°. 31′. 45″. in longitude. Now at London it is 2 minutes and 3 ſeconds paſt 1 o'clock, when at the other place it is only 12 o'clock. Therefore the reckoning of time is 1 hour 2 minutes and 3 ſeconds later at the other place than at London, and conſequently, by propoſition 297, the other place differs from London in longitude 15°. 30′. 45″. weſt; or the meridian of the other place is 15°. 30′. 45″. to the weſt of the meridian of London.

The eclipſes of jupiters ſatellites are conſtant and regular, ſo that the time, when thoſe eclipſes will happen, may be and has been determined beforehand, as well as the time, when an eclipſe of our own moon will happen. By this means tables of thoſe eclipſes have been made: and the inſtant, when any eclipſe of any ſatellite will happen in the meridian of London, is put down in the tables. Such tables as theſe will ſupply the place of an obſerver in London: for by looking into the tables we may know at what hour, minute, and ſecond, any eclipſe happens, as well as if we had an account of the time from an actual obſervation. Therefore in whatever part of the earth we are, if we obſerve at what time an eclipſe of any ſatellite happens there, we may know by the tables at what time the ſame eclipſe happened in the meridian of London: and conſequently from comparing our own obſervation with the tables, we ſhall know the difference of time at the place where we are, and in the meridian of London: and this difference of time once known will determine the difference in longitude.

This method of determining the longitude cannot be uſed with any certainty only at land: for as the appearance to be obſerved is over in an in-

inſtant, the motion of a ſhip at ſea will ſcarce ever give us an opportunity of making ſuch an obſervation. If ſuch an obſervation could be made, the time of the night, when the eclipſe happened at the place where the ſhip is, might be determined by the fixed ſtars.

For want of an opportunity to make ſuch obſervations as theſe, another method has been propoſed for determining the longitude at ſea; and that is by finding out ſome way of making a clock, which ſhall not be affected in its motion either by the heat and cold of different climates, or by the ſhaking of the ſhip, or by the difference of gravity in different parts of the earth. If ſuch a clock could be made, the longitude at ſea would be determined with very little trouble. Thus, for inſtance, if the clock, when the ſhip ſails from any part of England, was ſet for the meridian of London, it would conſtantly ſhew what hour of the day it is at London, in whatever part of the world the ſhip is. And then, either by the ſtars in the night-time or by the ſun in the day-time, the ſailors might know what hour of the day it is in the place where they are. Their obſervations compared with the clock would give them the difference in the reckoning of time in London and in the place where they are, and this difference in time would determine the difference in longitude.

299. *Thoſe, who live in oppoſite ſemicircles of the ſame meridian, but in the ſame circle of latitude, have oppoſite hours of the day, but have the ſame ſeaſons of the year: theſe in reſpect of one another are called perioeci by geographers.*

If EQ, Plat. XXVIII. fig. 1, is the earths equator, PS the axis, and HO the horizon of a place at *a*, PO is the elevation of the pole, and E*a* is the latitude. A perſon, who lives at *a*, is in the circle of latitude *ab*, and another perſon, who lives at *b*, is in the ſame circle of latitude. Conſequently, becauſe they are both on the ſame ſide of the equator, ſuppoſe the north ſide of it, and at the ſame diſtance from it, it follows that when the ſuns north declination is ſuch as, by propoſition 34, to make it ſummer in one of the places, it will be ſummer likewiſe in the other; and when the ſuns ſouth declination is ſuch as to make it winter in one of the places, it will be winter likewiſe in the other. So that theſe two places will have the ſame ſeaſons of the year. But then the place *a* and the place *b*, are in oppoſite ſemicircles of the meridian, and have oppoſite longitudes, that is, they differ in longitude 180°, and conſequently, by propoſition 296, they will have oppoſite times of the day, or when it is noon at one of the places, it will be midnight at the other. The ſame

might

might be ſhewn of two perſons one at *d* and the other at *f*; they are in the ſame circle of ſouth latitude *df*, and conſequently they will have the ſame ſeaſons: but they are in oppoſite ſemicircles of the ſame meridian, and for that reaſon their hours of the day will be oppoſite.

300. *Thoſe, who live in oppoſite circles of latitude, but in the ſame ſemicircle of the meridian, will have oppoſite ſeaſons of the year, but the ſame hours of the day: theſe in reſpect of one another are called by geographers antoeci.*

If one perſon is at *a*, Plat. XXVIII. fig. 1, in north latitude E*a*, and another at *d* in equal ſouth latitude E*d*; their ſeaſons will be oppoſite. Becauſe, by propoſition 34, when the ſun has declination towards the north pole P, it will be ſummer at the place *a* in north latitude, and winter at the place *d* in ſouth latitude: and on the contrary, when the ſun has declination towards the ſouth pole S, it will be ſummer at the place *d* in ſouth latitude, and winter at the place *a* in north latitude. But then as theſe places are under the ſame ſemicircle P*ad*S of the meridian, their hours of the day will be the ſame, by propoſition 292, becauſe their longitude is the ſame. This might be ſhewn in the ſame manner of two places *b* and *f*; they have oppoſite latitudes, and therefore will have oppoſite ſeaſons; but as they have the ſame longitude they will have the ſame reckoning of time as to the hours of the day.

301. *Thoſe, who live in oppoſite circles of latitude and in oppoſite ſemicircles of the meridian, have both oppoſite ſeaſons of the year, and likewiſe oppoſite hours of the day: theſe in reſpect of one another are called by geographers antipodes.*

If one perſon lives at *a*, Plat. XXVIII. fig. 1, and the other at *f*, they have oppoſite latitudes, for one is in north latitude E*a* and the other is in equal ſouth latitude Q*f*, conſequently they will have oppoſite ſeaſons, by propoſition 300. They have likewiſe oppoſite longitudes or the places differ in longitude 180°, for one is in the ſemicircle P*a*S, and the other is in the oppoſite ſemicircle P*f*S of the ſame meridian PESQ: therefore, by propoſition 299, they will have oppoſite reckonings as to the time of the day. The ſame might be ſhewn of any other two places *b* and *d*, which are in the like ſituation in reſpect of one another.

Two perſons, one at *a* and the other at *f* are called antipodes, becauſe as the feet of both are towards the earths center C, the feet of one will be oppoſite to the feet of the other. For two ſuch perſons are diametrically oppoſite to one another, ſo that the center of the earth is between them.

Thoſe,

Thoſe, who live in oppoſite ſemicircles of the meridian but in the ſame circle of latitude, as at *a* and *b*, are not antipodes: the feet of both of them are towards the earths center C, but then they are not diametrically oppoſite to one another, or the earths center is not between them. In like manner thoſe, who live in oppoſite circles of latitude but in the ſame ſemicircle of the meridian, as at *a* and *d*, are not antipodes; for their feet are not oppoſite to one another: the feet of both are towards the earths center, but as the two places, where they live, are not diametrically oppoſite to one another, the center of the earth is not between them.

302. *If twelve ſecondaries of the celeſtial equator are conceived to be drawn at equal diſtances from one another, they will divide the equator into twenty four equal parts, and each part will contain 15°. If the meridian of any place is made one of theſe twelve ſecondaries, they are then called the hour-circles of that place.*

Each ſecondary of the equator divides the equator into two parts, for one ſemicircle of any ſecondary cuts the equator on one ſide, and the other ſemicircle cuts it on the oppoſite ſide. But if each ſecondary cuts the equator into two parts, twelve ſecondaries will cut it into twelve times two or twenty four parts. As theſe ſecondaries are ſuppoſed to be drawn at equal diſtances from one another, each of the parts, into which they divide the equator, will be the 24th part of the equator or 15°. And in the ſame manner as they divide the equator, they will likewiſe divide any other circle, which is parallel to the equator into 24 equal parts of 15° each. Now the circles of the ſuns daily motion are either the equator or ſome circle, that is parallel to the equator, by propoſition 249. Therefore theſe 12 ſecondaries will divide all the circles of the ſuns daily motion into 24 equal parts, each part containing 15°. But ſince, by propoſition 289, the ſun in its daily motion moves at the rate of 15° in an hour, it follows that the ſun in every hour of its apparent daily revolutions will move out of one of theſe ſecondaries of the equator into the next. From hence it is that theſe ſecondaries are called hour-circles.

303. *If the ſphere of the world had an opake axis, the ſhadow of the axis would always be oppoſite to the ſun: and conſequently, when the ſun was on one ſide of any hour-circle, the ſhadow of the axis would fall upon the oppoſite ſide or oppoſite ſemicircle of the ſame hour-circle.*

All the hour-circles paſs through the two poles, by propoſitions 302, 19, becauſe they are ſecondaries of the celeſtial equator. Therefore the

axis

axis of the world is in the plane of every hour-circle. All opake bodies caſt a ſhadow oppoſite to the ſun, ſo that this ſhadow is always in the ſame plane with the ſun. Therefore, if the axis of the world was opake, then as the ſun is in the plane of any hour-circle, the ſhadow of the axis would be in the plane of the ſame circle and would be oppoſite to the ſun, that is, as the ſun is in one ſemicircle of any hour-circle, the ſhadow of the axis would fall upon the oppoſite ſemicircle.

When the ſun is in the meridian of any place or at 12 o'clock, the ſhadow of the opake axis would fall upon the oppoſite ſemicircle of the meridian. When the ſun is in the next hour-circle, which is 15°. on the weſt ſide the plane of the meridian, or at 1 o'clock, the ſhadow would fall upon the oppoſite ſemicircle, which is 15°. to the eaſt of the plane of the meridian. When the ſun is in an hour-circle 30°. on the weſt ſide of the plane of the meridian, the ſhadow would fall on the oppoſite ſemicircle of the ſame hour-circle, which oppoſite ſemicircle is 30°. on the eaſt ſide of the plane of the meridian. Thus as the ſun moves, at the rate of 15°. in an hour, on the weſt ſide of the plane of the meridian, the ſhadow of the opake axis would keep moving at the ſame rate on the eaſt ſide of that plane. When the ſun in its daily motion has paſſed the oppoſite ſemicircle of the meridian of any place and gets to the eaſt of the plane of the meridian, the ſhadow would ſtill continue oppoſite to the ſun: and conſequently as the ſun moved on at the rate of 15°. in an hour on the eaſt ſide of the plane of the meridian, the ſhadow would continue to move at the ſame rate on the weſt ſide of that plane.

This propoſition is the foundation of the art of dialling; and to give the reader ſome notion of dialling, we will in the next propoſition explane the conſtruction of ſome different ſorts of dials.

304. *In a dial the plane of the dial repreſents the plane of ſome great circle in the ſphere; the gnomon repreſents the axis of the world; and the hour-lines repreſent the hour-circles projected in right lines upon the plane of the dial.*

This propoſition will be beſt underſtood from a particular deſcription of ſome one ſort of dial. The plane of a horizontal dial repreſents a part of the plane of the horizon. Let WSEN, Plat. XXVIII. fig. 2, be ſuch a dial. W the weſt ſide, E the eaſt ſide, S the ſouth ſide, and N the north ſide. PC is the ſtyle or gnomon. Now this gnomon repreſents a part of the axis of the world, and conſequently it muſt make the ſame angle with the plane of the dial, that the axis of the world makes with the plane of the horizon in that place for which the dial is deſigned.

Therefore, by propoſition 246, the angle which the gnomon of an horizontal dial makes with the plane of it, muſt be equal to the latitude of the place. And farther, as the gnomon is to repreſent that part of the axis, which is above the plane of the horizon, and this in north latitude is the northern part of the axis, the gnomon muſt point due north. Now the ſun at noon is in the plane of the meridian, by propoſition 272, and as the gnomon is an opake axis, the ſhadow of it will fall towards the oppoſite ſemicircle of the meridian, or will be in the plane of the meridian, by propoſition 303. Let *m* and *m* be the points where the plane of the meridian interſects the plane of the horizon; if a line *mm* is drawn along the plane of the dial from one of theſe points to the other, this line will be in the plane of the meridian and likewiſe in the plane of the horizon, becauſe the points of interſection *m* and *m* are in both theſe planes. But the ſhadow is in the plane of the meridian; and as it falls upon the plane of the dial or plane of the horizon it muſt likewiſe be in the plane of the horizon. Therefore it will fall upon C*m* which is a part of the line *mm*. Here then we may obſerve that the line, where the ſhadow falls at 12 o'clock at noon or when the ſun is in the meridian, is a meridian line, by propoſition 275; for it is the common interſection of the horizon and meridian. Therefore in placing the dial care is to be taken to ſet the 12 o'clock hour-line C*m* due ſouth and north, or to ſet it ſo as to be a true meridian line. We may obſerve farther that ſince ſuch a line *mm*, as is drawn from one point of the meridians interſection with the horizon to the other point, is the line on which the ſhadow of the gnomon falls at noon, and ſince, by propoſition 303, this ſhadow falls in the plane of the meridian, the line *mm* ſo drawn is the meridian repreſented by a right line upon the plane of the horizon, or is a projection of the meridian upon a horizontal plane. From hence we may ſhew the method of drawing the other hour-lines. The ſun in an hours time will get from the meridian into an hour-circle, which is 15° to the weſt; let *a* and *a* be the points of this circles interſection with the horizon, then if the line *aa* is drawn from one point of interſection to the other, this line *aa* will be a projection of the circle upon the plane of the dial. Now as at 1 o'clock the ſhadow will be in the plane of this circle, by propoſition 303, it will likewiſe fall upon the plane of the dial, and as the ſun is to the weſt of the meridian *mm*, the ſhadow will be on the eaſt ſide of *mm*. But C1*a* is in the ſame plane with *aa*, and it is in the plane of the dial, and it is on the eaſt ſide of C*m*N. Therefore C1*a* will be the 1 o'clock hour-line. In like manner we might ſhew that *bb*, *cc*, *dd*, *ee*, *ff*, *gg*, *hh*, *ii*, *kk*, *ll*, are projections of the hour-circles upon the plane of the

dial,

dial, and that C62 is the 2 o'clock hour-line, C73 the 3 o'clock hour-line &c. These hour-lines on the east-side of the dial need be marked only to 8 in our latitude; because on the longest day of the year in our latitude the sun will set before 9 o'clock, and after that will not shine upon the dial or shew the hour. In the morning the sun is on the east side of the dial, and consequently the shadow will be on the west side: so that C64 on the west side will be the 4 o'clock hour-line in the morning. C65 the 5 o'clock hour-line &c. The lines on this side need not be marked earlier than 4 in our latitude, where the sun at the longest day does not rise so early as 3.

Every other dial is an imperfect sphere, as the horizontal dial is. Thus a direct south dial, such an one as is placed upon the south side of a wall, represents the south side of the prime vertical; the hour-lines are projections of the hour-circles upon the plane of the prime vertical; and the gnomon makes the same angle with the plane of the dial that the axis of the world makes with the plane of the prime vertical. Now the distance of the pole from the zenith or nadir is equal to the complement of latitude: and consequently the angle, which the axis makes with the plane of the prime vertical, is equal to the complement of latitude: this therefore is the angle, which the gnomon makes with the plane of a direct south dial. It is the southern part of the axis, which this gnomon represents in our latitude, and for that reason the gnomon points downwards or towards the south pole.

305. *When the moon is near the first of aries, and is moving towards the tropic of cancer; the time of its rising will vary but little for several days together.*

Under this proposition we shall explane an appearance of the moon, that is commonly taken notice of and is called the harvest moon. About the time of the autumnal equinox, in the months of august, september, and october, when the moon is at or near the full, it rises for several nights together at nearly the same time. Some, who are unacquainted with the reason of this and will not give themselves the trouble of observing it, seem inclined to question the truth of the fact: they expect that after the full the moon should rise near an hour later every night, as it does at other times of the year. But perhaps, if they were asked, why the moon after the full varys near an hour in the time of its rising at other seasons of the year, they would be as much at a loss to account for this, as they are to shew why the time of its rising after the full is nearly the same for several nights together in the autumnal season. Their

ignorance in one caſe is no more a reaſon for diſbelieving the matter of fact, than it is in the other. The fact however is true in both caſes, and from the nature of the moons motion and its ſituation in reſpect of the ſun, it cannot be otherwiſe.

The fact of the harveſt-moon ſeems ſtrange, only becauſe it is thought peculiar to this one ſeaſon of the year. In one reſpect indeed it is peculiar to this ſeaſon: for this appearance will happen at the full moon only near the time of the autumnal equinox: but then in another reſpect it is common to this with all other ſeaſons of the year: for in every other month, as well as in the three autumnal ones, the moon for ſeveral days together will vary the time of its riſing very little: this in the autumnal months happens about the time when the moon is at the full, in the vernal months about the time of the new, in the winter months about the time of the firſt quarter and in the ſummer months about the time of the laſt quarter.

We will firſt ſhew why the time of the moons riſing ſhould vary at all every day; ſecondly, that it will vary but little in any month, when the moon is moving from aries to cancer; and thirdly, that this ſmall variation in the time of its riſing will happen about the full moon in autumn, about the new moon in ſpring, about the firſt quarter in winter, and about the laſt quarter in ſummer.

When the moon is in conjunction with the ſun it riſes at the ſame time that the ſun riſes. But after the conjunction it leaves the ſun and gets to the eaſt of it, and comes round to conjunction again in ſomething more than 29 days, by propoſitions 135, 140. Now in leaving the ſun and coming round to the ſun again, it apparently deſcribes a circle or 360° in reſpect of the ſun: and conſequently, if in about 29 days it deſcribes 360° in reſpect of the ſun, it muſt in one day deſcribe ſomething more than 12°; that is, the moon leaves the ſun, or its apparent diſtance from the ſun encreaſes at the rate of ſomething more than 12° every day. Conſequently, if both the ſun and the moon were in the equator, ſince the difference between the ſuns place and the moons place varys every day at the rate of more than 12°, the moon would every day riſe later than the ſun at the rate of more than 12° of the equator. But by propoſition 289, 12° of the equator, when converted into time, are more than $\frac{1}{4}$ of an hour. Therefore, if the moon was to move in the equator, the time of its riſing would vary above $\frac{1}{4}$ of an hour every day.

But the moons orbit does not coincide with the plane of the equator, it is, by propoſition 183, inclined to the plane of the ecliptic at an angle of about 5°, ſo that it more nearly coincides with the ecliptic than with

the

the equator: and to make this matter as clear as we can, we will ſuppoſe that the moons orbit does coincide with the ecliptic, or that the moon moves in the ecliptic. Now we have ſhewn, in propoſition 267, that every year, whilſt the ſun is moving in the ecliptic from aries to cancer, its ſtay above the horizon is longer and longer every day, and conſequently it riſes every day ſooner and ſets later than it did the day before. For the ſame reaſon, every month, whilſt the moon is moving in its orbit from aries to cancer, its ſtay above the horizon will be longer and longer every day, and conſequently it will riſe every day earlier and ſet later than it did the day before. Thus we have ſeen that the moons motion from the ſun will make it riſe later every day, and that the motion in its orbit will make it riſe earlier every day, whilſt it is moving from aries to cancer. Therefore as the moon, whilſt it is moving from aries to cancer, is to riſe later every day upon one account and earlier every day upon another account, the time of its riſing will, upon both accounts together, vary but little. The reader I imagine, need not now be informed that the time of the moons riſing will vary the moſt every day, when it is moving from libra to capricorn. For as, by propoſition 267, the ſuns ſtay above the horizon ſhortens every day, when it is moving in the ecliptic from libra to capricorn, and conſequently it riſes every day later and ſets earlier than it did the day before; ſo for the ſame reaſon every month, when the moon is moving in its orbit from libra to capricorn, its ſtay above the horizon ſhortens every day, and conſequently it will riſe every day later and will ſet earlier than it did the day before. Therefore in this part of the month, the time of the moons riſing is later every day upon account of its motion from the ſun, and later likewiſe upon account of its moving from libra to capricorn; and conſequently the time of its riſing being later upon both accounts will vary more in that part of the month than in any other part.

Thus we have ſeen that it is not only the harveſt-moon, which varys ſo little every day in the time of its riſing. The ſame thing happens every month in the year as well as in the autumnal months, whilſt the moon is moving from aries to cancer, that is, whilſt its north declination is encreaſing. All that is ſingular in the autumnal moon is, that this ſmall variation in the time of its riſing happens in autumn at the time of the full. The ſun at the autumnal equinox is in libra, by propoſition 36, and conſequently the moon in oppoſition, or at the full, will be in aries. And in the three autumnal months the ſun is not far from libra, and conſequently in the three autumnal months the moon, when at the full, will not be far from aries. But whenever the moon is moving from

aries

aries to cancer, the time of its rising varys but little every day. Therefore about the time of the autumnal full moons it will rise for ſeveral evenings together at nearly the ſame time. At the vernal equinox the ſun is in aries, and for the three ſpring months it is not far from aries. Therefore the moon, when it is in or near aries in the ſpring, will be in conjunction. But when the moon is moving from aries to cancer the time of its riſing will vary but little every day. Therefore the moon in the vernal months will riſe nearly at the ſame time for ſeveral days together about the time of conjunction, or about the time of the new. At the winter ſolſtice the ſun is in capricorn, and for the three winter months it will not be far from capricorn: conſequently the moon, when it is in or near aries in the three winter months, will be about 90° from conjunction, or in the firſt quarter. But when it is moving from aries to cancer, the time of its riſing varys but little for ſeveral days together. Therefore in the three winter months this ſmall variation in the time of its riſing will happen about the firſt quarter. At the ſummer ſolſtice the ſun is in cancer, and in the three ſummer months it is not far from cancer: conſequently the moon, when it is in or near aries in the three ſummer months, will want about 90° of being in conjunction, or will be in the laſt quarter. But when the moon is moving from aries to cancer the time of its riſing varys but little for ſeveral days together. Therefore in the three ſummer months this ſmall variation in the time of its riſing will happen about the laſt quarter.

Now the moon at the full riſes when the ſun ſets, and this makes us take notice of the time of its riſing: at the new it riſes with the ſun, and we ſee nothing of it: at the firſt quarter it riſes about noon, and is not obſerved when it riſes: at the laſt quarter it is ſeldom ſeen at its riſing, becauſe it riſes about midnight. This is the reaſon why we take notice of this appearance in the harveſt-moon and in no other: we obſerve the time of the moons riſing at the full, and take but little notice of it at the new or either of the quarters. Though in ſome part of all other months the moon varys as little in the time of its riſing for ſeveral days together as it does in the autumnal months: yet in other months this happens when the moon is not near the full, and therefore it paſſes unobſerved.

CHAP.

PLATE XXVII.

Fig. 1.

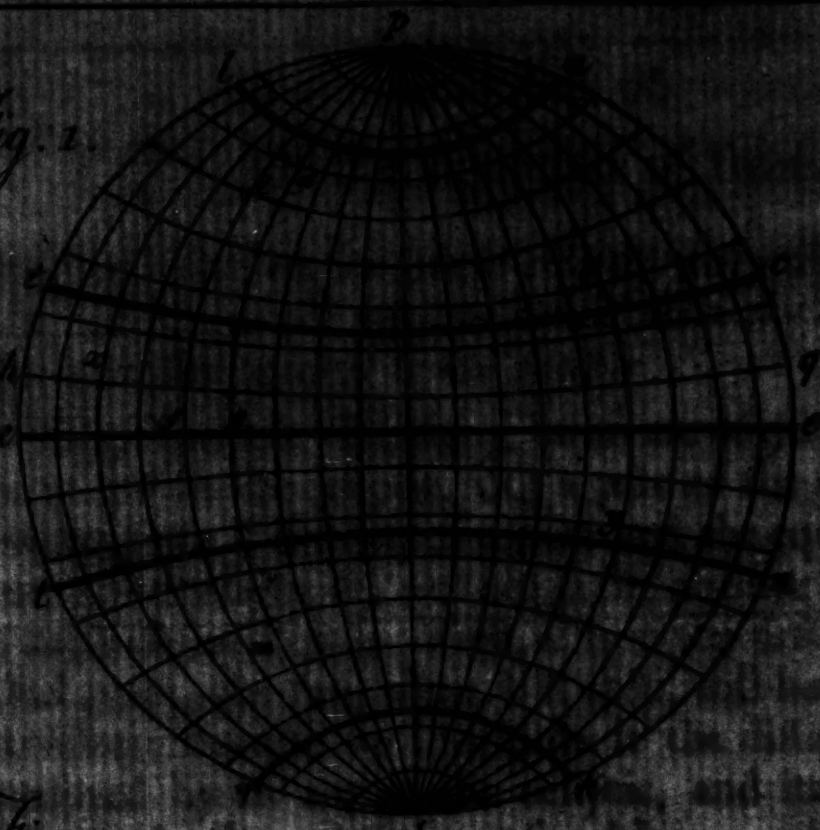


Fig. 2.

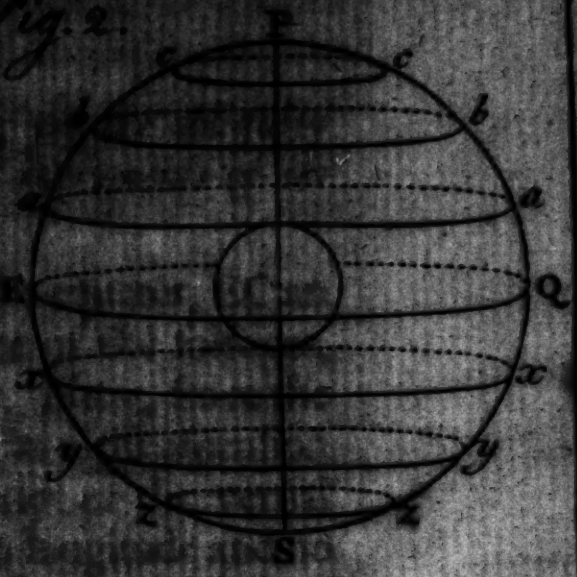


Fig. 3.

Fig. 4.

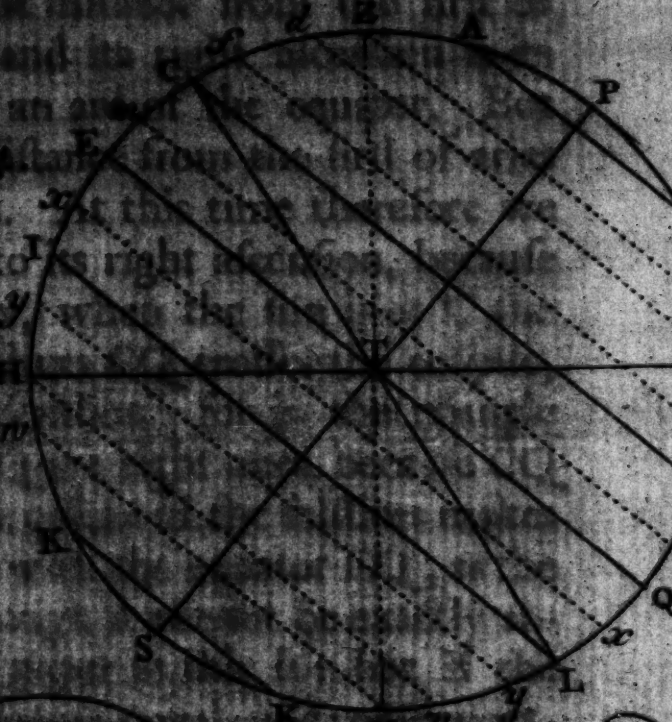


Fig. 5.

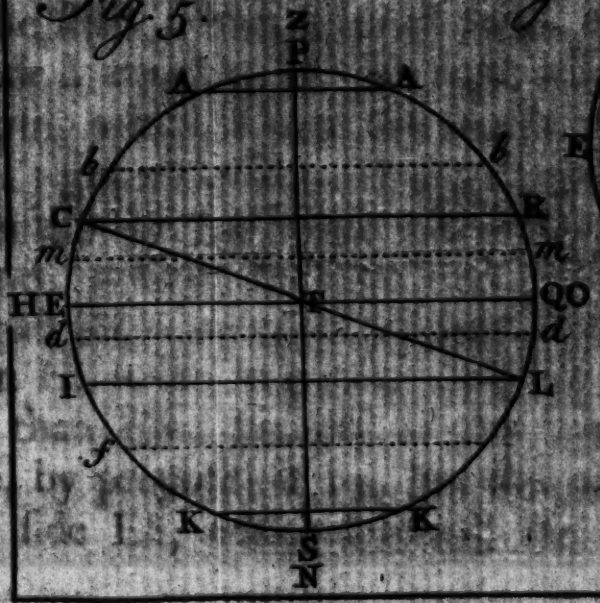


Fig. 6.

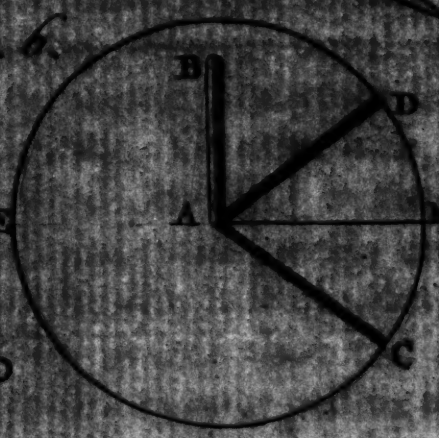
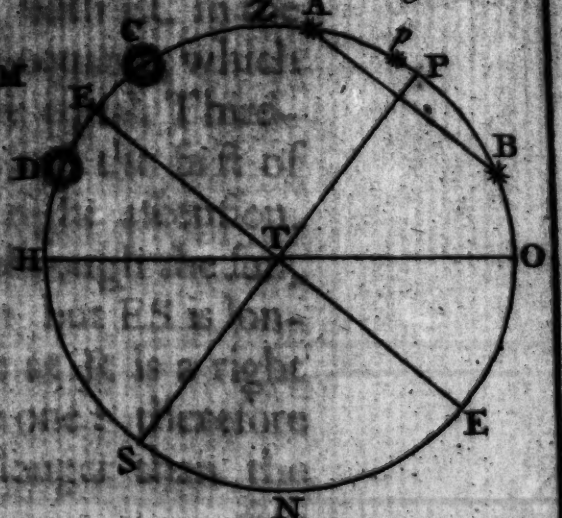


Fig. 7.



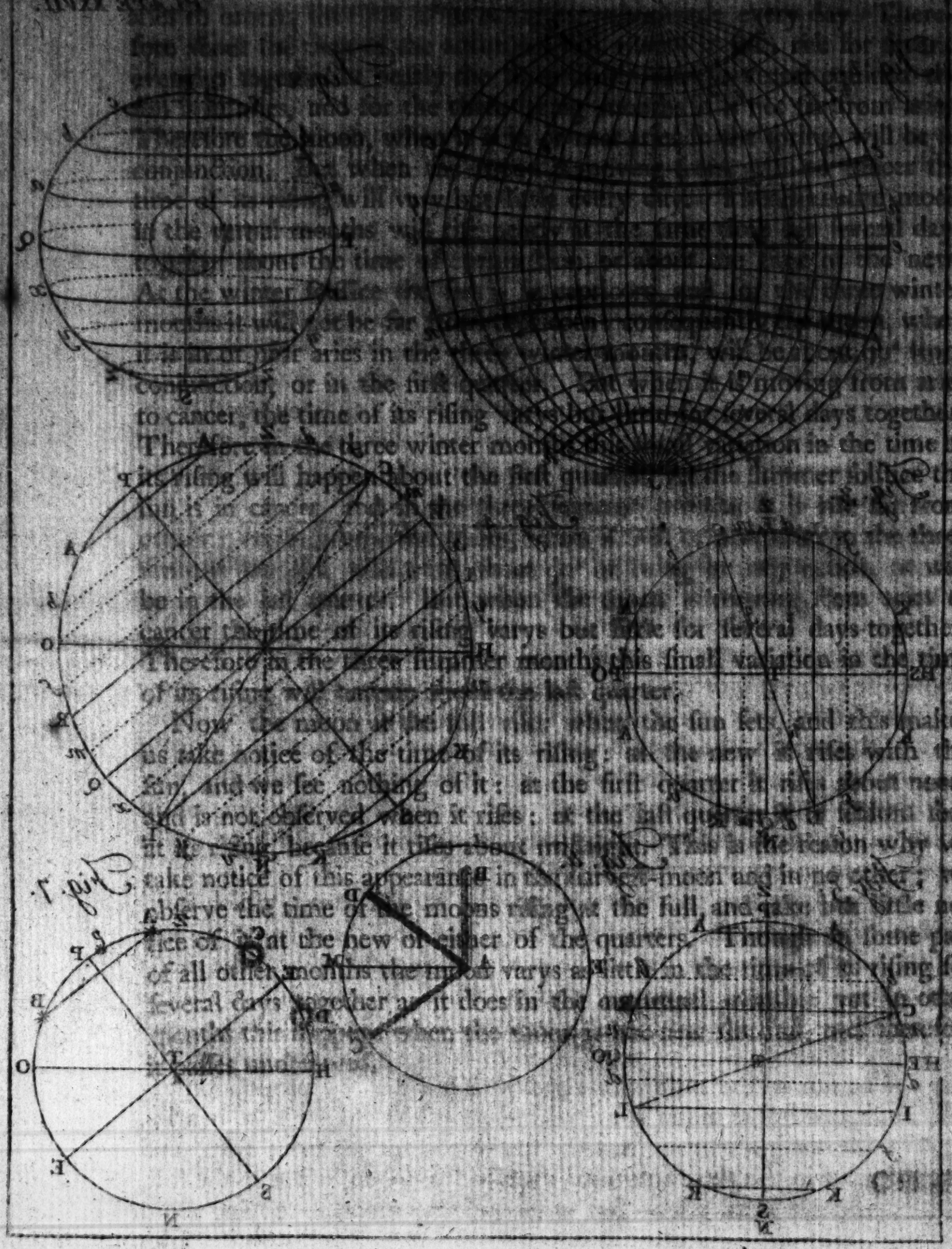


Fig. 7.

CHAP. X.

Of the equation of time.

306. *Though the ſuns motion in the ecliptic was uniform, yet the obliquity of the ecliptic to the equator would make the daily increments of its right-aſcenſion unequal to one another.*

LET EQ, Plat. XXVIII. fig. 3. be an arc of 90° in the equator, EC an arc of 90° in the ecliptic, CQ a part of the colure of the ſolſtices, which lies between C the firſt of cancer and the equator. Now, ſince the ſun moves in the ecliptic EC and not in the equator EQ, its right aſcenſion will not encreaſe uniformly. Firſt, if E is the firſt of aries or the vernal equinoctial point, the ſun, when it is at E, will have no longitude and no right aſcenſion: becauſe its longitude is the diſtance from the firſt of aries meaſured by an arc of the ecliptic, and its right aſcenſion is its diſtance from the ſame point meaſured by an arc of the equator. But when the ſun is at this point E, it is at no diſtance from the firſt of aries either in the ecliptic or in the firſt of aries. At this time therefore we may ſay the longitude of the ſun is equal to its right aſcenſion, becauſe they are either of them nothing. Secondly, when the ſun is at C the firſt of cancer, where the colure of the ſolſtices CQ croſſes the ecliptic, the longitude will be equal to the right aſcenſion: for in the triangle ECQ both the angle at C and the angle at Q are right ones, becauſe CQ is an arc in the colure of the ſolſtices, and the colure of the ſolſtices makes right angles both with the ecliptic EC and with the equator EQ, as the reader may ſee upon the artificial globe, though it cannot be exactly repreſented in a figure drawn upon a plane paper as this is: but if the angles at C and at Q are both right ones, they are equal; and conſequently the ſides, which ſubtend them, are equal; therefore EC the ſuns longitude is equal to EQ its right aſcenſion. Or otherwiſe, both EC in the ecliptic, which is then the ſuns longitude, and EQ in the equator, which is its right aſcenſion, are arcs of 90°, and conſequently are equal. Thirdly, when the ſun is any where between the firſt of aries and the firſt of cancer, as at S, the longitude will be greater than the right aſcenſion. For let SR be an arc of a ſecondary of the equator paſſing through the ſun, then ES is the longitude, and ER is the right aſcenſion: but ES is longer than ER: becauſe in the triangle ESR, the angle at R is a right one, by propoſition 19, but the angle at S is an acute one: therefore the ſide ES, which ſubtends the right angle at R, is longer than the ſide

ſide ER which ſubtends the acute angle at S. Now in the propoſition we ſuppoſe the ſun to move uniformly in the ecliptic from E to C, and conſequently the ſuns longitude or its diſtance from E meaſured in an arc of the ecliptic will encreaſe uniformly; it will deſcribe equal arcs in equal times, ſo that the daily encreaſes or increments of longitude will be equal to one another. But we have ſhewn that in the two extremes E and C, when the ſun is in the firſt of aries or the firſt of cancer, the longitude is equal to the right aſcenſion. And conſequently, ſince they are equal at each extreme and the longitude encreaſes uniformly, by the ſuppoſition, they would at all times be equal, if the right aſcenſion encreaſed uniformly too. But at S, or any where elſe between the firſt of aries and the firſt of cancer, the longitude and right aſcenſion are not equal to one another. Therefore the right aſcenſion does not encreaſe uniformly, that is, the daily increments of right aſcenſion are not equal to one another.

When the ſun is in C, Plat. XXVIII. fig. 4, or the firſt of cancer, its longitude is equal to its right aſcenſion, as has juſt been proved. And when the ſun comes to E the firſt of libra, its longitude will again be equal to its right aſcenſion; becauſe then it is $\frac{1}{2}$ a circle or 180° from the firſt of aries, whether this diſtance is meaſured in CE the ecliptic or in QE the equator. But when the ſun is any where between the firſt of cancer C and the firſt of libra E, its longitude is leſs than its right aſcenſion. For here, as before, the triangle SRE has a right angle at R and an acute one at S; and conſequently SE, which ſubtends the right angle is greater than RE which ſubtends the acute one; or SE the ſuns diſtance from libra meaſured in the ecliptic, is greater than RE its diſtance from libra meaſured in the equator. Now the farther the ſun is from libra the nearer it is to aries, and the nearer it is to libra the farther it is from aries. But SE the ſuns diſtance from libra meaſured in the ecliptic is greater than RE is diſtance from libra meaſured in the equator, or S the ſuns place in the ecliptic is farther from libra than R its place, when referred to the equator by the ſecondary SR, is from the ſame point. Therefore S the ſuns place in the ecliptic is nearer to aries, than R its place, when referred to the equator, is to the ſame point. Now the diſtance of S the ſuns place in the ecliptic from the firſt of aries is its longitude, and the diſtance of R its place, when referred to the equator, from the firſt of aries is its right aſcenſion: therefore, ſince S is nearer to aries than R, the ſuns longitude, when it is at S or any where between cancer and libra, is leſs than its right aſcenſion. Thus we have ſeen that in the two extremes C and E, or in the firſt of cancer and the firſt of libra, the ſuns longitude is equal to its right aſcenſion, and we have ſuppoſed

that

that the ſun moves uniformly in the ecliptic, or that its longitude encreaſes equally every day: therefore ſince the right aſcenſion is equal to the longitude at firſt and at laſt, if the right aſcenſion encreaſed equally every day, it would always be equal to the longitude. But when the ſun is at S, or any where between cancer and libra, its longitude is leſs than its right aſcenſion, as we have juſt proved. Therefore the ſuns right aſcenſion does not encreaſe equally every day, or the daily increments of right aſcenſion are not equal to one another.

Thus we have ſhewn, that, whilſt the ſun is in the firſt and ſecond quarters of the ecliptic, any where between the firſt of aries and the firſt of cancer, or between the firſt of cancer and the firſt of libra, its daily increments of right aſcenſion are not equal to one another. We might ſhew this in the ſame manner, when the ſun is in the third and fourth quarters of the ecliptic. For as in the firſt quarter ſo likewiſe in the third, whilſt the ſun is any where between the firſt of libra and the firſt of capricorn, its longitude is greater than its right aſcenſion; and as in the ſecond quarter ſo likewiſe in the fourth, whilſt the ſun is any where between the firſt of capricorn and the firſt of aries, its longitude is leſs than its right aſcenſion.

307. *If the ſun moved in the equator inſtead of the ecliptic, yet the daily increments of its right aſcenſion would not be equal to one another.*

If the ſuns apparent annual motion was in the equator inſtead of the ecliptic, then, as it moved from the firſt of aries in the equator, the daily increment of its right aſcenſion would be the arc, which it deſcribes every day by its apparent annual motion. But the apparent annual motion of the ſun is not uniform, by propoſition 53, that is, it deſcribes a longer arc ſome days and a ſhorter arc other days. Therefore the daily increments of right aſcenſion would not be equal to one another, though the ſuns annual motion was in the equator inſtead of the ecliptic.

308. *Any place upon the earths ſurface deſcribes more than a circle round the earths axis in a natural day: and the arc, which it deſcribes more than a circle, is equal to the ſuns daily increment of right aſcenſion.*

A natural day is the time that paſſes between the ſuns leaving the meridian of any place and coming round to the ſame meridian again. Now by propoſition 25, the ſun, after leaving the meridian of a place, takes up a longer time to come to the meridian of the ſame place again, than the earth takes up to turn once round its axis. But whilſt the earth is turning

ing once round upon its axis, any place upon the ſurface deſcribes a complete circle. Therefore, ſince a natural day is not completed in the time of one rotation, any place upon the earths ſurface muſt deſcribe more than a complete circle in the time of a natural day.

We affirm farther that the arc, which any place deſcribes more than a complete circle in the time of a natural day, is equal to the ſuns daily increment of right aſcenſion. In order to prove this we muſt remember that the meridian of any place is perpendicular to the equator, becauſe, by propoſition 230, it paſſes through the two poles. Therefore when the ſun is in the meridian of any place, the plane of the meridian coincides with the plane of that ſecondary of the equator, where the ſun is. Thus if the ſun is at S in the ecliptic EC, Plat. XXVIII, fig. 3, and SR is an arc in a ſecondary of the equator, whenever the ſun S comes to the meridian of any place, the arc SR will be in the plane of the meridian. Suppoſe the earth and the place along with it to turn once round, and then the ſame arc SR, after the rotation is completed, will again be in the plane of the ſame meridian. But the ſun will not then be at S, but will have moved forward in the ecliptic EC, and will be at T, ſo that TV will then be an arc in the ſecondary of the equator where the ſun is, and conſequently the place, beſides this complete rotation, muſt turn a little farther, till the meridian comes even with the ſecondary TV. Now any place upon the earths ſurface in the daily rotation of the earth deſcribes either the equator or ſome circle parallel to it. Therefore any place upon the earths ſurface between the ſuns leaving its meridian, and coming thither again, or before the natural day is completed, muſt deſcribe a circle parallel to the equator, and an arc more than a circle equal to RV, which is an arc in the equator that meaſures the diſtance between the ſecondary SR, where the ſun was when it left the meridian, and the ſecondary TV, where it is when it comes to the meridian again. But ſince ER was the ſuns right aſcenſion, when it left the meridian, and EV is its right aſcenſion, when it comes to the meridian again, RV is the quantity by which the right aſcenſion has encreaſed in a days time, or RV is the daily increment of right aſcenſion. Therefore, after the ſun has left the meridian before it returns thither again, or before a natural day is completed, any place upon the earths ſurface deſcribes a complete circle and an arc of a circle beſides, which arc is equal to the daily increment of the ſuns right aſcenſion.

309. *The natural days are not equal to one another.*

The time of a natural day, by propoſition 308, is the time that any place upon the earths ſurface takes up to deſcribe a complete circle by the

earths

earths daily rotation round its axis, and an arc beſides this complete circle, which is equal to the ſuns daily increment of right aſcenſion. Now the earths motion round its axis is uniform, it moves always at the ſame rate: therefore any place upon its ſurface would always deſcribe a complete circle and the additional arc in equal times, if this arc was always the ſame. But this additional arc is equal to the ſuns daily increment of right aſcenſion, and, by propoſitions 306, 307, the ſuns daily increments of right aſcenſion are not equal to one another, ſo that this additional arc will be ſometimes longer and ſometimes ſhorter. The longer this additional arc is, ſo much longer time any place upon the earths ſurface will take up to deſcribe a complete circle and this additional arc; and the ſhorter the arc is, ſo much leſs time any place upon the earths ſurface will take up to deſcribe a complete circle with this arc added to it. But a natural day is the time that any place upon the earths ſurface takes up to deſcribe a complete circle and this additional arc. Therefore the natural days are not equal to one another, but are ſometimes longer and ſometimes ſhorter.

A natural day is the time that paſſes between the ſhadows leaving the 12 o'clock hour-line upon a ſun-dial one day at noon, and returning thither again at noon the day following, by propoſitions 25, 304. For whenever the ſhadow falls upon the 12 o'clock hour-line the ſun is in the meridian. But ſince the natural days meaſured by the ſuns apparent daily revolutions are unequal to one another, it is plane that a ſun-dial is not an uniform meaſure of time, or the revolutions of the ſhadow do not divide time into equal parts: the apparent daily revolutions of the ſun, and, in conſequence of that, the revolutions of the ſhadow upon a dial divide time into natural days: but then theſe natural days are not equal parts of time, they are ſometimes longer and ſometimes ſhorter.

The motion of a clock, divides time into equal parts. When the hour-hand ſetting out from 12 has completed two revolutions, exactly 24 hours have paſſed: the time, which paſſes during two ſuch revolutions of the hour-hand, is never either longer or ſhorter than this, if the clock goes truly, that is, if its motion is uniform, as it is always deſigned to be.

From hence it follows, that a good ſun-dial and a good clock will ſeldom ſhew exactly the ſame time of the day. The natural days as ſet out by the motion of the ſhadow upon a dial are ſometimes equal to 24 hours: and when they are ſo, it will be 12 at noon both by the dial and by the clock at the ſame inſtant. But the natural days are ſometimes longer than 24 hours: and when they are, the ſhadow upon the dial will not be come round to the 12 o'clock hour-line till after it is 12 by the clock, that is, the clock will be faſter than the dial. Again at other times the natural

days are ſhorter than 24 hours: and when they are, the ſhadow upon the dial will come round to the twelve o'clock hour, ſome time before it is 12 by the clock, that is, the clock will go ſlower than the dial.

310. *Time, as it is meaſured by the ſuns motion is called apparent time, but if it is meaſured by any uniform motion, it is then called mean time.*

311. *A clock ought not to be ſet exactly with a ſun-dial at all times of the year.*

A clock is deſigned to move uniformly, the hour-hand, if the clock is made as it ought to be, will take up exactly 24 hours neither more nor leſs in performing two revolutions: and conſequently a clock is a meaſure of mean time, the parts, into which it divides time, are always equal to one another. The ſhadow of a ſun-dial does not perform its revolutions uniformly, the revolutions of it, like the apparent daily revolutions of the ſun, take up ſometimes more and ſometimes leſs than exactly 24 hours: when time is thus divided by the apparent motion of the ſun into unequal parts, we call it apparent time.

Now the ſuns daily revolutions are what we have in view, when we divide time into natural days; or theſe daily revolutions gave occaſion to our making this diviſion of time. Therefore ſince the motion of a clock is deſigned to be a meaſure of natural days, a clock ought to be regulated by the apparent motion of the ſun, or ought to be ſet by a dial. But then the ſun is not an uniform meaſure of time, its apparent motion makes the natural days ſometimes longer and ſometimes ſhorter; whereas a clock is, or ſhould be, an uniform meaſure of time, and its apparent motion will divide time into natural days of the ſame length at all times of the year. Therefore though a clock ought to be regulated by the ſun or by the dial, yet the clock ought not to be ſet exactly with the dial: ſince the clock, as it is deſigned to meaſure mean time, is to be ſet upon a ſuppoſition that all natural days are of the ſame length; and a dial is a meaſure of apparent time, or the natural days, as meaſured by the motion of the ſhadow, are not always of the ſame length. If the natural days as meaſured by the ſuns motion are longer than 24 hours, the clock ought to be ſet faſter than the dial, that it may be 12 by the clock before it is 12 by the dial: if the natural days as meaſured by the ſuns motion are ſhorter than 24 hours, the clock ought to be ſet ſlower than the dial, that it may be 12 by the dial before it is 12 by the clock: but if the natural days as meaſured by the ſuns motion are exactly 24 hours, the clock ought to be ſet with the dial, that it may be 12 by both of them exactly at the ſame inſtant.

312. *The*

312. *The equation of time is the difference between mean and apparent time.*

The mean length of a natural day is 24 hours, the apparent length of a natural day as meaſured by the ſuns motion is ſometimes more and ſometimes leſs than 24 hours. The difference between 24 hours and the length of a natural day as meaſured by the ſuns motion, is the difference between mean and apparent time, and this difference is called the equation of time.

313. *When we have found out the hour of the day by apparent time, the equation is ſometimes to be added to this and ſometimes to be ſubtracted from it, in order to determine what the hour is by mean time.*

When we have found that it is 12 o'clock by the dial, we have then found that it is 12 o'clock by apparent time. But we want to ſet our clock upon a ſuppoſition that all the natural days are juſt 24 hours long, that is, we want to ſet it by mean time. If we find farther that an apparent natural day is then 24 hours 1 minute long, or that an apparent natural day is 1 minute longer than a mean one, then 1 minute is the equation. Now in order to determine how we are to ſet our clock, we may reaſon in this manner. It is now 12 o'clock by the dial; but an apparent natural day is 24 hours 1 minute long; therefore if a natural day was only 24 hours long, it would have been 12 o'clock 1 minute ago: but we deſign to ſet our clock as if a natural day was 24 hours long, and conſequently we muſt ſet it, as if it had been 12 o'clock 1 minute ago, that is, we muſt ſet it 1 minute after 12. In this caſe then mean time precedes apparent time; for it is to be 1 minute paſt 12 by the clock, when it is exactly 12 by the dial: and here having found apparent time by the dial, in order to ſet our clock, or to determine what the hour is by mean time, we add the equation 1 minute to apparent time. The ſame is to be done in all other caſes of the ſame ſort. If mean time precedes apparent time, then having found the hour of apparent time by the dial, we add the equation to it in order to determine what the hour is by mean time or in order to ſet our clock.

Suppoſe we have found that it is 12 o'clock by the dial, and know that the apparent length of a natural day is then only 23 hours 59 minutes; ſince the mean length of a natural day is always 24 hours, the equation will then be 1 minute; for an apparent natural day is at that time 1 minute ſhorter than a mean one. Here again in order to ſet our clock we reaſon thus: It is 12 o'clock by the dial; but an apparent natu-

tu-

tural day at this time is only 23 hours 59 minutes long, therefore it is 12 o'clock by the dial 1 minute sooner than it would be if a natural day was 24 hours long; from whence we conclude that if a natural day was 24 hours long, it would at that instant want 1 minute of 12. But our clock is to be set so as to measure natural days of their mean length, or so as to shew the hour if a natural day was 24 hours long: and consequently, when it is 12 o'clock by the dial it ought to want 1 minute of 12 by the clock. In this case then mean time follows apparent time; or when it is 12 by apparent time, it is to want a minute of 12 by mean time: therefore here having found the hour of apparent time by the dial, in order to set our clock, we subtract the equation 1 minute from the hour of apparent time, and the remainder is the hour of mean time, to which hour we set our clock. The same is to be done in all other cases of the same sort. If mean time follows apparent time, then having found the hour of apparent time by the dial, we subtract the equation from it in order to determine what the hour is by mean time, or in order to set our clock.

The only difficulties remaining are, how we shall find the difference between mean and apparent time, and how we shall determine whether mean time precedes or follows apparent time. To know the equation we must know how much mean time and apparent time differ; and to know whether this equation is to be added or subtracted we must know whether mean time precedes or follows apparent time.

314. *If a planet moved uniformly round in the equator, once every year, that is, in the same time that the sun by its annual motion describes the ecliptic, the apparent daily motion of such a planet would be a measure of mean time.*

The natural days measured by the suns daily motion are unequal, first, because the suns annual motion is in the ecliptic and not in the equator, by proposition 306, and secondly, because the suns annual motion is not uniform, by proposition 307. But the imaginary planet revolves annually in the equator, and its motion is supposed to be uniform. Therefore the natural days measured by the apparent daily motion of such a planet would be always equal to one another. If so much time as passes between such a planets leaving the meridian and returning in its apparent daily motion to the same meridian again is called a natural day, these natural days will be always 24 hours long; so that such a planets motion will be a measure of mean time.

315. *The*

315. *The time, that paſſes between the appulſe of the ſuppoſed planet to the meridian of any place and the appulſe of the ſun to the meridian of the ſame place, is the equation of time.*

It is noon by mean time at any place when the ſuppoſed planet comes to the meridian of that place, by propoſition 314, and it is noon at the ſame place by apparent time, when the ſun comes to the ſame meridian, by propoſition 304. Therefore the time that paſſes between the appulſe of the ſuppoſed planet to the meridian and the appulſe of the ſun to the ſame meridian is the difference between the apparent time of noon and the mean time of noon: and this difference, by propoſition 312, is the equation of time.

316. *That point of the equator, which comes to the meridian with the ſun, is the point where the ſuns right aſcenſion ends: and that point of the equator, which comes to the meridian with the ſuppoſed planet, is the point where that planets right aſcenſion ends.*

If the ſun is at S, Plat. XXVIII. fig. 3. and EC is the ecliptic, EQ the equator, and E the firſt of aries; then if a ſecondary of the equator paſſes through the ſun, SR is an arc in it, and the arc ER is the ſuns right aſcenſion: this arc ER begins at E the firſt of aries and ends at R the point where the ſecondary SR cuts the equator. Such a point as R, where the ſuns right aſcenſion ends, always comes to the meridian of any place at the ſame time with the ſun S. For, by what has been ſaid, under propoſition 308, the ſecondary SR, and conſequently the point R, comes to the meridian with the ſun.

If the ſuppoſed planet is at P in the equator, EP is its right aſcenſion, which begins at E the firſt of aries, and ends at P the place where the planet is. But it is evident that this point P in the equator muſt come to the meridian of any place at the ſame time with the planet, which is at that point.

317. *The ſuns right aſcenſion may be found, if we know the latitude of the place where we are, and the ſuns meridian altitude.*

The elevation of the pole may be found, by propoſition 279, and this is equal to the places latitude, by propoſition 246. When we know the latitude we may determine the elevation of the equator, for it is equal to the complement of latitude, by propoſition 247. The ſuns meridian altitude may be determined with the help of a quadrant, by propoſition 234. And when we have found the elevation of the equator and the

ſuns

ſuns meridian altitude, we may determine the ſuns declination, by propoſition 276. Suppoſe therefore, Plat. XXVIII. fig. 3. EC to be the ecliptic, EQ the equator, E the firſt of aries, and let the ſun be at S: then if a ſecondary of the equator paſſes through the ſun, the arc SR in ſuch a ſecondary is the ſuns declination. The length of this arc SR is determined as above. Now in the ſpherical triangle ESR, the angle at R is known: for ſince SR is an arc in a ſecondary of the equator, the angle at R is a right one, by propoſition 19. The angle at E is likewiſe known, for it is the inclination of the ecliptic to the equator. And the ſide SR is determined. But from theſe particulars being known we can by the rules of ſpherical trigonometry determine the length of the other ſides ES and ER. But ER is the ſuns right aſcenſion. Therefore by knowing the latitude of a place, and the ſuns meridian altitude, we may from thence find its declination; and when its declination is found we may determine its right aſcenſion.

318. *By the ſuns middle longitude at any time of the year is meant the longitude, which the ſun would have at that time, if it moved uniformly in the ecliptic.*

The ſuns apparent annual motion in the ecliptic is ſometimes faſter and ſometimes ſlower, or its apparent velocity is ſometimes greater and ſometimes leſs. But if it was always to move uniformly with the middle velocity between the greateſt and the leaſt, its revolution would be completed in the ſame time that it is now. The ſuns revolution in the ecliptic is completed in a years time; and if it was to move uniformly at the rate of 59′. 8″. every day, it would deſcribe the whole ecliptic or 360° in a year. Therefore the ſuns middle velocity at any time means the longitude which the ſun would have at that time, if it was to move uniformly at the rate of 59′. 8″. every day.

Thus in one day after the vernal equinox the ſun moving at this rate would be 59′. 8″. from the firſt of aries, that is, its mean longitude 24 hours after the vernal equinox is 59′. 8″. Two days after the vernal equinox, its middle longitude is twice 59′. 8″. or 1°. 18′. 16″. In three days from the vernal equinox its middle longitude is thrice 59′. 8″. or 2°. 57′. 24″. and ſo on, as many days as are paſt ſince the vernal equinox, ſo many times 59′. 8″. is the middle longitude of the ſun, or the longitude, which it would have if it moved always with its middle velocity or at the rate of 59′. 8″. every day.

319. *The*

319. *The right aſcenſion of a ſuppoſed planet deſcribing the equator by an uniform motion once in every year would always be equal to the ſuns middle longitude.*

Since the planet is ſuppoſed to move round in the equator once every year with an uniform velocity, ſo as to deſcribe the equator in the ſame time that the ſun annually deſcribes the ecliptic; its velocity muſt be the ſame with the ſuns middle velocity: and conſequently it muſt move at the rate of 59′ 8″ every day, by what has been ſhewn under propoſition 318. Therefore ſuch a planets diſtance from the firſt of aries in the equator muſt be the ſame at any time that the ſuns diſtance from the firſt of aries in the ecliptic would be at the ſame time, if the ſun was to move with its middle velocity. But the planets diſtance from the firſt of aries in the equator is the planets right aſcenſion: and the ſuns diſtance from the firſt of aries in the ecliptic, if it was to move with its middle velocity, is its middle longitude. Therefore the right aſcenſion of ſuch a planet would be always equal to the ſuns middle longitude.

320. *Between the time of the ſuns appulſe to the meridian of any place and the time of the ſuppoſed planets appulſe to the ſame meridian, an arc of the equator paſſes under the meridian, which is equal to the difference between the ſuns right aſcenſion and its middle longitude.*

When the ſun comes to the meridian of any place, that point in the equator, where its right aſcenſion ends, comes to the meridian, and when the ſuppoſed planet comes to the meridian that point in the equator, where the planets right aſcenſion ends, comes to the meridian. Therefore between the time of the ſuns appulſe to the meridian, and the time of the planets appulſe to the ſame meridian, an arc of the equator paſſes under the meridian, which is equal to the difference between the ſuns right aſcenſion and the planets right aſcenſion. But the planets right aſcenſion is equal to the ſuns middle longitude, by propoſition 319. Therefore between the time of the ſuns appulſe and the planets appulſe to the ſame meridian, an arc of the equator paſſes under the meridian, which is equal to the difference between the ſuns right aſcenſion and the ſuns middle longitude.

Thus if the ſun is at S, Plat. XXVIII. fig. 3, ER is its right aſcenſion; and when the ſun comes to the meridian, the point R likewiſe comes to the meridian. Let P be the place of the ſuppoſed planet; then the planets right aſcenſion is EP; and when the planet comes to the meridian the point P comes to the meridian too. Conſequent-

ly between the appulſe of the ſun and the appulſe of the planet to the meridian the arc RP muſt have paſſed under the meridian. This arc RP is the difference between ER the ſuns right aſcenſion and EP the planets right aſcenſion. But EP the planets right aſcenſion is equal to the ſuns middle longitude, by propoſition 319. Therefore RP the arc of the equator, which paſſes under the meridian between the ſuns and the planets appulſe to it, is the difference between the ſuns right aſcenſion and its middle longitude.

321. *If an arc of the equator equal to the difference between the ſuns right aſcenſion and its middle longitude is converted into time, this will be the equation of time.*

Thus by reaſoning upon the apparent motion of a planet, ſuppoſed to deſcribe the equator once every year with an uniform velocity, we are led to the equation of time, though in fact there is no ſuch planet. For, by propoſition 314, it is apparent noon, when the ſun comes to the meridian of any place; and it is mean noon when the ſuppoſed planet comes to the meridian of the ſame place. But between the time of the ſuns appulſe and the ſuppoſed planets appulſe, or between apparent noon and mean noon, an arc of the equator paſſes under the meridian equal to the difference between the ſuns right aſcenſion and the ſuns middle longitude, by propoſition 320. Now both the ſuns right aſcenſion and its middle longitude may be determined, by propoſitions 317, 318: and conſequently we may know the difference between theſe two quantities; that is, though there is no ſuch planet, we can find the length of that arc in the equator, which muſt paſs under the meridian between the time of apparent noon and the time of mean noon. And if we can find out the time, which this arc will take up in paſſing under the meridian, this time will be the difference between apparent noon and mean noon. Now in the daily motion, any point of the equator deſcribes 15° in one hour of time, 15′ in one minute of time, 15″ in one ſecond of time, by propoſitions 289, 290, 291. Therefore if the length of the arc, which is to paſs between apparent noon and mean noon, is known; the time that it will take up to paſs may be determined.

Thus if the ſuns middle longitude is 1°. 58′. 16″, and its right aſcenſion is 1°. 43′. 1″, then the difference between them is 15′. 15″. So that between apparent noon and mean noon, an arc in the equator of 15′. 15″. is to paſs in the daily motion under the meridian. But an arc of 15′. 15″. in the equator paſſes in 1 minute and 1 ſecond of time, or this arc of 15′. 15″. when converted into time, is 1 minute and 1 ſecond. Therefore

fore 1 minute and 1 ſecond of time will be the equation, or the difference between apparent time and mean time; by propoſitions 315, 320. If the ſuns middle longitude is greater than its right aſcenſion, then mean time follows apparent time: but if the ſuns middle longitude is leſs than its right aſcenſion, then mean time precedes apparent time.

322. *When we have found the hour of the day by apparent time, the equation is to be ſubtracted from this, in order to find the hour by mean time, if the ſuns middle longitude is greater than its right aſcenſion.*

We may now reaſon about the motion of a planet ſuppoſed to deſcribe the equator once every year with an uniform motion, ſo as to determine whether the equation is to be ſubtracted from the hour of apparent time, or added to it, in order to find the hour of mean time. For though there is no ſuch planet, yet we may always determine whereabouts in the equator it would be, if there was ſuch an one: becauſe, by propoſition 318, the ſuns middle longitude may be found, and, by propoſition 319, ſuch a planets right aſcenſion or diſtance from the firſt of aries in the equator is always equal to the ſuns middle longitude. That point of the equator, where the ſuppoſed planet muſt be, if there was any ſuch planet, is that point where the planets right aſcenſion ends, and the diſtance of this point from the firſt of aries is equal to the ſuns middle longitude. Now the right aſcenſion both of the ſun and the planet is reckoned from weſt to eaſt. Conſequently if the ſuns middle longitude is greater than its right aſcenſion, or, which is the ſame thing, if the planets right aſcenſion is greater than the ſuns right aſcenſion, the place of the planet in the equator will be to the eaſt of that point where the ſuns right aſcenſion ends. Therefore at apparent noon, when in the daily motion, which is from eaſt to weſt, that point of the equator where the ſuns right aſcenſion ends, is come to the meridian, by propoſitions 272, 320, the place of the ſuppoſed ſtar will be to the eaſt, or will not yet be come to the meridian. But it is not noon by mean time till the place of the ſuppoſed ſtar is come to the meridian, by propoſition 320. Therefore mean noon will be later than apparent noon, when the place of the ſuppoſed ſtar is to the eaſt of that point where the ſuns right aſcenſion ends, or when the ſtars right aſcenſion is greater than the ſuns right aſcenſion, or when the ſuns mean longitude is greater than the ſuns right aſcenſion. Now when mean noon thus follows apparent noon, if we have found that it is 12 by the dial, it ought not to be 12 by the clock: ſo that having found the hour by the ſun, we are, as in propoſition 313, to ſubtract the equation from the

apparent hour ſo found, and the remainder will be the hour of mean time, to which we are to ſet the clock.

323. *When we have found the hour of the day by apparent time, the equation is to be added to this in order to find the hour by mean time, if the ſuns middle longitude is leſs than its right aſcenſion.*

If the ſuns middle longitude is leſs than its right aſcenſion, then by propoſition 319, the ſuppoſed planets right aſcenſion is leſs than the ſuns right aſcenſion. Therefore, ſince the right aſcenſion is reckoned from weſt to eaſt, the point in the equator, where the planet would be, if there was ſuch an one, is leſs to the eaſt or more to the weſt than the point where the ſuns right aſcenſion ends. Now it is noon by apparent time, when that point of the equator, where the ſuns right aſcenſion ends, comes to the meridian, by propoſition 320: but when that point is in the meridian the place of the ſuppoſed planet being to the weſt of that point, will be to the weſt of the meridian; that is, the place of the ſuppoſed planet muſt in the apparent daily motion have paſſed the meridian ſome time before; or ſince, by propoſition 320, it is noon by mean time, when the place of that ſuppoſed planet was in the meridian, noon by mean time muſt have happened before it was noon by apparent time. Therefore when we have found that it is 12 by the dial, we know that it is paſt 12 by apparent time: conſequently if, as in propoſition 313, to the hour of the day found by the dial we add the equation, the ſum will be the hour of mean time, to which we are to ſet the clock.

CHAP. XI.

Of the twilight.

324. *That light, which we have from the ſun, before it riſes in a morning, and after it ſets in an evening, is called the twilight.*

325. *The earths atmoſphere above the horizon is enlightened by the ſun, whilſt the ſun itſelf is below the horizon: and this light in the atmoſphere is the twilight.*

LET ADL, Plat. XXVIII. fig. 5, be the earths ſurface, CFMB the outſide of the atmoſphere that ſurrounds it, A any place upon the earth, PAN the ſenſible horizon, and HEO the rational horizon. When the ſun is at G below the horizon HO, whether before it riſes in a morning or after it ſets in an evening, it will not be ſeen by a ſpectator at A, by

by propoſition 21, or the rays of it cannot fall directly upon the place A, by propoſition 34 of optics; for no ſtrait line can be drawn from G to A without paſſing through the earth at R: and conſequently no ray of light can paſs from the ſun at G to the place A without being ſtopped or intercepted by the earths convex ſurface at R. But though the ſun does not ſhine directly upon the place A, when it is below the horizon, yet it will ſhine upon the atmoſphere BCA: for as this part of the atmoſphere is above the earths ſurface, the rays can paſs from the ſun at G to this part of the atmoſphere, without being intercepted by the earth. Therefore this part of the atmoſphere will be enlightened by the ſun, though the ſun is below the horizon.

Now this enlightened part of the atmoſphere CAB is above the ſenſible horizon, and conſequently a ſpectator at A will ſee it. Thus, though the ſun is below the horizon, ſome part of the atmoſphere, which is above the horizon, will be enlightened; and by reflection from this part of the atmoſphere we receive ſuch light, in a morning before the ſun riſes and in an evening after it is ſet, as makes the morning and the evening twilight.

When the ſun appears to a ſpectator at A, it muſt be above his rational horizon HO: but the enlightened part of the atmoſphere ACB will not appear, unleſs it is above his ſenſible horizon PN, by propoſition 21: becauſe the ſun is a very remote body; and the enlightened part of the atmoſphere ACB is not very remote from the ſpectator. This is the reaſon why in ſpeaking of the ſuns riſing or ſetting we uſe the rational horizon HO, but in ſpeaking of the appearing or diſappearing of the twilight we uſe the ſenſible horizon PN. The reader ſhould however obſerve, by the way, that the ſuns depreſſion below the ſenſible or rational horizon is the ſame, becauſe, by propoſition 21, in reſpect of ſo remote an object, as the ſun is, the ſenſible and rational horizons coincide. For this reaſon, though PN is the ſenſible horizon, yet, when the ſun is in the line BS, we ſhall call the angle NBS the ſuns depreſſion, though this angle is its depreſſion below the ſenſible horizon PN. Indeed ſince PN the ſenſible horizon and HO the rational one are parallel to one another, whatever angle the line BS makes with one horizon it will neceſſarily make the ſame with the other, Euc. b. I. prop. 29. Therefore the angle of the ſuns depreſſion below the rational horizon is equal to the angle of its depreſſion below the ſenſible one. In ſpeaking of the twilight, we cannot make uſe of the rational horizon, but muſt uſe the ſenſible one only: becauſe the enlightened atmoſphere, which makes the twilight, is a near object; and for this reaſon its appearance or diſappearance depends upon it being above or below the ſenſible horizon.

326. *When*

326. *When the evening twilight ends, or the morning twilight begins, a ray of the ſun, that is reflected from the higheſt part of the atmoſphere, deſcribes after reflection a line that is in the plane of the ſenſible horizon.*

If the ſun is at G below the horizon, ſo that its rays fall upon the top of the atmoſphere at C, and are reflected in the line CA, the evening twilight cannot be at an end: becauſe all the atmoſphere from CA to B will then be enlightened: and conſequently the light of the twilight will extend itſelf above the horizon from B to C. The twilight ends, when the light of it does but juſt appear at B in the ſenſible horizon. Now if the twilight does but juſt appear in the ſenſible horizon at B, the rays of the twilight muſt come to a ſpectator at A along the line BA in the plane of his horizon. But it is evident that the top or higheſt parts of the atmoſphere, which are fartheſt from the earths ſurface, will be enlightened by the ſun longer than any other parts; and, by propoſition 325, the rays of the twilight are reflected rays. Therefore, when the twilight ends, a ray reflected from B the top of the atmoſphere deſcribes the line BA in the plane of the ſenſible horizon.

In like manner, when the morning twilight begins, the light of it juſt appears in the horizon, or the rays of the twilight, which, by propoſition 325, are reflected rays, come along the plane of the ſenſible horizon. But the top of the atmoſphere is the firſt part that is enlightened by the ſuns rays. Therefore, when the morning twilight begins, a ray, that is reflected from the top of the earths atmoſphere, deſcribes a line in the plane of the ſenſible horizon.

327. *When the evening twilight ends, or the morning twilight begins, a line drawn from the ſun to the top of the atmoſphere is a tangent to the earths ſurface.*

The evening twilight ends or the morning twilight begins, when a ray, that is reflected from the top of the atmoſphere, deſcribes a line BA, Plat. XXVIII. fig. 5, in the plane of the ſenſible horizon, by propoſition 326. Now if from E the common center both of the earth and the atmoſphere a line EB is drawn to the point of reflection B, this line will be perpendicular to the ſurface of the atmoſphere or reflecting ſurface at B the point of reflection. Therefore ABE is the angle of reflection. From hence it follows, that the ſun at that time muſt be at ſome point S, ſo that the incident ray, being SB, may make SBE the angle of incidence equal to ABE the angle of reflection, by propoſition 201 of optics. But

the

the ſenſible horizon, and conſequently any line drawn in the plane of it from B to A, touches the earths ſurface. Therefore when the evening twilight ends or the morning twilight begins, a ray of the ſun, that is reflected from the top of the atmoſphere, deſcribes after reflection a tangent to the earths ſurface. We are to prove that as the reflected ray BA, ſo likewiſe the incident ray SB, deſcribes a tangent to the earths ſurface: for we are to prove that a right line drawn from S the ſun to B, the top of the atmoſphere, is a tangent to the earths ſurface, and, by propoſition 34 of optics, the incident ray SB is ſuch a line. Imagine a line, as before, to be drawn from B the point of reflection, which is in the ſurface of the atmoſphere, to E the common center of the earth and atmoſphere, and this line BE will be perpendicular to the reflecting ſurface. Conſequently the angle SBE is the angle of incidence, and the angle ABE is the angle of reflection. Imagine farther a ſemidiameter of the earth AE to be drawn from the place of the ſpectator to the center; and another line as ED to be drawn from the center to the incident ray SB, ſo as to make the angle DEB equal to the angle AEB. Now in the triangles BAE, BDE, the angle ABE in one is equal to the angle DBE in the other, by propoſition 201 of optics, for ABE is the angle of reflection, and SBE or DBE is the angle of incidence. In the ſame triangles the angle AEB is equal to DEB, by the conſtruction; and the ſide BE is common. Therefore ſince there are two angles in one triangle reſpectively equal to two angles in the other triangle, and in each the ſide BE next to theſe two equal angles is the ſame, the two triangles will be equal to one another in all reſpects. Euc. b. I. prop. 26. Particularly the ſide ED in one triangle is equal to the ſide AE in the other. But AE is a ſemidiameter of the earth. Therefore ED is likewiſe a ſemidiameter of the earth. The angle BAE in one triangle is likewiſe equal to the angle BDE in the other. But ſince BA is a tangent and AE is a ſemidiameter drawn from A the point of contact, the angle BAE is a right one. Euc. b. III. prop. 18. Therefore BDE is likewiſe a right angle. Now if a right line as DB drawn from the extremity D of a ſemidiameter ED makes right angles with the ſemidiameter ED, the line DB and conſequently the line SB is a tangent to the circle. Euc. b. III. prop. 16. corol. Therefore SB a ray, which comes from the ſun S and falls upon the top of the atmoſphere B, is a tangent to the earths ſurface, when the reflected ray deſcribes a line in the plane of the ſenſible horizon, that is, by propoſition 326, when the evening twilight ends or the morning twilight begins.

328. *If the time is known, when the morning twilight begins or the evening twilight ends, the height of the atmoſphere may be determined.*

By

By knowing the time when the morning twilight begins and the evening twilight ends, it is found that at either time the ſun is 18°. below the horizon. Therefore if S, Plat. XXVIII. fig. 5, is the place of the ſun at that time and PAN is the horizon, the angle SBN is then 18°. Now, by propoſitions 326, 327, if B is the top of the atmoſphere the incident ray SB and the reflected ray BA touch the earth at D and A, when the morning twilight begins or when the evening twilight ends. But if, from the points of contact A and D, ſemidiameters of the earth AE and DE are imagined to be drawn, the angle AED, which theſe ſemidiameters contain at the center, is equal to NBS the ſuns depreſſion. For in the quadrilateral figure ABDE the four angles taken together are equal to four right ones. In conſequence of Euc. b. I. prop. 32. And, ſince the angles at A and D are both right ones, becauſe AB and DB are tangents, and AE and DE are ſemidiameters; Euc. b. III. prop. 18; it follows that the two remaining angles ABD and AED taken together are equal to two right ones, or ABD is the complement of AED to two right angles. But ABD is likewiſe the complement of SBN to two right angles. Euc. b. I. prop. 14. Therefore, ſince the ſame angle ABD is the complement either of AED or of SBN to two right ones, it follows that AED is is equal to SBN. So that when the morning twilight begins or the evening twilight ends AED is an angle of 18°. Indeed when the ſuns place below the horizon is found by the time either between the beginning of the twilight and the ſun riſing in a morning, or between the ſun ſetting and the end of the twilight in an evening, this is its true place: but its apparent place is raiſed about 30′ higher by the refraction, by propoſitions 139, 140, of optics. Now S, at the beginning or end of the twilight is the place from whence a ray, which is reflected in the line BA, appears to come, that is, by propoſition 137 of optics, S is the ſuns apparent place. Therefore ſince, when the ſuns true diſtance from the horizon is 18°, its apparent place is 30′ higher, and ſince S at the beginning of one twilight or the end of the other is its apparent place, the conſequence is that the angle NBS and likewiſe its equal AED, inſtead of being at that time 18°. is only 17°. 30′. The angle AEB is the half of AED, that is, when the morning twilight begins or the evening twilight ends, AEB is an angle of 8°. 45′. From hence we may determine RB the height of the atmoſphere.

In the angle AEB, AE is the radius, AB the tangent, and EB the ſecant. But RE is equal to AE, for they are both ſemidiameters of the earth. Conſequently RB the height of the atmoſphere is the difference between RE the radius and BE the ſecant of an angle of 8°. 45′. Therefore

fore as RE the radius, to RB the difference between the radius and ſecant of an angle of 8°. 45′, ſo is the number of miles contained in RE the ſemidiameter of the earth to the number of miles contained in RB the height of the atmoſphere. Now in the tables, the radius is to the difference between the radius and ſecant of an angle of 8°. 45′. as 10000 to 117; and the earths ſemidiameter in round numbers is 4000 miles. But as 10000 to 117, ſo is 4000 to 46 nearly. Therefore the height of the atmoſphere is about 46 miles.

329. *The twilights continue very long in a parallel ſphere.*

The twilight does not end till the ſun is 18° below the horizon. Now the ſuns depreſſion in a parallel ſphere is equal to its declination towards the depreſſed pole, by propoſition 260, and its declination towards the depreſſed pole can never be more than 23½ degrees, by propoſition 34. Therefore till the ſuns declination towards the depreſſed pole becomes 18° the twilight will laſt: from that time the twilight will ceaſe, only whilſt the ſuns declination is encreaſing from 18° to 23° ½ and decreaſing again in its return from the tropic, till in its decreaſe it becomes 18°. Thus the twilight will ceaſe but for a ſhort time in the half years night at either pole.

The ſun does not deſcend below the horizon of a parallel ſphere by its daily motion, by propoſition 259, for the daily motion carrys it in circles parallel to the horizon. It ſinks below the horizon only by its yearly motion: and as this motion will carry it but ſlowly below the horizon, the twilight will be long before it ends after the ſun is down, and will begin again as long before the ſun riſes.

330. *The twilights are of the ſhorteſt continuance in a right ſphere.*

The ſun in a right ſphere is always carryed by the daily motion in a circle perpendicular to the horizon. So that, when it ſets, it is carryed directly downwards by the whole daily motion, and conſequently gets 18° below the horizon the ſooneſt poſſible. For this reaſon the evening twilights, which end when the ſun is 18° below the horizon, will be very ſhort: and the morning twilights will likewiſe be as ſhort; for as long as the evening twilights laſt after the ſun ſets, ſo long the morning twilights begin before it riſes.

331. *The twilights in an oblique ſphere are ſhorter than in a parallel one, but longer than in a right one.*

The daily motion makes the ſun ſet below the horizon of an oblique ſphere, by propoſition 263, and conſequently it will get 18° below the

horizon ſooner, than if it had deſcended only by the annual motion, as it does in a parallel ſphere. But then it deſcends obliquely, and conſequently will not get 18° below the horizon ſo ſoon, as if it deſcended directly or at right angles, as it does in a right ſphere. Therefore in an oblique ſphere the twilight, after the ſun is ſet, continues for a leſs time than in a parallel one, but for a longer time than in a right one: for the twilight does not end till the ſun is 18° below the horizon.

The morning twilight continues as long before the ſun riſes as the evening twilight continues after it is ſet: ſo that what we have proved of the evening twilight in an oblique ſphere is equally true of the morning twilight.

In our own latitude, the twilight laſts all night for great part of the ſummer: for in great part of the ſummer the ſuns declination towards the north pole, which is our elevated pole, is ſo great, that it is never 18° below our horizon.

CHAP. XII.

Of the parallax.

332. *The parallax of the heavenly bodies is the change of their apparent ſituation in reſpect of one another, as the ſpectator views them from different ſtations.*

WE have ſhewn, in propoſition 135 of optics, that two objects, which are at different diſtances, will change their apparent ſituation in reſpect of one another, when they are viewed from different ſtations. In like manner ſome of the heavenly bodies, as a ſpectator views them from different ſtations, will change their apparent ſituations in reſpect of one another. This change of apparent place is in either caſe called the parallax. A ſpectator upon the earth changes his ſtation either by the earths daily motion round its axis, or by the earths annual motion in its orbit.

333. *The diurnal parallax of a heavenly body is the change of its apparent ſituation, in reſpect of ſome other body, ariſing from the ſpectators having changed his ſtation by the earths diurnal motion round its axis.*

It will be here neceſſary to obſerve that the fixed ſtars have no diurnal parallax, that is, they do not change their apparent ſituation in reſpect of one another, from any change that is made in the ſpectators ſtation by the earths daily rotation round its axis. The moon has a conſide-

ſiderable diurnal parallax; that is, from the ſpectators changing his ſtation, as the earth turns round upon its axis, the moon changes its apparent place, among the fixed ſtars. Suppoſe ABD, Plat. XXVIII. fig. 9, to be the earth: then let H be the moon, ZT a vertical circle, and A the place of a ſpectator. The moon H appears in the ſpectators horizon, and is ſeen in the line AHT, ſo that, by propoſition 135 of optics, it will be ſeen near the fixed ſtar at T. After the earth has turned one quarter round its axis, and the ſpectator is come to B, ſuppoſe the moon at H to be in his zenith, he then ſees it in the line BHO, ſo that it appears near a different fixed ſtar at O. Thus by the ſpectators changing his ſtation in reſpect of the moon, it has changed its apparent place amongſt the fixed ſtars: in one ſtation of the ſpectator it appeared near the fixed ſtar T, and in the other ſtation it appears near a different fixed ſtar O.

334. *When a ſpectator in his firſt ſtation ſees a planet in his horizon, and in his ſecond ſtation ſees the ſame planet in his zenith; the parallax is the ſame, as if his firſt ſtation had been at the earths ſurface, and his ſecond ſtation at the earths center.*

If a ſpectators firſt ſtation is at A, Plat. XXVIII. fig. 9, he ſees the planet H in his horizon, near the fixed ſtar T; and if by the earths daily rotation he comes to his ſecond ſtation at B, ſo as to have the planet in his zenith, it then appears near the fixed ſtar O. The change of apparent place then, or the parallax, is TO. But if the ſpectators firſt ſtation had been at A, and the planet H had appeared, as before, near the fixed ſtar at T; then ſuppoſe that without any diurnal motion, the ſpectator could immediately have taken his ſecond ſtation at C the earths center, and that he could have had a view of the ſame planet H from thence; at this ſecond ſtation he would ſee it in the line CHO near the fixed ſtar O, juſt as he would have done, if he had been at B: ſo that this change of ſtation from A to C would produce juſt the ſame parallax TO, as the other change of ſtation from A to B produced. From hence we ſhall get another definition of the diurnal parallax.

If the planet is at G when the ſpectator at A makes his firſt obſervation, its apparent place in the heavens will be at N: for the apparent place is always determined by a line AGN drawn from the ſpectator through the planet to the concave ſphere of the heavens: the ſpectators eye is at A one end of this line, the planet G is in the middle of this line, and the apparent place in the heavens is at N the other end of this line. But as the earth ADB turns round upon its axis, till the ſpectator comes to P, the planet, though it has had no real motion in the mean time, will appear

to have changed place among the fixed ſtars. Indeed, when the ſpectator is at A, the planet G is not in his zenith; and when he is come to P in the daily rotation, it will be in his zenith: but this is not the change of place that we are now ſpeaking of, for this is a change in reſpect of the earths ſurface only. The change, that we are ſpeaking of, is a change of apparent place in reference to the fixed ſtars. Thus the planets apparent place is cloſe to the fixed ſtar N, when the ſpectator is at A, but it is cloſe to a different fixed ſtar M, when the ſpectator is arrived at P: this change of apparent place from M to N ariſing from the earths diurnal rotation is called the planets diurnal parallax.

But whether the ſame ſpectator has an opportunity in the earths diurnal motion of viewing the planet G at two different times from two different ſtations A and P, or whether at the ſame time one ſpectator views the planet from one part of the earths ſurface A, and another ſpectator views it from a different part P; in either caſe the change of apparent place or the parallax will be the ſame. In one caſe the ſpectator by comparing his firſt obſervation with his ſecond will find that the parallax is NM; and in the other caſe the two ſpectators by comparing their obſervations with one another will find that he, who was at A, ſaw the planet cloſe to the fixed ſtar N, and that he who was at P, ſaw it cloſe to a different fixed ſtar M: ſo that the difference of the planets apparent place, as obſerved by theſe two ſpectators, will be NM. This difference of a planets apparent place, as obſerved by two ſpectators in different ſtations, is likewiſe called its diurnal parallax, becauſe the change is juſt the ſame, as if the firſt ſpectator had in the earths diurnal motion been transferred to the place of the ſecond. In this ſenſe we ſay that all meteors in the earths atmoſphere have a diurnal parallax, and that comets have no diurnal parallax. If the ſame meteor is viewed at one and the ſame time by different ſpectators in different parts of the earth, they will ſee it in different parts of the heavens. But if the ſame comet is thus viewed, all the ſpectators will ſee it in the ſame place among the fixed ſtars.

In like manner if the ſpectator had been at A, when he made his firſt obſervation upon a planet G, and at P, when he made his ſecond, the parallax, by propoſition 333, would have been NM. But if he had been at A the earths ſurface, when he made his firſt obſervation, and at C the earths center, when he made his ſecond, in this caſe likewiſe the change of the planets apparent place would be NM.

The reaſon, why the parallax ariſing from the diurnal motion is the ſame with the parallax, that would ariſe from the ſpectators making his firſt obſervation at the ſurface, and his ſecond at the center, is this. The

ap-

apparent place of any planet H in the heavens is determined by a line AHT, one end of which is at the ſpectators eye A, the other end T is at the heavens, and ſome intermediate point is at H the planet. Now if the planet has no real motion of its own, or if this real motion, during the time that the ſpectator changes his ſtation, is ſo ſmall as to be inſenſible, then it is evident that in the motion of the ſpectator from A to B, or whilſt the end of the line, which is next the ſpectators eye, is transferred with him from A to B, the point, which is at the planet H, will reſt: and therefore the line AHT will turn upon H as upon a center of motion, and the parallax will be proportional to the angular motion of the other end of it from T to O. Now whenever the angular motion of the end A is given on one ſide of the planet, the angular motion of the other end T will be given on the other ſide of the planet. But the angular motion of the end A is AHB, when the ſpectator is transferred by the diurnal motion from A to B, and the angular motion of this end A will be AHC =AHB, if the ſpectator was to be transferred from A the ſurface to C the center. Therefore the angular motion of T the other end of this line will be THO in either caſe: ſo that TO the parallax will likewiſe be the ſame in either caſe.

Or otherwiſe: the ſpectators change of ſtation is the ſame in reſpect of the planet H, whether he is transferred by the earths diurnal motion from A to B, or could transfer himſelf from A to C: for in either caſe he paſſes out of his firſt line of view AH into the line CBH. But if in either caſe the change of the ſpectators ſtation is the ſame, the apparent change of the planets ſituation in reference to the fixed ſtars, or the parallax TO, will likewiſe be the ſame.

335. *The diurnal parallax is the change of a heavenly bodys apparent place ariſing from the ſpectators having changed his ſtation from the earths ſurface to its center.*

This definition of the diurnal parallax amounts to the ſame with that already given in propoſition 333, as has been ſhewn, in propoſition 334. This is the definition we ſhall commonly refer to hereafter. One uſe of the other definition was to ſhew the analogy between the parallax of the heavenly bodies and the parallax that was explaned in propoſition 135 of optics. Another uſe was to ſhew the reaſon why this ſhould be called a diurnal parallax.

336. *If a ſemidiameter of the earth is drawn from the place of the ſpectator upon the earths ſurface; the parallax of any planet will always be proportional to the angle, which this ſemidiameter would ſubtend, if it was to be viewed from that planet.*

If the ſpectator is at A, Plat. XXVIII. fig. 9, and AC a ſemidiameter of the earth is drawn from thence, the parallax of any planet will be proportional to the angle, which this ſemidiameter of the earth would ſubtend, if it was to be viewed from the planet. Suppoſe the planet at H; then if the ſpectator was at A on the earths ſurface, AHT would be his line of view, and the planet would appear to him cloſe to the fixed ſtar T. If he was transferred from A the ſurface to C the center of the earth, the end of his line of view next to his eye would be transferred thither with him; the point H in this line would reſt, by what has been ſaid in propoſition 334, and the other end T of this line would be transferred from T to O: ſo that the planet, if it could be ſeen from the earths center, would then appear in the line CHO cloſe to the fixed ſtar O: and conſequently TO would be the parallax. Now TO ſubtends, and conſequently is proportional to, the angle THO. But as THO and AHB are vertical angles they are always equal to one another. Euc. b. 1. prop. 15. Therefore TO is proportional to AHB. But TO is the parallax, and AHB is the angle, which AC the earths ſemidiameter would ſubtend, if it was viewed from the planet H. Therefore the parallax is proportional to the angle, which a ſemidiameter of the earth drawn from the place of the ſpectator would ſubtend, if it was viewed from the planet.

337. *The parallax of a planet makes it appear lower in a vertical circle.*

That is, by propoſition 334, when a planet is viewed from the earths ſurface, it appears farther from the zenith or nearer to the horizon, than when viewed from the center, if it has any parallax. Let TZ*p* be a vertical circle: if the ſpectator is at A on the earths ſurface, the planet H appears cloſe to the fixed ſtar T, but if he was at C the earths center, the ſame planet H would appear cloſe to a different fixed ſtar O. Its apparent diſtance from the zenith or vertex Z is ZT in one caſe and ZO leſs than ZT in the other: ſo that it is apparently farther from the vertex Z or is lower in a vertical circle, when it is viewed from the ſurface, than when it is viewed from the center.

The reaſon of this is obvious from what has been ſaid, under propoſition 334. For if the ſpectater is at C the earths center, and the planet at H, his line of view is CHO: and conſequently O is the planets apparent

place

place, and ZO is its apparent distance from the vertex Z. But if the spectator was to rise from C the center to A the surface of the earth, the end C of his line of view would rise with him: and as the point H in this line is immoveable, the line would turn upon H as upon a center of motion: and consequently the motion of the end O next the planet would be contrary to the spectators motion: so that whilst the spectator rises from C to A, the end of his line of view next the planet sinks from O to T. But the apparent place of the planet among the fixed stars is at the end of his line of view next the planet. Therefore as he rises from the center to the surface the apparent place of the planet among the fixed stars sinks farther from the zenith or vertex; so that the planets apparent distance from the vertex is greater, when the planet is viewed from the surface, than when it is viewed from the center, or the parallax makes the planet appear lower in a vertical circle.

338. *The diurnal parallax in a vertical circle is the difference between the apparent distance of the planet from the vertex when it is viewed from the surface, and when it is viewed from the center.*

Thus if the planet G, Plat. XXVIII. fig. 9, is seen from A the earths surface, the line of view is AGN, and ZN is the apparent distance from the vertex Z: but if a spectator could have a view of the same planet from C the earths center, CGM would be the line of view, and ZM would be its apparent distance from the vertex Z: the difference between ZN and ZM, or MN is the planets diurnal parallax in the vertical circle pZT.

339. *The diurnal parallax of any planet, at a given distance from the earth, is the greatest of all, when the planet is in the horizon, and decreases as the planets altitude encreases: till at last, if the planet was to be in the zenith, the parallax would be nothing.*

If CH, Plat. XXVIII. fig. 9, is the distance of a planet H from C the earths center; the diurnal parallax OT will be the greatest, when the planet is at H in the horizon of a spectator at A: when the planet is at G, its parallax MN will be less: as its altitude encreases the parallax will decrease; for if the planet was at F, its parallax would be IL: and if the planet was at E in the vertex, its parallax would then be nothing, for it would appear in the same line of view CAEZ, whether the spectator was at A the surface or at C the center of the earth.

The parallax is proportional to the angle which AC a semidiameter of the earth drawn through the place of the spectator would subtend if it

was

was to be ſeen from the planet, by propoſition 336. Now the length of AC is given, for it is a ſemidiameter of the earth: and the diſtance from whence this ſemidiameter is ſuppoſed to be viewed is likewiſe given, for it is ſuppoſed to be viewed from the planet; and the planets diſtance is the ſame, whether it is in the horizon, or at any altitude above the horizon. Therefore the ſemidiameter AC, if it was viewed from the planet, would always ſubtend the ſame angle, by propoſition 129 of optics, provided it was always viewed at the ſame obliquity. But by an eye placed at the planet this diameter AC would be viewed at different obliquities, according to the different altitudes of the planet. When the planet is at H in the horizon, an eye at the planet would ſee the ſemidiameter AC directly or at right angles, and it would ſubtend the angle AHC: when the planet is at G, an eye placed there would ſee the ſame ſemidiameter AC obliquely, and for that reaſon, by propoſition 129 of optics, it would ſubtend a leſs angle AGC: when the planet is at F, ſo that its altitude is ſtill greater, an eye placed there would ſee the ſame ſemidiameter AC ſtill more obliquely, and therefore it would ſubtend a ſtill leſs angle AFC: and if the planet came to the vertex E the end of this ſemidiameter AC would be towards an eye placed at the planet, and conſequently its apparent length would be nothing, or it would ſubtend no angle at all. Thus the apparent length of AC ſeen from the planet decreaſes, as the planets altitude encreaſes, and becomes nothing, when the planet is in the vertex. But the parallax is proportional to the apparent length of AC or to the angle, which it would ſubtend if it was to be viewed by an eye placed at the planet. Therefore the parallax is the greateſt of all, when the planet is in the horizon, and decreaſes, as the planet riſes: ſo that if the planet comes to the zenith its parallax will be nothing.

One method amongſt many for finding out this parallax we may explane here. Let HMO, Plat. XXVIII. fig. 7, be an arc of the horizon; APVM an arc in the meridian; P the elevated pole; V the vertex; E the apparent place of a planet, when ſeen from the earths ſurface; and S its place if it could be ſeen from the earths center: then ES is the diurnal parallax in the vertical circle VE. When the planet is firſt obſerved at E, its altitude above the horizon muſt be determined; and then the complement of its altitude, or VE, which is its apparent diſtance from the vertex will be known. At the ſame time its diſtance from the meridian or the angle EVM, which is the diſtance between the vertical EV, where the planet is, and the meridian VM, muſt be determined; this diſtance is called the planets azimuth. This firſt obſervation is to be made before the ſtar comes to the meridian. A ſecond obſervation is to be made, after the planet

net has paſſed the meridian, when its altitude, and conſequently its complement of altitude, is the ſame that it was at the firſt obſervation, that is, the ſecond obſervation is to be made when the planet is at ſome point *e*, where *e*V its apparent diſtance from the vertex is equal to EV. Now if E is the apparent place of the planet, when at the time of the firſt obſervation we view it from the earths ſurface, and S would be its place, if at the ſame time we could view it from the earths center; and if *s* is its apparent place, when at the time of the ſecond obſervation we view it from the ſurface, and *s* would be its place, if at the ſame time we could view it from the center; then, though we know neither how far S is from the vertex V, nor how far *s* is from the ſame vertex, yet we may be ſure of this, that the parallax ES is equal to the parallax *es*, becauſe the planets altitude was the ſame at the time of both obſervations; and conſequently, ſince EV is equal to *e*V, SV is likewiſe equal to *s*V. So that, if PS is a ſecondary of the equator, which paſſed through the planet at the time of the firſt obſervation, and P*s* is another ſecondary, which paſſed through it at the time of the ſecond obſervation, the planet between the times of the firſt and ſecond obſervation muſt have deſcribed the arc S*s* in a circle of daily motion. By the time that has paſſed between the two obſervations we may find out this arc S*s*: by propoſitions 289, 290, 291. Thus ſuppoſe 4 hours to have paſſed then S*s* is equal to 4 times 15° or to 60°, by propoſition 289. Therefore there are 60°, in the circle of daily motion between S and *s*; and conſequently, ſince by propoſition 249, any circle of daily motion is parallel to the equator, there will likewiſe be as many degrees in the equator between the ſecondaries PS and P*s*. From hence then we may in the ſame manner always determine the angle SP*s*. Let us now ſee how many particulars are known in the ſpherical triangle PVS. Firſt, the angle PVS is known; becauſe EVM is known: for PVS is the complement of EVM to two right angles. Secondly the angle VPS is known; becauſe at the two obſervations the planet was at equal altitudes, and for that reaſon, by propoſition 272, it was at equal diſtances from the meridian APVM; therefore the meridian divides the angle SP*s* into two equal parts: and as the whole angle SP*s* is known, the half of it VPS muſt be known too. Thirdly the ſide PV is known, if the latitude of the place is known; becauſe PV is the diſtance of the elevated pole from the vertex: for the diſtance of the vertex from the horizon is 90°, and conſequently PV the poles diſtance from the vertex is the complement of the poles diſtance from the horizon, or, by propoſition 246, the complement of latitude. But when two angles and one ſide of ſuch a triangle are known, the rules of ſpherical trigonometry will determine

the length of any of the other ſides, and conſequently the ſide SV may thus be determined. But EV is known already. Therefore if SV is taken from EV, the remainder is SE or the planets parallax.

340. *The diurnal parallax of a planet, when it is in the horizon, is called its horizontal parallax.*

341. *At a given altitude of different planets, their diurnal parallaxes are inverſely as their diſtances from the earths center.*

If a planet is at *f*, Plat. XXVIII. fig. 9, where its altitude is *f*A*g*, and another planet is at *b* having an equal altitude *b*A*m*, their diſtances from the earths center are *f*C and *b*C, and their parallaxes will be inverſely as theſe diſtances: the nearer planet will have a greater and the more remote will have a leſs parallax. If the planet *f* is viewed from A the earths ſurface, the line of view is A*fr*, and *r* will be its apparent place among the fixed ſtars: if the ſame planet could be viewed from C the center C*ft* would be the line of view, and its apparent place among the fixed ſtars would be *t*: ſo that, by propoſition 338, its parallax is *rt*. In the ſame manner we might ſhew that *rs*, which is leſs than *rt*, is the parallax of the planet *b*.

Now if A is the place of the ſpectator, the parallax of each planet *f* and *b* is proportional to the angle, which AC would ſubtend, if it was viewed from the planet, by propoſition 336. But the real length of AC is given, becauſe it is a ſemidiameter of the earth: and it would be viewed at an equal obliquity by an eye placed at either planet *f* or *b*, becauſe each planet, by the ſuppoſition, is at an equal altitude. Therefore, by propoſition 126 of optics, the apparent length of AC, or the angle, which AC would ſubtend to an eye placed at either planet, would be inverſely as the planets diſtance from C: and conſequently the parallaxes of theſe planets are inverſely as their diſtances from C the earths center.

342. *The ſemidiameter of the earth is to the diſtance of any planet from the earths center, as the ſine of the planets parallax is to the ſine of its apparent diſtance from the vertex.*

Suppoſe that a planet is at F, Plat. XXVIII. fig. 9, then if the ſpectator is at A, where the line of view is AFL, the planet will appear at L, and ZAL will be its apparent diſtance from the vertex Z. Suppoſe farther that the parallax TL, or that the angle AFC, which, by propoſition 338, is proportional to the parallax, is known. Then AC the ſemidiameter

meter of the earth is to CF the diſtance of the planet from the earths center, as the ſine of the parallax AFC to the ſine of the apparent diſtance from the vertex ZAL. Here we muſt obſerve that, ſince LAC or FAC is the complement of ZAL to two right angles, FAC and ZAL have the ſame ſine. Therefore we are to prove that AC is to FC as the ſine of the angle AFC, or the ſine of the parallax, is to the ſine of the angle FAC, which is the ſame with the ſine of the planets apparent diſtance from the vertex. Now in the plane triangle ACF, the ſides are to one another as the ſines of the oppoſite angles, by propoſition 32 of optics. Therefore AC the ſemidiameter of the earth is to FC the planets diſtance from the earths center, as the ſine of the angle AFC, or the ſine of the parallax, is to the ſine of the angle FAC, which is the ſame with the ſine of the planets apparent diſtance from the vertex.

The horizontal parallax is of all the greateſt, by propoſition 339, and conſequently the horizontal parallax of the ſun or of a planet may be found with more exactneſs, than its parallax, when it has any altitude. Suppoſe that only the horizontal parallax is known, or the parallax, when the planet is at H. The planet is then 90°, from the vertex, or ZAT is a right angle; and ſo likewiſe is TAC or TAH its complement to two right ones. Therefore in the triangle AHC, AC the earths ſemidiameter is to HC the planets diſtance, as the ſine of the horizontal parallax AHC is to the ſine of HAC, which is then a right angle. Now the ſine of a right angle is the radius. Therefore the earths ſemidiameter is to the planets diſtance, as the ſine of the horizontal parallax is to the radius.

The ſuns horizontal parallax, according to the lateſt obſervations is about 10″. Therefore as the ſine of an angle of 10″, in the tables of ſines, is to the radius, or as 1 to 20600, ſo is the earths ſemidiameter to the diſtance of the ſun: that is, the diſtance of the ſun is equal to 20600 ſemidiameters of the earth, ſuppoſing its horizontal parallax to be 10″. But in obſervations upon ſo ſmall a parallax it is moſt likely that there ſhould be ſome error: and an error of only 2″ or 3″ would make a very conſiderable difference in the ſuns diſtance. The ſuns horizontal parallax 10″, is the angle, which, in propoſition 170, we ſaid was ſo ſmall that we might neglect it.

The moſt certain way of determining the ſuns diſtance is by the parallax of venus. For the larger the parallax of any planet is the leſs liable we are to errors in obſerving it: and, by propoſition 341, the nearer any planet is to us, the greater will be its parallax. Now venus in its inferior conjunction is nearer to the earth than any other planet ever is, by pro-

position 117. Therefore, when venus is in the inferior conjunction, it has a greater parallax than the ſun, or than any other planet has. But venus at its inferior conjunction is too near the ſun for us to ſee it, unleſs it happens to be in one of its nodes at the ſame time and ſo goes acroſs the ſuns diſc, by propoſitions 102, 103. Such a tranſit of venus acroſs the ſun will happen on may 26, 1761. Therefore at that time the parallax of venus may be obſerved at the time of its inferior conjunction. From the parallax of venus known we may determine its diſtance from the earth, when it is in its inferior conjunction. And, by propoſition 88, the diſtance of venus from the earth, when in the inferior conjunction, is to the diſtance of the earth from the ſun, as 276 to 1000. Conſequently if venus's diſtance from the earth at that time was known, we might determine the earths diſtance from the ſun by the rule of proportion: for as 276 to 1000, ſo is the number of ſemidiameters of the earth contained in venus's diſtance from the earth to the number of ſemidiameters of the earth contained in the earths diſtance from the ſun.

The other primary planets, except mars, are ſo remote as to have no ſenſible parallax. Indeed mars's parallax is very ſmall, it is commonly reckoned 25″ ½.

The reader, from propoſition 341, will obſerve the reaſon of what we mentioned at firſt, that the fixed ſtars have no diurnal parallax: this is owing to their remoteneſs: and conſequently for want of ſuch a parallax, we cannot determine their diſtance from the earth by the method laid down in this propoſition.

343. *The diurnal parallax of a planet in a vertical circle produces a parallax of declination and of right aſcenſion: unleſs the planet is in the meridian, for then there is only a parallax of declination.*

When a planet is ſeen from the earths ſurface, it does not appear in the ſame part of a vertical circle, or at the ſame diſtance from the vertex, that it would appear at, if it could be ſeen from the earths center, by propoſition 337. And from this parallax in a vertical circle it follows, that neither the declination nor the right aſcenſion of a planet is the ſame when we view it from the earths ſurface, that it would be, if we could view it from the center. This difference of declination and right aſcenſion, according to the different ſtation of the ſpectator, is called the parallax of declination and right aſcenſion.

Let HO, Plat. XXVIII. fig. 8, be the horizon, EC an arc of the equator, which cuts the horizon at C, P the pole of the world, Z the zenith, ZV a vertical circle, F a planets apparent place in the vertical circle ZV,

if

if the planet is ſeen from the earths ſurface, and I its apparent place in the ſame vertical circle, if we could view it from the earths center; then, by propoſition 338, FI is the parallax in a vertical circle. When the planets apparent place is F, PFA is a ſecondary of the equator paſſing through it; and when I is its apparent place, PIB is a ſecondary paſſing through it. Therefore AF is the planets declination, when it appears at F, and BI is its declination, when it appears at I. But by the parallax in a vertical circle, it appears at F, when ſeen from the ſurface, and at I, when ſeen from the center. Therefore its declination, when ſeen from the ſurface, is AF, and when ſeen from the center, is BI: the difference between AF and BI, which is DI, is the parallax of declination, that is, DI is the change of apparent declination ariſing from the different ſtation of the ſpectator at the ſurface or at the center of the earth.

When the planets apparent place is F, and PFA is the ſecondary of the equator, that paſſes through it, the diſtance of A from the firſt of aries is its right aſcenſion. But when the planets apparent place is I, and PIB is the ſecondary, that paſſes through it, the diſtance of B from the firſt of aries is the right aſcenſion. The difference of right aſcenſion, according as the planet appears at F or at I, is AB. But by the parallax FI in a vertical circle, the planet, which appears at F, when viewed from the earths ſurface, would appear at I, if it could be viewed from the earths center. Therefore the parallax FI in a vertical circle makes a difference or parallax of right aſcenſion AB.

But if the planet is in the meridian PZH, which is a vertical circle perpendicular to the equator; the parallax in ſuch a vertical circle will occaſion only a parallax of declination and no parallax of right aſcenſion. For ſuppoſe L to be the planets apparent place, if it is viewed from the earths ſurface, and N to be its apparent place if it could be viewed from the earths center. Then, by propoſition 338, LN is its diurnal parallax in a vertical circle. But then this vertical circle is perpendicular to the equator EC, becauſe it is the meridian. Therefore LE is the planets declination, when viewed from the ſurface, and NE is its declination, if it could be viewed from the center: and conſequently its parallax of declination is then LN equal to its parallax in a vertical circle. In ſuch a ſituation of the planet there can be no parallax of right aſcenſion. For the vertical circle PZH is a ſecondary of the equator, becauſe it is perpendicular to the equator. Therefore in whatever part of this vertical circle the planet appears, whether at L or N, it is in the ſame ſecondary of the equator, which cuts the equator at the point E: and conſequently the right aſcenſion of the planet will be the diſtance of the point E from the firſt

first of aries, whether the planet appears at L or at N: that is, the parallax in a vertical circle will make no difference of parallax of right aſcenſion.

344. *The diurnal parallax of a planet in a vertical circle produces a parallax of latitude and of longitude: unleſs the vertical circle is a ſecondary of the ecliptic, for then there will be only a parallax of latitude.*

If HO, Plat. XXVIII. fig. 8, is the horizon, P the pole, not of the world as in propoſition 343, but of the ecliptic, EC an arc in the ecliptic, which cuts the horizon at C, ZV a vertical circle, then we may prove this propoſition in the ſame manner as we proved the laſt. Let FI be the planets parallax: PFA a ſecondary of the ecliptic, which paſſes through it, when it is viewed from the earths ſurface and appears at F; and PIB a ſecondary of the ecliptic, which would paſs through it, if it could be viewed from the earths center, when it would appear at I. When the planet in the vertical circle ZV appears at F, its latitude is AF, and when it appears at I in the vertical circle, its latitude is BI. The difference then of latitude, according as the planet appears at F or at I, is DI or the difference between AF and BI. But by the parallax FI in a vertical circle, the planet will appear at F, when ſeen from the earths ſurface, and would appear at I, if it could be ſeen from the earths center. Therefore the parallax FI in a vertical circle produces a difference or parallax of latitude DI.

When the planet appears at F in the ſecondary of the ecliptic PFA, its longitude is the diſtance of the point A from the firſt of aries. But when the planet appears at I in the ſecondary of the ecliptic PIB, its longitude is the diſtance of the point B from the firſt of aries. So that there is a difference of longitude AB, according as the planet appears at F or at I in the vertical circle ZV. Now by the parallax FI in a vertical circle the planet will appear at F, when ſeen from the ſurface, and would appear at I, if it could be viewed from the center of the earth. Therefore the parallax FI in a vertical circle produces a difference or parallax of longitude AB.

But if the planet is in a vertical circle PZH, which paſſes through P the pole of the ecliptic and conſequently is a ſecondary of the ecliptic; its parallax in ſuch a vertical circle will produce only a parallax of latitude, but no parallax of longitude. For let LN be the parallax in this vertical circle: then if the planet is ſeen from the earths ſurface, where its apparent place is L, its latitude or diſtance from the ecliptic is LE; and if it could be viewed from the earths center, where its appa-

rent

rent place would be N, its latitude would then be NE: ſo that the difference or parallax of latitude in theſe different ſtations of the ſpectator would be LN, or the parallax of latitude is then equal to the whole parallax in a vertical circle. But ſince the vertical circle PZH is perpendicular to the ecliptic, by the ſuppoſition, in whatever part of the vertical circle the planet appears, whether at L or at N it is in the ſame ſecondary of the ecliptic; becauſe the vertical circle itſelf is a ſecondary of the ecliptic: the diſtance between the firſt of aries and the point E where this ſecondary cuts the ecliptic is the planets longitude. But whether the planet is viewed from the earths ſurface and appears at L, or whether it is viewed from the earths center and appears at N, it is ſtill in the ſame ſecondary of the ecliptic, which cuts the ecliptic at E: and conſequently its longitude is not altered by the different ſtation of the ſpectator.

345. *The menſtrual parallax of the ſun is the angle, which the ſemidiameter of the moons orbit would ſubtend, if it was viewed from the moon.*

The word *parallax* properly ſignifies *a change* of apparent place according to the different ſituation of the ſpectator; we have ſhewn in propoſition 333, that this word is uſed in its proper ſenſe, when we ſpeak of the daily parallax of the ſun or any planet: and, in propoſition 338, we have ſhewn farther that the daily parallax of any planet is proportional to the angle, which a ſemidiameter of the earth would ſubtend, if it was to be viewed from that planet. In conſequence of this, the angle, which the ſemidiameter of the lunar orbit would ſubtend if it was to be viewed from the ſun is likewiſe called a parallax of the ſun: but then as this ſort of parallax is referred to the ſemidiameter of the moons monthly orbit, it is called the monthly or menſtrual parallax.

346. *The menſtrual parallax of the ſun might be determined, if we could tell at what inſtant of time exactly half the moons diſc is enlightened.*

Suppoſe S, Plat. XXVIII. fig. 6, to be the ſun, T the earth, ALQ an arc in the moons orbit; and let the moon be at L, when juſt half its diſc is enlightened.

Firſt then, if we imagine lines as ST, SL, LT, which joyn the centers of the ſun, earth, and moon, and form a triangle, the external angle SLB at the moon is always equal to the angle, which the circle of light makes with the circle of the diſc, by propoſitions 107, 143. But when the moons diſc is half enlightened, the circle of light makes a right angle with the circle of the diſc, by propoſition 143. Therefore at that inſtant

the

the angle SLB is a right one. And when SLB is a right angle, SLT muſt be a right angle too. Euc. b. I. prop. 14.

Secondly, when the moon is at L, it appears from the earth T in the line TL, and the ſun appears in the line TS, ſo that the angle STL is the moons apparent diſtance or elongation from the ſun. By knowing the inſtant, when the moons diſc is half enlightened, we may determine what its elongation is at that inſtant. For the moon leaves the ſun, after it has been in conjunction, at the rate of about 733′ every day. Therefore by knowing what time has paſſed between the conjunction and the inſtant, when the moons diſc is half enlightened, we may determine how far the moon has left the ſun between the time of conjunction and that inſtant; and conſequently we may determine the moons elongation at that inſtant. By this means the angle STL is known.

Thirdly, all the angles in the triangle TSL taken together are only equal to two right ones. Euc. b. I. prop. 32. But the angle at L is one right angle. Therefore the two angles at T and S together are equal to another right angle: and conſequently the angle TSL is the complement of the angle STL. But the angle STL is known. Therefore its complement TSL is known. Now TSL is the angle, which TL a ſemidiameter of the lunar orbit would ſubtend, if it was to be viewed from the ſun, or TSL, which is thus found, is the ſuns menſtrual parallax.

Thus ſuppoſe the moon to be half enlightened when its elongation is 89°. 50′. the complement of this or 10′ is the ſuns menſtrual parallax. This is the ſmall angle, which we ſaid might be neglected, in propoſition 143.

From this menſtrual parallax we might determine the ſuns daily parallax. For if a ſemidiameter of the moons orbit ſubtends an angle of 10′ at the ſun; a ſemidiameter of the earth, which is a 60th part of a ſemidiameter of the lunar orbit would ſubtend an angle at the ſun equal to the 60th part of 10′, or an angle of 10″.

347. *When the ſuns menſtrual parallax is known, we may determine what proportion the ſemidiameter of the lunar orbit bears to the ſuns diſtance from the earth.*

In the triangle TSL, Plat. XXVIII. fig. 6, the angle at L is a right one when the moons diſc is half enlightened, by what has been ſaid under propoſition 346, and the angle TSL is the menſtrual parallax of the ſun. Conſequently if we make TS the radius, FL is the ſine of the menſtrual parallax. Therefore as the ſine of the parallax is to the radius, ſo is TL a ſemidiameter of the lunar orbit to TS the ſuns diſtance from the earth.

Thus

Thus ſuppoſing the menſtrual parallax to be 1°; the ſine of 1° is to the radius, about as 1 to 343. Therefore TS the ſuns diſtance from the earth is equal to about 343 ſemidiameters of the lunar orbit.

This method of determining the ſuns diſtance, however exact it may appear to be in theory, is of little or no uſe in practice. For it depends entirely upon our determining the very inſtant, when the moons diſc is exactly half enlightened. But the appearance of the moons diſc is the ſame, as far as our eyes can diſtinguiſh, at the inſtant it is exactly half enlightened, and ſome time before that inſtant and ſome time after it. Therefore it is not poſſible to be ſufficiently exact in determining the very inſtant, that we want to determine.

If any one thinks that the moon is exactly half enlightened when it is at Q, where its elongation is 90°; he may correct his miſtake from propoſition 143; where we ſhewed that the angle made by the circle of light and circle of the moons diſc is ſomething greater than the moons elongation; and conſequently they will make a right angle with one another, or the moons diſc will be half enlightened, before its elongation is 90°.

348. *The annual parallax of a heavenly body is the change of its apparent place as the ſpectators ſtation is altered by the annual motion of the earth.*

We have ſeen, in propoſition 122, that any ſuperior planet, if it had no motion of its own, would yet apparently move or change place among the fixed ſtars upon account of the earths annual motion in its orbit. Particularly, Plat. XXIII. fig. 8, whilſt the earth is moving in its orbit from *g* through *k* to *n*, the planet P, though it had no motion of its own, would yet appear to move from G through K to N among the fixed ſtars. This change of the planets apparent place from G to N ariſes from the earths annual motion; and therefore it is called the planets annual parallax. The arc GKN, which the planet thus deſcribes is its arc of regreſs, by propoſition 122: but the retrograde motion and conſequently the arc of regreſs or annual parallax will be the ſame, whether the planet reſts at P or revolves in an orbit of its own, as the earth does, provided the planet revolves ſlower than the earth, by propoſition 126.

When the earth is at A in its orbit, Plat. XXIX. fig. 1, if *ps* is its axis, the fixed ſtar E will appear in or near the pole of the world: the earth by its annual motion will paſs in its orbit ADBC round the ſun S from A to B, in half a years time; its axis *ps* will ſtill be inclined to the plane of its orbit, when at B, juſt as it was, when at A, by propoſition 31; and

conſequently this axis ρs will always be parallel to itſelf. In this poſition of the earth at B a different fixed ſtar F ſhould appear to be in or near the pole of the world, and the ſtar E ſhould appear to be removed to ſome diſtance from the pole towards the ſide A, or towards that part from whence the earth has moved.

The earth and its axis ρs in half a years time is transferred by the annual motion of the earth from any point A in its orbit, to the oppoſite point B. But, by propoſition 17, the pole of the world is always in the axis of the earth continued. Therefore by the earths annual motion the pole of the world changes its place in the heavens, and in half a years time is transferred from ſome ſtar E to ſome other ſtar F. A ſpectator upon the earth ſuppoſes his own place to be at reſt, and, in conſequence of that, ſuppoſes the pole of the world to be at reſt: therefore when he obſerves that the fixed ſtar E, which was near the pole at one time of the year, is at ſome diſtance from the pole at the oppoſite time of the year; he will impute this change of the ſtars apparent place in reſpect of the pole to a motion not of the pole but of the ſtar itſelf: and conſequently, whilſt the earth in its orbit is moving from A to B, and the pole of the world is moving from E to F, any fixed ſtar will appear to him to have moved the contrary way.

Since ſuch a change in the apparent place of a fixed ſtar changes its apparent diſtance from the pole, it muſt likewiſe change the complement of its apparent diſtance from the pole, which complement is the ſtars apparent diſtance from the equator, or the ſtars declination. Therefore if the fixed ſtars have any annual parallax their declination muſt be variable: and the change of declination muſt be in a direction contrary to the earths motion.

349. *The annual parallax of any heavenly body is proportional to the angle which a diameter of the earths orbit would ſubtend if it was to be viewed from that body.*

In propoſition 347, we have ſhewn that the arc of regreſs is the annual parallax of a ſuperior planet, and, in propoſition 123, we have ſhewn that half the arc of regreſs is proportional to the angle, which a ſemidiameter of the earths orbit would ſubtend, if it was to be viewed from the planet. Therefore the whole arc of regreſs, or the annual parallax, of any ſuperior planet is proportional to the angle, which a diameter of the earths orbit would ſubtend, if it was to be viewed from that planet.

If, when the earth is at A, Plat. XXIX. fig. 1, the fixed ſtar E appears at or near the pole of the world, and when the earth has moved to the oppo-

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PLATE XXVIII.

Fig. 1.

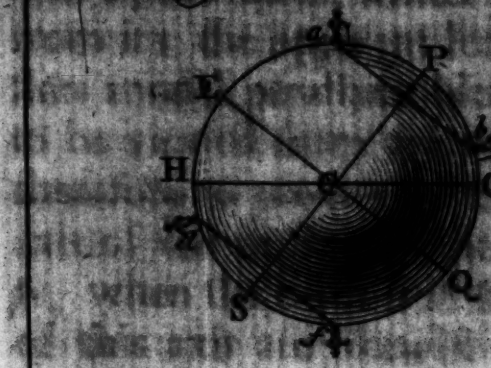


Fig. 2.

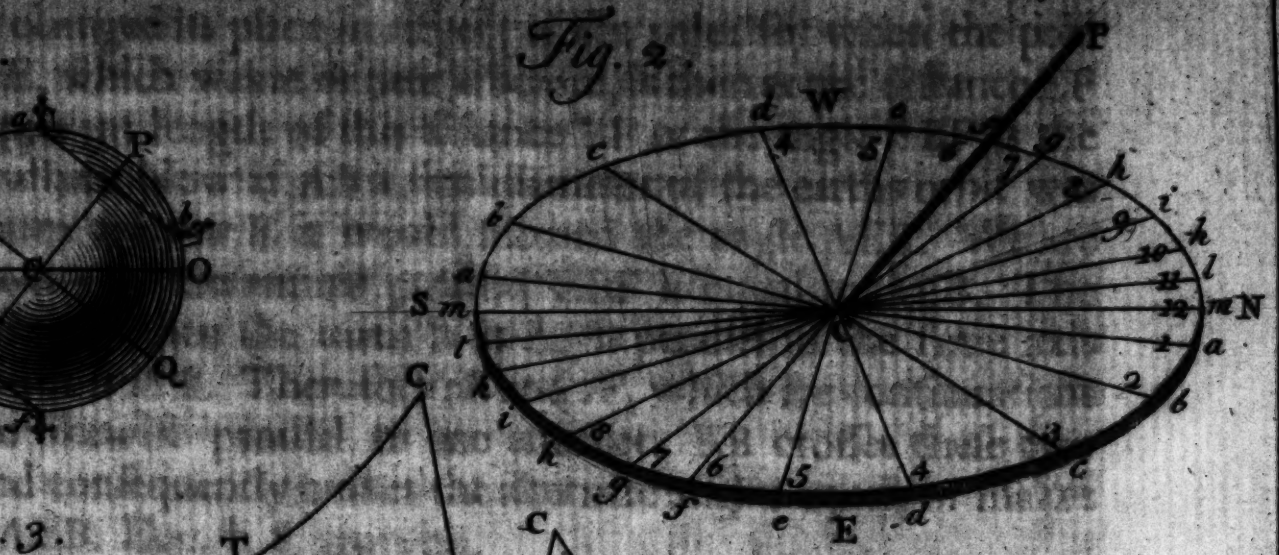


Fig. 3.

Fig. 4.

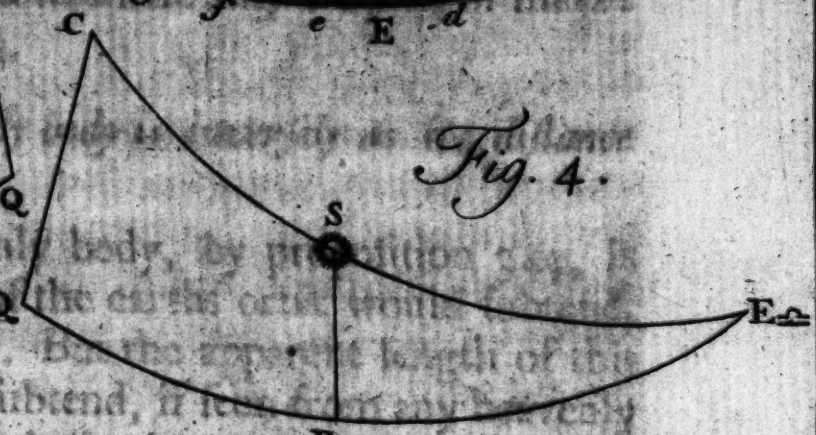


Fig. 5.

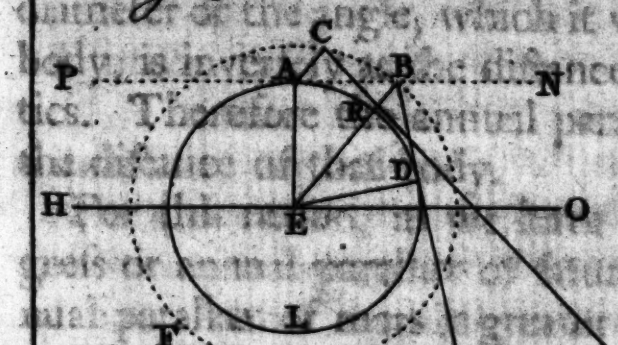


Fig. 6.

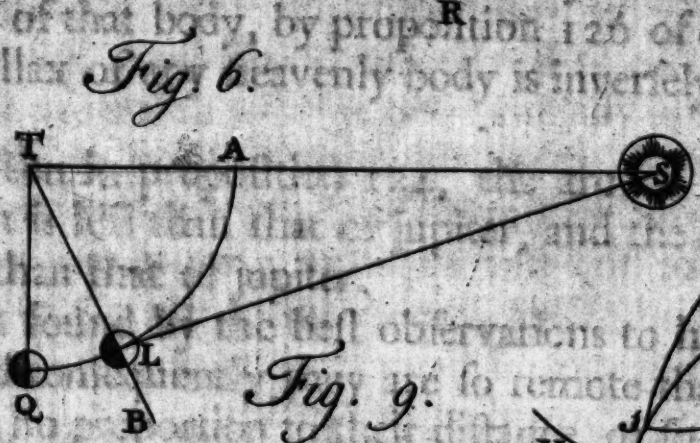


Fig. 7.

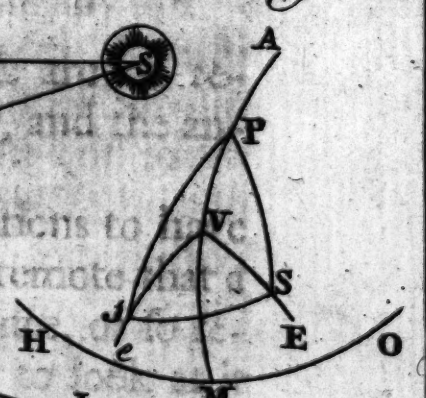


Fig. 8.

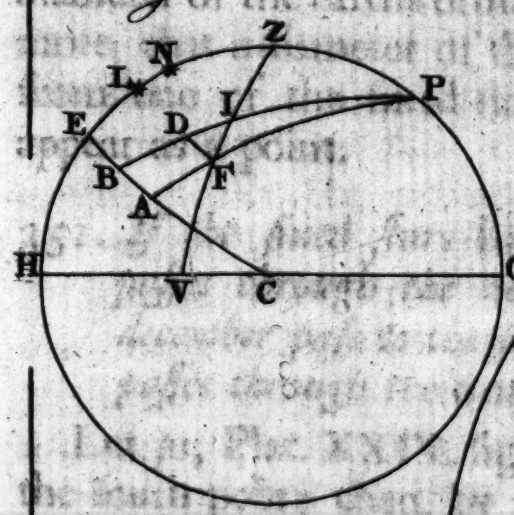
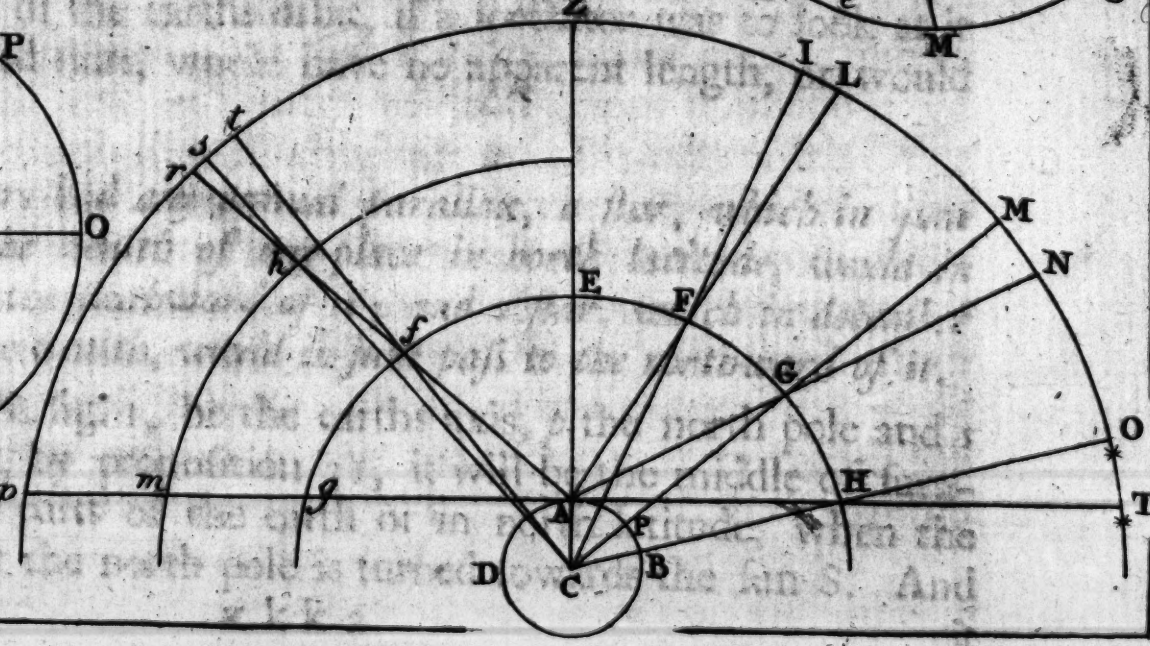
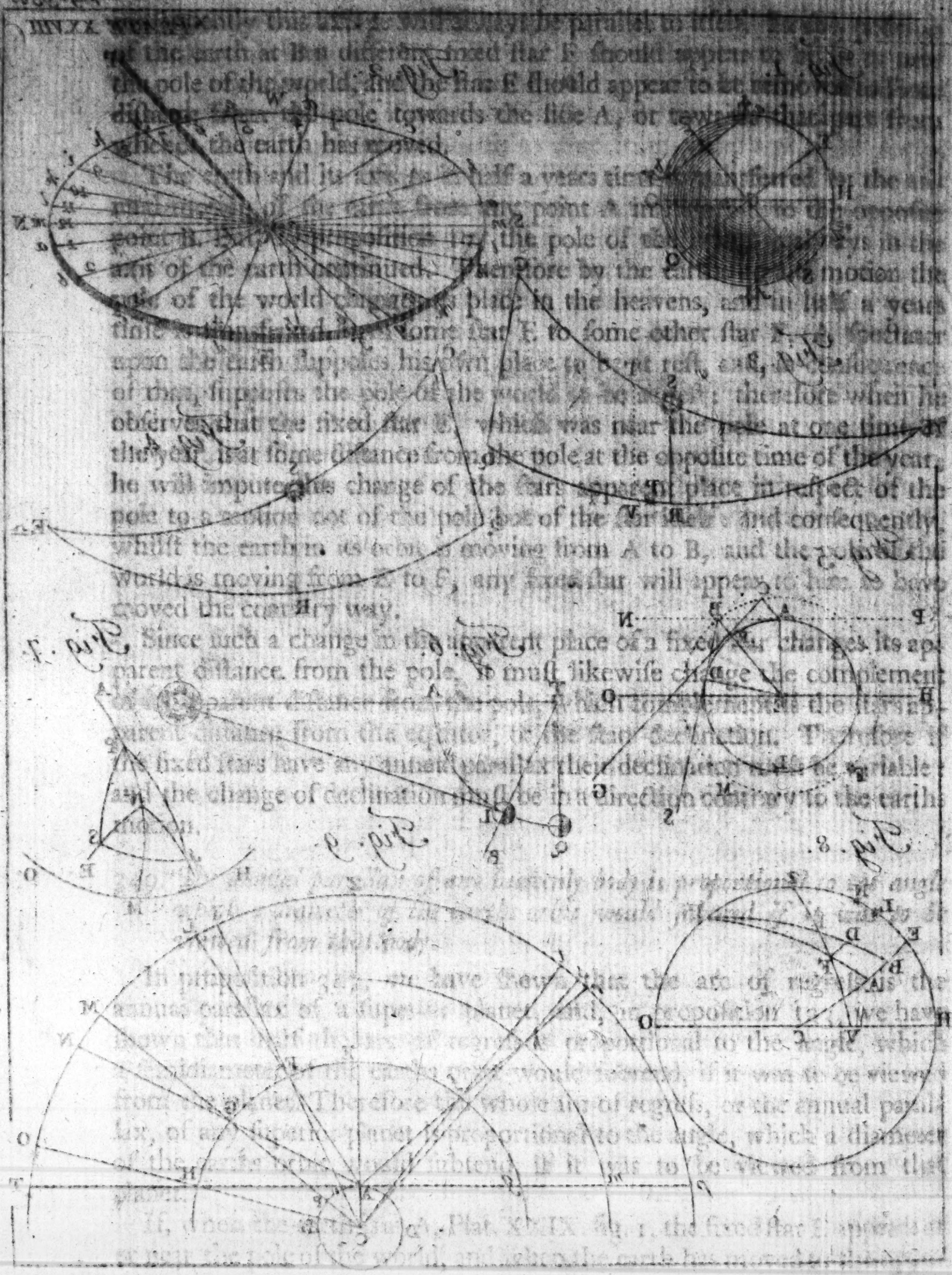


Fig. 9.





fite point B, a different fixed ſtar F appears at or near the ſame pole; the ſtar E will have changed its place in reſpect of the pole: for when the pole is at F, the ſtar E, which was at or near it before, is then at the diſtance EF from it: the apparent length of this diſtance EF or the angle EBF is the ſtars annual parallax. Now if AB a ſemidiameter of the earths orbit was to be viewed from the ſtar E it would ſubtend the angle AEB. We are therefore to prove that the annual parallax EBF is equal to AEB. The axis of the earth *ps*, when the earth is at A, is parallel to the ſame axis *ps*, when the earth is at B. Therefore *s*E and *s*F, which are continuations of this axis are likewiſe parallel to one another. EB croſſes theſe two parallel lines, and conſequently makes the alternate angles equal, or makes EBF equal to AEB. Euc. b. I. prop. 29.

350. *The annual parallax of any heavenly body is inverſely as its diſtance from the earth.*

The annual parallax of any heavenly body, by propoſition 349, is equal to the angle, which a diameter of the earths orbit would ſubtend, if it was to be viewed from that body. But the apparent length of this diameter or the angle, which it would ſubtend, if ſeen from any heavenly body, is inverſely as the diſtance of that body, by propoſition 126 of optics. Therefore the annual parallax of any heavenly body is inverſely as the diſtance of that body.

For this reaſon, as we have ſeen in propoſition 124, the arc of regreſs or annual parallax of ſaturn is leſs than that of jupiter, and the annual parallax of mars is greater than that of jupiter.

The fixed ſtars have not been found by the beſt obſervations to have any ſenſible annual parallax: and conſequently they are ſo remote that a diameter of the earths orbit bears no proportion to their diſtance, or ſo remote, that a diameter of the earths orbit, if a ſpectator was to look at it from one of the fixed ſtars, would have no apparent length, or would appear as a point.

351. *If the fixed ſtars had any annual parallax, a ſtar, which in june paſſes through the zenith of any place in north latitude, would in december paſs to the ſouthward of it; and a ſtar, which in december paſſes through the zenith, would in june paſs to the northward of it.*

Let *ps*, Plat. XXIX. fig. 1, be the earths axis, *p* the north pole and *s* the ſouth pole: then, by propoſition 31, it will be the middle of ſummer in the northern parts of the earth or in north latitude, when the earth is at A, ſo that the north pole is turned towards the ſun S. And

it will be the middle of winter there, when the earth is at B, ſo that p the north pole is turned from the ſun S. Therefore A is the place of the earth in june, and B is its place in december. Or, becauſe p the north pole points towards B, we may call B the northern ſide of the orbit: and becauſe s the ſouth pole points towards A, we may call A the ſouthern part of the orbit. Or we may call B the northern ſide of the orbit and A the ſouthern ſide; becauſe when the earth is at B, its heliocentric place is in the northern ſigns of the ecliptic, and when it is at A, its heliocentric place is in the ſouthern ſigns.

In june, when the earth is at A in the ſouthern part of its orbit ACBD, ſuppoſe a ſtar G to paſs through the zenith of a place L, which is in north latitude: then after the earth, by its annual motion, has gone half round its orbit, and is come to B the northern part of its orbit, the ſtar H will be in the zenith of the place L, and the ſtar G will have changed its apparent place in reſpect of the zenith by the annual parallax. And as this change of apparent place makes the ſtar appear to have moved the contrary way to the earth, by what has been ſaid under propoſition 348; therefore ſince the earth by moving from A to B has moved northward, the ſtar G will in reſpect of the zenith have apparently moved ſouthward: and conſequently in december, when the earth is at B, the ſtar G will be to the ſouthward of the zenith.

For the ſame reaſon, if in december the ſtar H is in the zenith of any place L in north latitude, then, as the earth moves from B to A, it moves from north to ſouth; and conſequently the annual parallax will make the ſtar apparently move the contrary way, or from ſouth to north. Therefore in june, when the earth is come to A, the ſtar H having moved from ſouth to north in reſpect of the zenith will be to the northward of the zenith.

Or otherwiſe. When a ſtar G paſſes through the zenith of a place L in north latitude, its northern declination is equal to the places latitude, by propoſition 277. But, by what has been ſaid under propoſition 348, the annual parallax changes a ſtars declination, and this change is made in a direction contrary to the earths motion. Therefore as the earth moves from A the ſouth part of its orbit to B the north part, the north declination of the ſtar G will decreaſe; and by this means the north declination of this ſtar will become leſs than the latitude of the place L: conſequently the ſtar G will then not paſs through the the zenith of the place, but to the ſouthward of it; and its apparent diſtance from the zenith will be proportional to the angle GBH=AGB, which

which is the angle that AB a diameter of the earths orbit would ſubtend, if it was viewed from the ſtar G.

In like manner when a ſtar H paſſes through the zenith of a place L, in north latitude, its northern declination is equal to the latitude of the place. But the annual parallax changes a ſtars declination the contrary way to the earths motion. Therefore as the earth moves from B the north part of its orbit to A the ſouth part, the north declination of the ſtar H will encreaſe, and by this means, the north declination of this ſtar will become greater than the latitude of the place L: conſequently the ſtar H will then not paſs through the zenith of the place, but to the northward of it.

352. *If a fixed ſtar, that has no annual parallax, paſſes through the zenith of a place at one time of the year, it ſhould paſs through the zenith of the ſame place at all other times of the year.*

If any ſtar G, Plat. XXIX. fig. 1, paſſes through the zenith of a place L, when the earth is at A, the ſame ſtar, provided it has no parallax, ſhould paſs through the zenith of the ſame place L at the oppoſite time of the year, when the earth is at B. For when the earth is at B, H is the zenith of the place L; and the diſtance of the ſtar G, from H the zenith is GBH. But GBH is equal to AGB the ſtars annual parallax, by propoſition 349. Therefore, if the ſtar has no annual parallax, that is, if the angle AGB is nothing, the angle GBH will likewiſe be nothing, or G will be cloſe to H, ſo that the ſame ſtar, which paſſes through the zenith G, when the earth is at A, ſhould likewiſe paſs through the zenith H, when the earth at the oppoſite time of the year is at B.

353. *The motion of the earth and the progreſſive motion of light will make a fixed ſtar, which has no parallax, apparently deviate from its true place in the ſame direction that the earth moves.*

If a ſtar S, Plat. XXIX. fig. 2, paſſes through the zenith of a place, when the earth is at B, it ſhould likewiſe, by propoſition 352, paſs through the zenith of the ſame place, when the earth is at D, or at A, ſuppoſing that BA is a diameter of the earths orbit and that the ſtar has no annual parallax. Now in order to ſee a ſtar through a teleſcope, when it is in the zenith, the teleſcope muſt be perpendicular to the horizon: and conſequently, if the ſtar S has no parallax, it might be ſeen through ſuch a vertical teleſcope, either when the earth is at B or when it is at D. This would be the caſe if the motion of light was inſtantaneous

neous, that is, if a ray of light paſſed through any ſpace SD or CD in an inſtant. But, by propoſition 35 of optics, the motion of light is progreſſive, or a ray will take up ſome time to deſcribe the ſpace CD. And from hence we are to prove that by the motion of the earth from B to D, the ſtar S will deviate from its true place, and will appear at s, ſo that, if B is the northern ſide and A the ſouthern ſide of the earths orbit, whilſt the earth is moving from B to D, or from north to ſouth, the ſtar S will appear to have moved the ſame way: and conſequently, if in the daily motion it paſſed through the zenith of any place, when the earth was at B, it will paſs to the ſouthward of the zenith, when the earth is come to D. Such an apparent deviation as this was obſerved by the ingenious and accurate profeſſor Bradley: and ſuch a deviation cannot be accounted for by any annual parallax of the ſtar; for any deviation cauſed by an annual parallax is made in a direction contrary to the earths motion, by propoſition 348; but this, that we are ſpeaking of, is made in the ſame direction with the earths motion.

When the earth is at B, the ſtar S appears in our zenith, and conſequently will be ſeen through a vertical teleſcope, or a teleſcope, which is in the direction BP perpendicular to our horizon. The reader may here imagine that a ſtar S could not be ſeen through a teleſcope in the direction BP, when the earth is at B, becauſe ſuch a teleſcope is not pointed directly towards the ſtar. But he may correct his miſtake if he recollects, that we ſuppoſe the ſtar to have no annual parallax, and conſequently that BD a ſemidiameter of the earths orbit, bears no proportion to SD the ſtars diſtance; or, in reſpect of SD, BD is nothing. Now he will readily perceive that, if it was not for the reaſon we are going to explane, when the earth is at D, a ſtar S might be ſeen through a vertical teleſcope in the direction DC. But, in reſpect of DS the ſtars diſtance, DB is nothing, or B and D are cloſe together. Therefore, if the ſtar has no parallax, it might be ſeen through a vertical teleſcope in the direction BP, when the earth is at B, as well as when the earth is at D. Thus the reader will find that his miſtake, if he has fallen into one, was owing to an incorrectneſs in the figure, which could not be avoided: for BD ought to have been infinitely ſhort in reſpect of DS, as the ſtar S is ſuppoſed to have no parallax: but this could not be done, unleſs we had either made DS infinitely long, which is impoſſible, or had made BD only a point, and then BD would not have anſwered our future purpoſes. Thus we have ſhewn, that, when the earth is at B, a ſtar S in our zenith would be ſeen through a teleſcope in the direction BP. And when the earth is come to D, the ſame ſtar G might be ſeen

through

through a telescope in the same vertical position DC, if the star had no parallax; and the motion of light was not progressive. What alteration the progressive motion of light will make in the stars apparent place, we are next to enquire.

Let the velocity of light be to the velocity of the earth as CD to BD, and considering these as the two sides of a parallelogram, draw CB the diagonal: then the direction of a telescope, in order to see the star S, when the earth comes to D must be DH, parallel to this diagonal BC. Suppose that BC was a very long slender telescope, through which only one ray of light can pass at a time, or suppose it to be the axis of a telescope, that is as long as this, but not so slender. The star S cannot be seen through this telescope BC, but through the telescope BP, when the earth is at B; as has been shewn already. But if the telescope was to continue in the same position, and in this position to move along with the earth to D, so as to be in the line HD, when the earth arrives at D, the star S might then be seen through it. For if a ray of light was to enter the upper end C of the telescope, when the earth is at B, it would just be come to the lower end of it at the instant the earth arrives at D. The rays strait course is the line CD: and if it comes to the eye without interruption it must be always in the line CD, by proposition 34 of optics. The ray cannot come strait along CB the axis of the telescope and arrive at the eye in this axis, unless it is always in the axis. But if the ray in order to come to the eye must be in the line CD and in the line CB, it must always be in the common intersection of these two lines. C is this common intersection, when the earth is at B: *c* is the common intersection, when the earth is at F, and the telescope is at FG: and D is the common intersection, when the earth comes to D and the telescope is in the line HD. Thus the common intersection descends down the line CD, whilst the earth moves from B to D: and since the velocity of light is to that of the earth as CD to BD, a ray of light will likewise have descended down CD, whilst the earth was moving from B to D. Therefore in the whole motion of the telescope the ray will have been in the common intersection of the line CD and the axis of the telescope: and consequently it will have passed along the axis of the telescope and will come without interruption to the eye at D.

From hence it appears that by the progressive motion of light, a ray, which comes through the telescope HD will arrive at the eye, when the earth is at D: and for that reason, by proposition 113 of optics, the eye at D will see the star S, through such a long telescope in the direction HD. But the length of a telescope makes no difference in the direction

that

that is neceſſary to ſee a ſtar through it. Therefore, when the earth is at D, the axis of any teleſcope muſt be in the direction DH in order to ſee the ſtar S. Now when a ſtar is ſeen in the axis of any teleſcope, it muſt be directed towards the apparent place of that ſtar; and *s* is the point, towards which the axis of a teleſcope lying in the line HD is directed: therefore, when the earth is at D, *s* is the apparent place of the ſtar S. Thus the ſtars apparent place *s* has deviated from its true place S, in the direction S*s*, which is the ſame as the direction BD; or this deviation is made in the ſame direction, that the earth is moving in from B to D. So that if a motion in BD is from north to ſouth, the ſame ſtar S, which appeared in the zenith of a place, when the earth was at B, will appear to the ſouthward of the zenith, when the earth is come to D.

In the ſame manner we may ſhew, that if any ſtar S appears in the zenith of a place, or through a vertical teleſcope AP, fig. 3, when the earth is at A, the ſame ſtar by the progreſſive motion of light and the motion of the earth from A to D, or from ſouth to north, will deviate from the zenith northwards: ſo that, when the earth is come to D, the ſtar will appear in the axis of a teleſcope lying in the direction DL and pointing to *n*; and therefore the ſtars apparent place will be at *n* on the north ſide of the zenith. For firſt, the axis of a teleſcope, which is a right line drawn down the middle of it, may be conſidered as a ſlender tube, through which only one ray of light can paſs at once: and whenever a ſtar appears in the axis, it is made viſible by a ſucceſſion of rays that come along in the axis. Secondly, the rays, that come from the ſtar S to the point D, deſcribe the right line SD. Now if the earth is at A, and a teleſcope was placed with its axis in the direction AC, the ſtar S could not be ſeen in the axis of that teleſcope: for the ſtar, by the ſuppoſition is in the zenith, and conſequently can only be ſeen in the axis or middle of a vertical teleſcope in the direction AP. But ſuppoſe, when the earth is at A, that a ray of light in its progreſs down S, is juſt arrived at C: then if a teleſcope was placed with its axis in the line AC, the ray would be both in the line CD, and likewiſe in the line AC, which is a continuation of the axis of the teleſcope: for C is the common interſection of CD and CA, and conſequently is in both theſe lines. As the earth moves on from A to D, and the teleſcope moves on with it in a direction parallel to AC, the common interſection of the lines CA and CD will deſcend along the line CD, ſo that, when the earth arrives at D, the common interſection of the lines CA and CD will likewiſe be at D. Conſequently the velocity of the earth is to the velocity of this interſection as AD to CD, by propoſition 10 of mechanics. Therefore if

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the velocity of the earth is likewiſe to the velocity of light as AD to CD, the velocity of any ray deſcending along the line CD will be the ſame as the velocity of that common interſection. From hence then it follows that any ray, which was at C in the common interſection of the lines AC and DC, when the earth was at A, will in its deſcent along CD keep pace with this common interſection, ſo that the ray will be always in the common interſection. Therefore, when the earth comes to D, the ray will be at D too. Thus in the whole motion of the earth and teleſcope the ray has always been in the common interſection of the axis of the teleſcope and the line of its own direction CD, that is, it will never have been out of the lines AC, CD in the whole motion of this line from AC to DL: ſo that when the eye comes to D, the ſtar will appear in the axis of a teleſcope that lies in the direction DL. But a teleſcope, whoſe axis is in the direction DL, is directed to the point *n*; and the ſtar, ſeen in the axis of the teleſcope, appears in that place towards which the axis is directed. Therefore *n* will be the apparent place of the ſtar. And conſequently, whilſt the earth moved from A to D or from ſouth to north, the ſtar S will have deviated in the ſame direction; if it appeared at P in the zenith, when the earth was at A, it will appear at *n* to the northward of the zenith, when the earth is arrived at D.

354. *If the earth had no motion, or if the propagation of light was inſtantaneous; a fixed ſtar, that paſſes through our zenith at one time of the year, would never deviate from its true place, but would paſs through our zenith at all other times of the year.*

Firſt if we ſuppoſe that the earth at B, Plat. XXIX. fig. 2, has no motion, and that the difference of ſeaſons is made by the ſuns revolving round the earth, as is ſuppoſed in the Ptolemaic ſyſtem; it is evident that, if a fixed ſtar is ſeen to paſs through our zenith at any one time of the year, the ſame ſtar muſt be ſeen to paſs through our zenith at any other time of the year. For as the ſtar is fixed or has no motion, and as the earth likewiſe, by the ſuppoſition, has no motion, it is impoſſible that any alteration ſhould be made in the apparent place of the ſtar. So that if it could be ſeen in the axis of a vertical teleſcope BP in december, it would be ſeen in the axis of the ſame vertical teleſcope in march, or june, or ſeptember. In this caſe we may farther obſerve that whether the propagation of light is ſucceſſive or inſtantaneous, that is, whether it takes up ſome time to paſs through the line CS, or paſſes through it in an inſtant, the velocity of light muſt be infinite in reſpect of the earths velocity. Becauſe the earths velocity is nothing, by the ſuppoſition, and conſequently

it will bear no proportion to the velocity of light, whether the velocity of light is great or ſmall.

Secondly if the earth does move, yet if the fixed ſtars have no parallax and the propagation of light is inſtantaneous; any fixed ſtar will never deviate from its true place: ſo that, if a fixed ſtar could be ſeen in our zenith or in the axis of a vertical teleſcope BP, when the earth is at B, it would likewiſe be ſeen in the axis of a vertical teleſcope DC, when the earth is at D. For if the propagation of light was inſtantaneous, it would take up no time to paſs from one place to another, however great the diſtance might be between thoſe places: and conſequently a ray of light would run to an infinite diſtance, whilſt the earth was deſcribing the finite line BD. Therefore the velocity of light would be infinite in reſpect of the earths velocity; or, which is the ſame thing, the earths velocity would be nothing in reſpect of the velocity of light. In this caſe if CD is to BD as the velocity of light to the velocity of the earth; either CD muſt be infinite or BD muſt be nothing. But either way BD bears no proportion to CD, or, in reſpect of CD, BD is nothing. And if BD is nothing, the angle BCD vaniſhes. Therefore if the propagation of light was inſtantaneous, the angle BCD would be nothing. Now DH the direction, in which the axis of a teleſcope muſt be placed to ſee the ſtar S, when the eye arrives at D, muſt be parallel to BC, by propoſition 353. Therefore the angle HDC muſt be equal to the alternate angle BCD. Euc. b. I. prop. 29. But if the propagation of light was inſtantaneous, the angle BCD and conſequently the alternate angle HDC would be nothing. Therefore DH and DC would coincide. So that, if the propagation of light was inſtantaneous, the axis of a teleſcope muſt be vertical in order to ſee the ſtar S, as well when the earth is at D, as when it was at B. Therefore in this caſe the ſtar S would not deviate at all from its true place.

By Bradleys obſervations the ſtar marked γ in the dragon paſſed very near his zenith december 17. From that time till march it deviated ſouthwards, and in march it paſſed 20″ more to the ſouth of his zenith than it did at his firſt obſervation. From that time till june its ſouthern deviation decreaſed, and about the beginning of june it paſſed at the ſame diſtance from his zenith as in december. It then began to deviate northwards, and in ſeptember it paſſed about 20″ more to the north than it did in june. Its northern deviation then kept decreaſing, till it arrived in the following december to the ſame place where he had ſeen it at his firſt obſervation.

Thus from december to june, whilſt the earth was moving from north to ſouth, the ſtar deviated ſouthwards: and from june to december, whilſt

whilſt the earth was moving from ſouth to north, it deviated northwards. That is, its deviation was always in the ſame direction, in which the earth was moving. Such a deviation could not happen, unleſs the earth moved and the motion of light was progreſſive. Therefore the earth does move, and the motion of light is progreſſive.

The reaſon, why the ſtars ſouthern deviation ſhould encreaſe from december to march and then decreaſe again till june; and why its northern deviation ſhould encreaſe from june till ſeptember, and then decreaſe again till december, ſhall be ſhewn in the following propoſitions.

355. *The ſlower the earth moves either ſouthward or northward, the leſs a fixed ſtar will deviate from its true place; and the faſter the earth moves either ſouthward or northward, the more the ſtar will deviate.*

This follows from the firſt part of propoſition 354. For we have there proved, that, if the earth had no motion either ſouthward or northward, a fixed ſtar would not deviate at all. And it ſeems to be an eaſy conſequence from hence that the ſlower the earth moves, the leſs the ſtar will deviate; and the quicker the earth moves, the more the ſtar will deviate.

356. *The ſouthern deviation of a fixed ſtar in or near our zenith will encreaſe from december till march, and will decreaſe from march till june: its northern deviation will encreaſe from june till ſeptember, and will decreaſe from ſeptember till december.*

If B, Plat. XXIX. fig. 4, is the northern, and A the ſouthern point of the earths orbit, and the earth revolves in its orbit, according to the order of the letters BHIKLMANOPQR; it will be at B in december, by what has been ſaid in propoſition 351, and at A in june; and conſequently at one of the equinoxes, or in march, it will be at K, and at the other equinox, or in ſeptember, it will be at P. Though the earth in its annual motion revolves in its orbit BKAP round the ſun at D, yet in half a years time, its motion from north to ſouth may be reduced to the line BA. For though in half a years time from december to june, its motion is not along the line BA but in the curve BKA, yet in that time it is carryed directly from north to ſouth only from B to A. Conſequently its motion from north to ſouth is to be eſtimated by a line drawn from B to A, or by the line BA, and its velocity from north to ſouth will be proportional to the ſpaces deſcribed in equal times upon the line BA. Thus, let us ſuppoſe the earths orbit to be circular, and the ſun at D to be in its center: for though this is not exactly true, by propoſition 53, yet the

earths orbit is ſo nearly circular and the ſun is ſo near the center, that in the preſent caſe we may neglect the difference: and farther let us ſuppoſe, that the earths motion in its orbit is uniform, for though this, by propoſition 53, is not true, yet the earths velocity is ſo nearly the ſame at all times that we may neglect the difference. Upon theſe two ſuppoſitions, the earth will deſcribe an equal part of its orbit every month: ſo that if the whole orbit BKAP is divided into twelve equal parts, ſuch as BH, HI, IK, &c. the earth will deſcribe the firſt of theſe parts BH in december, the ſecond HI in january, the third IK in february, &c. ſo that at the beginning of march it will be at K.

From hence we may ſee that its motion referred to the line BA or its motion from north to ſouth is accelerated from the beginning of december till the beginning of march: and is retarded from the beginning of march till the beginning of june. For when it has deſcribed the arc BH in december, it will have gotten ſouthward of the point B, or, in reference to the line BA, it will have gotten forward only through the ſpace BC: when it has deſcribed the arc HI, it will have gotten from north to ſouth or will have moved, in reſpect of the line BA, through the ſpace CE: when it has deſcribed the arc IK, it will have moved from north to ſouth, or in reſpect of the line BA, through the ſpace ED. In like manner we may ſhew that in each of the three next months, from the beginning of march to the beginning of june, the earths motion from north to ſouth, or in reſpect of the line BA, will be through the unequal ſpaces DF; FG, GA. Now the ſpaces, which the earth deſcribes in reference to the line BA, or from north to ſouth, encreaſe from B to D, and then decreaſe from D to A. Therefore the earths velocity from north to ſouth is firſt accelerated and then retarded: it is accelerated whilſt the earth is going through BK the firſt quarter of its orbit, or from the beginning of december till the beginning of march, and is retarded, whilſt it is going through KA the ſecond quarter of its orbit or from the beginning of march till the beginning of june. In like manner we may ſhew, that, whilſt the earth is moving northwards from A through P to B, its velocity northwards or in reference to the line AB is accelerated, whilſt it is deſcribing AP the third quarter of its orbit, or from the beginning of june till the beginning of ſeptember, and then is retarded again, whilſt it is deſcribing PB the fourth quarter of its orbit or from the beginning of ſeptember till the beginning of december.

The reaſon of this difference of the earths velocity from north to ſouth, and from ſouth to north, at different times of the year, may eaſily be underſtood, if the reader will obſerve, that, when the earth is in any point of its

its orbit, its motion as to the direction of it, whilſt at that point is the ſame as if it was moving in a tangent to that point. Thus when the earth is at B, the direction of its motion is the ſame in its orbit, as if it was moving in the tangent B*b*; when it is at K, the direction of its motion is the ſame, as if it was moving in the tangent K*k*, &c. Now ſince, when the earth is at B, the tangent B*b* is perpendicular to BA, a motion in this tangent B*b* would carry the earth from weſt to eaſt, but would not carry it forwards at all in reſpect of the line BA, or from north to ſouth: ſo that the earth, when it is at B is ſtationary, as to any motion ſouthwards. But when the earth is at K, the tangent K*k* is parallel to BA; therefore at that time the whole motion of the earth is parallel to BA, or its whole velocity carrys it from north to ſouth. For the ſame reaſon, when the earth is at A, where the tangent A*a* is from eaſt to weſt, the earths whole velocity is in that direction; and conſequently no part of its motion carrys it forwards in the direction AB or from ſouth to north: but when the earth is at P, the tangent P*p* is parallel to AB, and for that reaſon the whole velocity will carry it forwards in the direction AB, or from ſouth to north. Hence it appears that when the earth is at B or A it moves neither ſouthwards nor northwards, when it is at K it moves ſouthwards the faſteſt, and when it is at P it moves northwards the faſteſt. As the earth is moving from B towards K, the direction of the tangent is conſtantly changing, till from being perpendicular it becomes parallel to BA: the velocity of the earth from ſouth to north is the greateſt, when this tangent becomes parallel to BA: therefore its velocity from north to ſouth encreaſes, as the tangent becomes more nearly parallel to BA, that is, as the earth moves from B towards K. As the earth is moving from K towards A, the direction of the tangent is conſtantly changing, till from being parallel it becomes perpendicular to BA: the velocity of the earth from north to ſouth is nothing, when this tangent becomes perpendicular to BA: therefore its velocity from north to ſouth decreaſes, as the tangent becomes more nearly perpendicular to BA, that is, as the earth moves from K towards A. In like manner we might ſhew, that the earths velocity from ſouth to north encreaſes, as it moves from A to P, and decreaſes again, as it moves from P to B.

Now, by propoſition 355, the ſouthern deviation of a ſtar will encreaſe and decreaſe, as the earths velocity ſouthwards encreaſes and decreaſes. But we have juſt ſhewn, that the earths velocity ſouthwards encreaſes from december till march, and decreaſes from march till june. Therefore if a ſtar paſſes through our zenith in december, it will deviate more and more to the ſouthward of our zenith till march, and then it will

will be at its utmoſt ſouthern limit: and from march till june its ſouthern deviation will decreaſe, ſo that in june it will paſs through our zenith again. In like manner we have ſhewn, that the earths velocity northwards encreaſes from june till ſeptember, and decreaſes from ſeptember till december. Therefore a ſtar, which paſſes through our zenith in june, will deviate more and more to the northward of our zenith till ſeptember, and then it will be at its utmoſt northern limit: and from ſeptember till december its northern deviation will decreaſe, ſo that in december it will again paſs through our zenith as at firſt.

357. *A fixed ſtar always deviates from its true place towards a point which is* 90° *behind the ſuns place in the ecliptic.*

The direction of the earths motion in any point of its orbit is the ſame as if it was moving in a tangent to that point. Thus when the earth is at B, Plat. XXIX. fig. 4, its direction is in the tangent B*b*; for at the point of contact B the orbit and tangent coincide. At the point K the earths direction is in the tangent K*k*. And ſo on in every other point of the orbit. Therefore when the earth is in any point of its orbit, if a tangent is drawn from that point the ſame way that the earth is moving, the deviation of any fixed ſtar from its true place will be in the direction of that tangent: for the deviation of a fixed ſtar, by propoſition 353, is always in the ſame direction with the earths motion.

Now the direction of a tangent to that point of the orbit where the earth is, will always be 90°. forwarder than the earths heliocentric place. For when the earth is at B, if the ſun is at D, the earths heliocentric place will be in the line DB ♋: but the tangent B*b* makes a right angle or an angle of 90° with DB. Euc. b. III. prop. 18. Therefore the direction of this tangent will be 90°. forwarder than the earths heliocentric place. But the ſuns geocentric place is 180°. forwarder than the earths heliocentric place. For when the earth is at B and its heliocentric place is in the line BD ♋, the ſuns geocentric place will be on the oppoſite ſide of the ecliptic in the line BDA ♑. Since then the ſuns geocentric place is 180°. forwarder than the earths heliocentric place, and ſince the direction of the tangent is only 90°. forwarder than the earths geocentric place, it follows that the direction of the tangent, and conſequently the point towards which a fixed ſtar deviates, will always be 90°. behind the ſuns geocentric place. Thus likewiſe when the earth is at K ♎ the direction of the tangent K*k* will be 90°. behind the line KDP ♈, which determines the ſuns geocentric place. When the earth is at A: the ſuns geocentric place is in the line ADB ♋, and the direction of the tangent A*a* is 90°. behind it. When the earth is at P ♈,

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the ſuns geocentric place is in the line PDK ♑, and the direction of the tangent *Pp* is 90°. behind it. And in the ſame manner we might ſhew that in every other point of the orbit the direction of the tangent, and conſequently the point towards which a fixed ſtar deviates, is 90°. behind the ſuns geocentric place.

358. *A fixed ſtar, whoſe true place is in the pole of the ecliptic, will in its annual deviation appear to deſcribe a ſmall circle round the pole of the ecliptic.*

A ſtar deviates towards a point which is 90°. behind the ſuns geocentric place, by propoſition 357. But the ſuns geocentric place, and conſequently a point 90°. behind it, is always in the plane of the ecliptic, by propoſition 28. Therefore a ſtar in the pole of the ecliptic will always deviate towards the plane of the ecliptic. The deviation of ſuch a ſtar is about 20″: and conſequently the ſtar will at all times of the year appear 20″ nearer to the plane of the ecliptic than its true place. So that the ſtars true latitude being 90°. becauſe it is ſuppoſed to be in the pole of the ecliptic, its apparent latitude will be only 89°. 59′. 40″. But then as the ſun moves round in the plane of the ecliptic from weſt to eaſt, the point 90°. behind the ſuns geocentric place moves round in the ſame manner. But this is the point towards which a ſtar deviates, by propoſition 357. Therefore as this point is moving round from weſt to eaſt the ſtar in its deviation muſt apparently move the ſame way. Now ſince the ſtars apparent place is always 20″ from its true place in the pole of the ecliptic; and ſince its apparent place annually moves round from weſt to eaſt; ſuch a deviation will make the ſtar appear annually to deſcribe a circle round the pole of the ecliptic at the diſtance of 20″ from it. The diſtance between the ſtars apparent places at two oppoſite times of the year is equal to a diameter of this circle, or 40″. It will be 40″ more to the ſouth at one ſolſtice than it is at the other, and 40″ more to the eaſt at one equinox than it is at the other.

359. *A fixed ſtar, whoſe true place is in the plane of the ecliptic, will in its annual deviation appear to deſcribe a ſmall arc in the plane of the ecliptic.*

A fixed ſtar in the plane of the ecliptic will have no deviation in latitude. For it always deviates towards a point that is 90°. behind the ſun, and conſequently by propoſition 28, towards a point which is in the plane of the ecliptic. Therefore, ſince the ſtars true place is in the plane of the ecliptic, by the ſuppoſition, and ſince it always deviates towards ſome

ſome point in the plane of the ecliptic; its apparent place will always be in the ſame plane with its true place. Yet, by propoſition 353, its apparent place at ſome one time of the year will be 20″ on one ſide of its true place, and at the oppoſite time of the year it will be 20″ on the other ſide; ſo that, its apparent places at oppoſite times of the year will be at 40″ diſtance from each other. But the deviation of ſuch a ſtar is in the plane of the ecliptic, as has juſt been proved. Therefore ſuch a ſtar in its deviation will appear every year to deſcribe an arc of 40″ in the ecliptic from weſt to eaſt and from eaſt to weſt alternately.

360. *A fixed ſtar, whoſe true place is any where between the pole of the ecliptic and the plane of it, will in its annual deviation appear to deſcribe a ſmall ellipſis round its true place.*

The diameter of the circle of deviation, when a ſtars true place is in the pole of the ecliptic, is 40″ from north to ſouth, by propoſition 358. But there is no deviation from north to ſouth or from ſouth to north in reſpect of the ecliptic, that is, there is no deviation in latitude, when a ſtars true place is in the plane of the ecliptic, by propoſition 359. Since then a ſtars deviation in latitude is the greateſt, if its true place is in the pole of the ecliptic, and is nothing, if its true place is in the plane of the ecliptic; the conſequence is that the deviation in latitude will be ſo much the leſs as the ſtar is nearer to the plane of the ecliptic.

The diameter of the circle of deviation is 40″ from weſt to eaſt, when a ſtars true place is in the pole of the ecliptic, by propoſition 358: and the arc of deviation is 40″ in the ſame direction, when a ſtars true place is in the plane of the ecliptic, by propoſition 359. Since then a ſtars deviation from weſt to eaſt or from eaſt to weſt is the ſame whether its true place is in the pole or in the plane of the ecliptic, the conſequence is that the deviation from weſt to eaſt, or from eaſt to weſt will be the ſame in any intermediate place, or when a ſtars true place is any where between the pole and the plane of the ecliptic.

From what has heen ſaid it appears, firſt, that the curve of deviation is as broad from north to ſouth, as it is long from eaſt to weſt, when the ſtars true place is in the pole of the ecliptic, that is, the curve of deviation will then be a circle, having the ſtars true place in its center. Secondly, if the ſtars true place is in the plane of the ecliptic, the curve of deviation is an arc, the breadth of which is nothing, and the length of which from weſt to eaſt is 40″, having the ſtars true place in the middle of it. Therefore thirdly, if the ſtars true place is any where between the pole and the plane of the ecliptic, the breadth of the curve of deviation will be leſs than 40″ and greater than nothing: conſequently

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the curve is longer than it is broad: but its length will be 40″: half the breadth of this curve, and likewiſe half the length of it, will be on one ſide of the ſtars true place; and the other half both of its breadth and length will be on the other ſide. The length of this curve is from weſt to eaſt parallel to the ecliptic, and the breadth of it is, in reſpect of the ecliptic, from north to ſouth. Now ſuch a curve as this is an ellipſis having the ſtars true place in its center; the breadth of it is the ſhorter axis, and the length of it is the longer axis of the ellipſis. The longer axis of this ellipſis will be the ſame, wherever the ſtars true place is: but the ſhorter axis of it will be ſo much the longer, as the ſtars true place is nearer to the pole of the ecliptic, and ſo much the ſhorter, as the ſtars true place is nearer to the plane of the ecliptic: for if the ſtars true place was in the pole of the ecliptic, the ſhorter axis of the ellipſis would be equal to the longer, and the curve of deviation would be a circle: and if the ſtars true place was in the plane of the ecliptic, the ſhorter axis of the ellipſis would be nothing, and the curve of deviation would be an arc of the ecliptic equal in length to the longer axis.

CHAP. XIII.

Of the comets.

361. *The comets are at a greater diſtance than the moon.*

THE moon has a diurnal parallax, and the comets are not found to have any. Therefore by propoſition 341, the comets are farther from the earth than the moon is. This is ſufficient to ſhew the miſtake of thoſe philoſophers, who imagined that comets were meteors floating in the earths atmoſphere. For if this opinion was true, comets would be much nearer to us than the moon is, and conſequently would have a very conſiderable diurnal parallax.

362. *The comets are not ſo remote as the fixed ſtars, but deſcend into the planetary ſyſtem.*

When the earth is between the ſun and a comet, though the comet is ſo far off as to be juſt about to diſappear, its motion is either retrograde, or elſe becomes ſlower. Therefore, by propoſition 348, it has an annual parallax, and by propoſition 350, is not ſo far off as the fixed ſtars. The annual parallax of ſome comets, when they are thus diſappearing, has been found greater than thoſe of jupiter or mars: and conſequently at that time they muſt have been nearer to the earth than thoſe planets. In like manner

may be ſhewn, that, when they are on the other ſide of the earth towards the ſun, they are ſometimes found to deſcend lower or to move nearer to the ſun than the inferior planets.

363. *The comets are opake bodies.*

A comet, at a given diſtance from the earth, ſhines much brighter, when it is on the ſame ſide of the earth with the ſun, than when it is on the contrary ſide. If a comet was a luminous body, it would appear equally bright at equal diſtances, on which ever ſide it was. But ſince, at a given diſtance, it appears the brighteſt, when it is on the ſame ſide with the ſun; we may conclude that it owes its brightneſs to the nearneſs of the ſun; and therefore that it is in itſelf an opake body, and ſhines only by reflecting the ſuns rays.

364. *The comets are fixed and durable bodies like the planets.*

The comet, which appeared in 1680, was ſo near the ſun in its perihelion that its heat was 2000 times greater than that of red-hot iron. For, upon december 8, its diſtance from the ſun was to the earths middle diſtance about as 6 to 1000. The ſquares of theſe diſtances are 36 and 1000000. The heat of the ſun, like the light of it, at theſe different diſtances, is as theſe ſquares inverted, by propoſition 22 of mechanics, that is, the heat at the comet was to the heat of a ſummers day as 1000000 to 36 or as 28000 to 1. Now the heat of boyling water is about 3 times greater than the heat of dry earth expoſed to a ſummers ſun. Therefore the heat of the comet was to the heat of boyling water about as 28000 to 3. The heat of red-hot iron is about 4 times greater than that of boyling water. Therefore the heat of the comet was to the heat of red-hot iron, about as 28000 to 12, or about as 2000 to 1. In ſuch a heat as this any vapours or exhalations would have been conſumed or diſſipated: but the comet remained entire after it had paſſed its perihelion: ſo that a comet cannot conſiſt of vapours or exhalations, but muſt be a fixed and durable body, as the planets are.

365. *The comets ſeem to deſcribe very eccentric ellipſes round the ſun placed in one of the focuſes.*

If GHKT, Plat. XXIX. fig. 5, is the earths orbit, which is nearly though not exactly circular; the places of a comet, as obſerved in the heavens, will agree to an elliptical orbit ABDEC, which is very eccentric and has the ſun at S in one of its focuſes; that is, a comet appears to move, juſt as it would do, if its orbit was ſuch an eccentric ellipſis. Now ſince,

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by propoſitions 363, 364, the comets are in all other reſpects like the planets; it is moſt agreeable to the ſimplicity of nature, and therefore moſt probable, that the comets deſcribe orbits of the ſame ſort with thoſe deſcribed by the planets. But the planets deſcribe elliptical orbits having the ſun in one of their focuſes. Therefore ſince the places of a comet, as obſerved in the heavens, will agree very well to an elliptical orbit; it is moſt probable that the comets deſcribe ſuch orbits.

Indeed Kepler from his own obſervations has concluded that the trajectory or path of a comet is a right line: which he might eaſily do, if he obſerved the comet only in one or two parts of a very eccentric elliptical orbit. When a comet is deſcending from its aphelion to its perihelion, if the orbit is a very eccentric ellipſis, the part DB will differ little from a right line. Whilſt the comet is deſcribing the more curve part of its orbit BAC, it is in its perihelion, and may be ſo near to the ſun as not to be ſeen by us, upon account of the ſuns greater light. And when it emerges out of the ſuns rays at C, its trajectory CE will, as before, differ little from a right line. Now when the comet has gone through its perihelion and emerges out of the ſuns rays, it appears ſo very different from what it did before, that it might eaſily paſs for a different comet: for before it came to its perihelion, it would have little or no tail, and after it has gone through its perihelion, the tail will be very long. And thus the ſame comet deſcribing an elliptical orbit might be miſtaken for two different comets, one of which deſcribed the right line DB towards the ſun, and the other the right line CE from the ſun. The appearance would be the ſame, whether two different comets deſcribed the trajectories DB, and CE, or one comet deſcribed the trajectory DBACE, which could only be ſeen, whilſt it was in the lines DB or CE, and was hidden in the ſuns rays, whilſt it was in the curve BAC that connects the other two parts DB and CE.

Perhaps the two focuſes S and F may be at an infinite diſtance. If ſo the trajectory of a comet inſtead of being an elliptical orbit will be a parabola. And as a parabola is a curve, which does not return into itſelf, it will follow from this ſuppoſition, that a comet deſcribing ſuch a curve muſt come down from an infinite diſtance BP, fig. 6, to its perihelion, and then muſt aſcend to an infinite diſtance PA, ſo as never to return. But it is more probable from what has been ſaid concerning the analogy between planets and comets, that comets deſcribe elliptical orbits, or curves returning into themſelves, and conſequently, that the ſame comet returns at ſtated periods: the obſervations of this ſort have been made too ſeldom, and till lately with too little accuracy to determine the periods of any of the comets.

Halley is of opinion that the comet, which Appian obſerved in the year 1531, was the ſame that Kepler obſerved between 75 and 76 years afterwards in the year 1607, and that he himſelf obſerved after another period of the ſame length in the year 1682. This opinion is confirmed from the appearance of a comet in the year 1456, which is about 75 years before Appians obſervations. And from another appearance of a comet in the year 1305, which is 151 or about twice 75 years before the appearance in 1456. Thus therefore the acount of this comet ſtands, if it was indeed the ſame comet. The firſt appearance mentioned in hiſtory was in the year 1305. Whether it appeared 75½ years afterwards is not ſaid. Its next appearance, that is taken notice of, was after twice 75½ years, in the year 1456. It appeared again in the years 1531, 1607, and 1682. And, if it was the ſame comet, it will appear again after another period of between 75 and 76 years, that is, in the year 1758. If it ſhould appear at that time, there will be no doubt of its being the ſame comet, of its deſcribing an orbit, which returns into itſelf, and of its having a ſtated periodical time as the planets have.

A remarkable comet is ſaid to have appeared 44 years before Chriſt in the year that Julius Cæſar was murdered. After 575 years, that is, in the year after Chriſt 531, as Malela affirms, there was ſuch another comet. After 575 years more in the year 1106, in the reign of our Henry I, ſuch another comet appeared. And after about 575 years more in the year 1680, a very remarkable comet appeared again. The correſpondence of theſe periods makes it probable, that it was the ſame comet: and if it was, it will appear again after another period of 575 years or in the year 2255.

The comet, that appeared in the year 1680, according to Halleys calculation, was not above the length of a ſemidiameter of the ſun to the northwards of the earths way, november 11 at 1 o' clock in the afternoon: and if the earth had then been in that part of its orbit, the comet would have had a diurnal parallax equal to that of the moon. And he adds that he leaves it to thoſe, who are ſtudious in phyſical matters, to conſider what might be the conſequences of ſo near an approach; or of a contact; or laſtly of a ſhock of the celeſtial bodies, which is by no means impoſſible to come to paſs.

We have obſerved, in propoſition 29, that the inclination of the orbits of all the planets to the plane of the ecliptic is ſo ſmall, that though the planets do not move in the ecliptic, yet they are always within the zodiac. But the orbits of the comets make much greater angles with the plane of the ecliptic. The inclination of the orbit of that

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comet which appeared in 1680, was 60°. 56′. The orbit of a comet ſeen in 1682 was inclined to the plane of the ecliptic at an angle of 17°. 56′. Indeed not more than one or two have been obſerved, whoſe orbits have not been inclined to the plane of the ecliptic at greater angles than any of the planets orbits are.

The planets, as in propoſition 7, move round the ſun from weſt to eaſt, and ſo do ſome of the comets; but there are other comets which move the contrary way from eaſt to weſt.

366. *The tail of a comet is ſmoke, which riſes from the nucleus or head in a line oppoſite to the ſun.*

We have ſhewn, in prop. 126 of hydroſtatics, by what means ſmoke is made to riſe in the earths atmoſphere. When a body is on fire, a ſtream of rarefyed air is conſtantly aſcending from it: and by the impulſe of this air ſmall particles of the body are conſtantly made to aſcend. The ſtream of air is the vehicle; and the ſmall particles of the body, which are impelled upwards by that ſtream, are the ſmoke. The nucleus of a comet is ſurrounded with an atmoſphere: and the ſmall particles of this atmoſphere, when they are impelled upwards, compoſe the tail: for it is obſerved that after a comet has paſſed its perihelion, where the tail is the longeſt, the atmoſphere, which ſurrounds the nucleus, is ſenſibly diminiſhed: therefore ſince the atmoſphere is leſs as the tail is longer, it follows that the tail conſiſts of a part of the comets atmoſphere. We are therefore to enquire in what vehicle the particles of the comets atmoſphere are raiſed, and for what reaſon they ſhould always riſe in oppoſition to the ſun.

Firſt as to the vehicle, by which the particles of a comets atmoſphere are raiſed; we are to obſerve, that moſt of the comets, at their firſt appearance, have no tails, and that the tails begin to riſe as they approach the ſun. Now if there is a fine and ſubtile fluid diffuſed through the heavens, though it reaches to a great diſtance from the ſun, we may conſider it as the rarer parts of the ſuns atmoſphere. But this ſolar atmoſphere will be more heated in thoſe parts, where a comet paſſes, than in any other part at the ſame diſtance from the ſun. For the other parts are heated only by the ſuns direct rays; and the heat of direct light, where there is no refraction and no reflection, is very inconſiderable. But thoſe parts, where the comet paſſes, are heated both by the direct rays, and by the rays, which are reflected from the nucleus: and beſides, the nucleus itſelf is heated in its approach to the ſun; to how intenſe a degree it may be heated, when it comes to the perihelion, has been

been ſhewn, in propoſition 364. But ſince the parts of the ſuns atmoſphere, where the nucleus paſſes, are more heated, than the other parts, they will likewiſe be more rarefyed than the others. Thus the parts of the ſolar atmoſphere, which immediately ſurround the nucleus, and are mixed with the comets atmoſphere, become ſpecifically lighter than any other parts at the ſame diſtance from the ſun. Upon this account they will begin to move, as was ſhewn in a ſimilar caſe in propoſition 126 of hydroſtatics. The ſame cauſe, which put the ſolar atmoſphere about the comet in motion, will continue, provided the heat continues: for the parts next to the comet being more heated, and conſequently more rarefyed, than the reſt, will conſtantly move; and the freſh parts of the atmoſphere, which ſucceed in their place, will likewiſe be heated in their turn, and will move the ſame way. Thus there will be a conſtant ſtream of this rarefyed celeſtial matter, which I have here called the ſolar atmoſphere. Such a ſtream as this is the vehicle of the tail; it impells the particles of the comets atmoſphere, with which it is mixed, and makes them move along with it: and the ſmoke or tail aſcending from the nucleus conſiſts of thoſe particles of the comets atmoſphere impelled by this ſtream and floating in it. The ſmoke, which thus riſes from the comet, will reflect the ſuns rays, as the ſun ſhines upon it, and the brightneſs of the tail is owing to this reflected light.

Now as the celeſtial matter or ſolar atmoſphere gravitates towards the ſun; its denſity will be the greateſt near the ſun, by propoſition 50 of hydroſtatics. Therefore, by propoſition 3 of hydroſtatics, its ſpecific gravity will be the greateſt at the ſun, and will be the leaſt in thoſe parts, which are the remoteſt from the ſun. But a ſtream of this celeſtial matter, when it is rarefyed, will move towards thoſe parts, which have the leaſt ſpecific gravity, by propoſitions 31, 126 of hydroſtatics; that is, it will move directly from the ſun. And the tail of a comet riſes the ſame way that this ſtream moves. Therefore the tail of a comet will riſe directly from the ſun, or in a line oppoſite to the ſun. Let C, Plat. XXIX. fig. 6, be the comet, S the ſun, and SCO a line drawn from the ſun through the comet. The tail riſes directly from the ſun, that is, it riſes from the nucleus in the line CO. But the ſun S is on one ſide of the comet and the line CO is on the oppoſite ſide. Therefore the tail begins to aſcend in a line oppoſite to the ſun. From hence we may obſerve that the tail, becauſe it aſcends in a line oppoſite to the ſun, will always be farther from the ſun than the nucleus is. Conſequently, if the nucleus is at C moving towards its perihelion P, that is, towards the ſun, the tail being farther from the ſun than the nucleus will follow the nucleus.

cleus. Or if the nucleus is at I moving from its perihelion P or from the ſun, the tail being farther from the ſun than the nucleus will go before the nucleus, and will aſcend in the line IN. But if the nucleus is at P or in its perihelion, the tail will neither follow the nucleus nor go before it, but will be even with it, and will aſcend in the line PF.

367. *In the progreſſive motion of the nucleus of a comet the tail will not continue directly oppoſite to the ſun, but will be made to deviate towards that part of the trajectory, which the nucleus has left.*

If a line is conceived to be drawn from the ſun through the comet we may call that part of the line, which is beyond the comet, the line of oppoſition. As the comet is in different parts of its trajectory C, G, X, I, Plat. XXIX. fig. 6, CO, GT, XE, IN are the lines of oppoſition. The tail will not lie along in this line of oppoſition, but will deviate from it towards that part of the trajectory, which the comet has left. When the nucleus is at G, the tail GH will not lie in the line of oppoſition GT, but will bend from it towards CG that part of the orbit or trajectory, through which the nucleus has juſt been paſſing.

When the nucleus was at C, ſuppoſe that the tail was in the line of oppoſition CO; it is plane that it cannot paſs from thence into the line GT, whilſt the nucleus paſſes from C to G, unleſs it was to revolve round the ſun S with a greater velocity than the nucleus. For ſince the tail is at a greater diſtance from the ſun than the nucleus is, by propoſition 366, every part of the tail, as it revolves round the ſun with the nucleus, muſt deſcribe a greater orbit than the nucleus: if any particle of the tail was at O, in the line of oppoſition, when the nucleus was at C, this particle could not be in the line of oppoſition, when the nucleus comes to G; unleſs, whilſt the nucleus is deſcribing the arc CG, the particle could have deſcribed the ſimilar arc OT in an orbit, which is more remote from the ſun, and therefore larger. Now the particle cannot poſſibly deſcribe the larger arc OT, whilſt the nucleus is deſcribing the ſmaller arc CG, unleſs the tail revolved with a greater velocity than the nucleus. But this and every other particle of the tail revolves with the ſame velocity, that the nucleus revolves: this would certainly have been the velocity of the particle, if it had continued in the comets atmoſphere, and had not been raiſed from thence; becauſe then it would have revolved along with the nucleus: and when it is raiſed, the force which raiſes it, does not change the velocity, with which it revolves; for the projectile force and its gravity toward the ſun will be the ſame, whether it continues in the atmoſphere or is raiſed into the tail. Since then

then the velocity, with which any particle in the tail revolves, is the ſame with the velocity of the nucleus, a particle of the tail cannot deſcribe the greater arc OT in the ſame time that the nucleus deſcribes the ſmaller arc CG. Therefore if any particle was riſing in the line of oppoſition CO, when the nucleus was at C, this particle will not be come forwards faſt enough to be in the line of oppoſition GT, at the ſame time that the nucleus has arrived at G; that is, this particle will be left behind the line of oppoſition, or will deviate from this line towards C the place where the nucleus was, when the particle began to riſe. The ſame, which has been ſhewn of one ſingle particle, might be ſhewn of every other particle in the tail. Therefore the whole tail will deviate from the line of oppoſition towards that part of the trajectory, which the nucleus has left.

This would be the only cauſe of the tails deviation if the medium, in which the tail moves, was quite void of reſiſtance. For upon this ſuppoſition the tail would always revolve with the ſame velocity that the nucleus has: and then it could be left behind the line of oppoſition, only upon account of its deſcribing a greater orbit than the nucleus deſcribes. That the medium, in which the tail commonly moves, makes an exceeding ſmall reſiſtance, is evident: becauſe the planets perform their revolutions in the ſame medium without being ſenſibly retarded. But there muſt be ſome reſiſtance in this medium; for otherwiſe a ſtream of it, when it is rareſyed and aſcends from the nucleus, could not by its impulſe put the particles of the comets atmoſphere in motion, and carry them up along with it, as has been ſhewn in a like caſe under propoſition 126 of hydroſtatics. Therefore, by propoſition 366, if the medium had no reſiſtance, the comets would have no tails. Now though this reſiſtance may be ſo ſmall as not to retard ſuch large bodies as the planets; yet it will retard ſuch minute bodies as the particles are, which the tail of a comet conſiſts of, by propoſitions 118, 119 of hydroſtatics: and for the ſame reaſon likewiſe each particle of the tail, upon account of its minuteneſs, will be more retarded than the larger body or nucleus of the comet. This reſiſtance and the retardation ariſing from it is conſiderable, when a comet in its perihelion comes very near the ſun, and penetrates into the denſer parts of the ſolar atmoſphere. Sir Iſaac Newton obſerves that, when the comet of the year 1680 was in its perihelion, its diſtance from the ſun was not above a ſixth part of the ſuns diameter, and that upon account of the denſity of the ſuns atmoſphere ſo near to the ſun, and upon account of the great velocity of the comet in its perihelion, the comet itſelf would be a little

retarded

retarded, by propoſitions 116, 112, of hydroſtatics. He takes notice of an effect of this retardation, which we have referred to in propoſition 223. The gravitation of the comet towards the ſun, will, as the comet is retarded, make it approach nearer to the ſun: and thus in every revolution its diſtance from the ſun, when it comes to its perihelion, will be diminiſhed, till at laſt it falls into the ſun. So that though in a long courſe of years the quantity of matter in the ſun muſt decay by its conſtant emiſſion of light, yet ſuch a ſupply of freſh matter will repare the decay. And this may poſſibly be one uſe of the comets. However though this reſiſtance is the greateſt, and may retard the nucleus itſelf, when it comes into the denſeſt parts of the ſolar atmoſphere; yet at all times there muſt be ſome reſiſtance, for otherwiſe there could be no tail: and a ſmall reſiſtance will retard the minute particles, of which the tail conſiſts, more than the nucleus is retarded. From hence it follows, that the tail has not only a larger orbit to deſcribe than the nucleus has, but likewiſe moves with a leſs velocity than the nucleus: and conſequently the tail will upon both accounts be left behind the line of oppoſition, or will deviate from the line of oppoſition, towards that part of the trajectory, through which the nucleus has been paſſing.

If CL is the diſtance, to which any particle would be left behind the nucleus, in the time that the ſame particle would take up to aſcend from the nucleus C through the ſpace CO; then we may ſay, that the velocity, with which that particle would deviate from the oppoſition, is to the aſcending velocity as CL to CO. We here ſay that the velocity of deviation is to the aſcending velocity as CL to CO; for though the nucleus moves on, and the tail deviates, becauſe it is left behind the nucleus; yet the effect upon any particle, whilſt it aſcends, is the ſame as if the nucleus reſted at C, and was acted upon by two forces CL and CO. Now any particle thus acted upon by theſe two forces, will deſcribe CD the diagonal of a parallelogram, whoſe two ſides are CL and CO, by propoſition 16 of mechanics. Therefore any particle, which riſes from the nucleus, when it is at C, will not begin to aſcend in the line of oppoſition CO, but in a diagonal CD, which deviates from CO one ſide of the parallelogram towards CL the other ſide. In like manner when the comet is at I, if the aſcending velocity is to the velocity of deviation as IN to IR, any particle, which is beginning to riſe, will neither aſcend directly from the ſun in the line of oppoſition IN, nor will it follow the comet in the line IR, but will begin to aſcend in the line IM, which is the diagonal of a parallelogram, whoſe two ſides are IN and IR; that is, the particle in its aſcent will deviate from the line of oppoſition IN, towards IR

the part of the trajectory, through which the nucleus had been paſſing, when it came to I.

368. *The tail of a comet, when it deviates from the ſuns oppoſition, is always bent: that ſide of the tail, which is towards the line of oppoſition is convex, and the other ſide is concave.*

If the comet is at C, and the velocity of deviation is to the velocity of aſcent as CL to CO, the particles will begin to riſe in the line CD, by propoſition 367, and if this proportion was to continue the ſame, whilſt each particle riſes from the nucleus to the top of the tail, then the tail would be ſtrait, and would lie in the direction CD.

But as the particles aſcend the proportion of CL to CO, or of the velocity of deviation to the aſcending velocity encreaſes. Firſt let us ſuppoſe that the aſcending velocity continues the ſame during the whole time of a particles aſcent from the nucleus, till it comes to the top of the tail: the proportion then of CL to CO can encreaſe no otherwiſe than by an encreaſe in the velocity of deviation. Now the velocity of deviation depends upon two particulars, it is proportional to the diſtance, at which the particle is left behind the neucleus, by propoſition 367: and the diſtance, at which any particle is left behind the nucleus depends, partly upon the retardation of the particle in its progreſſive motion, and partly upon the greatneſs of the orbit which the particle is to deſcribe: the retardation of any particle is the ſame whether it has juſt begun to aſcend, or whether it is arrived at the top of the tail; and conſequently this retardation will not change the velocity of deviation during the aſcent of the particle. But then as the particle aſcends from the nucleus it has a greater orbit to deſcribe: whilſt it is contiguous to the nucleus, it deſcribes the ſame orbit with the nucleus; but when it has aſcended to O, for inſtance, it is to deſcribe the arc OT of a greater orbit with the ſame velocity that the nucleus is to deſcribe GC a ſimilar arc in a leſs orbit: and ſince the orbit, which the particle is to deſcribe, encreaſes in this manner as the particle riſes, it is plane, from what has been ſaid under propoſition 367, that the farther the particle has riſen from the neucleus the more it will be left behind, or that the velocity of deviation will encreaſe, as the particle aſcends from the nucleus to the top of the tail. Therefore, though the aſcending velocity was to continue the ſame during the whole aſcent, yet the proportion of the velocity of deviation to the velocity of aſcent or of CL to CO would keep encreaſing as the particle aſcends.

Secondly, if the velocity of deviation was to continue the ſame, yet in the aſcent of the particles the proportion of the velocity of deviation

to

to the velocity of aſcent or of CL to CO will encreaſe. Now ſince we have ſuppoſed the velocity of deviation to continue the ſame, the proportion, which it bears to the velocity of aſcent, can encreaſe no otherwiſe than by a decreaſe in the velocity of aſcent: and ſuch a decreaſe will happen as the particles aſcend. The particles, which compoſe the tail are buoyed up by a ſtream of the ſolar atmoſphere, which being rarefyed aſcends directly from the ſun, by propoſition 366, into the rarer parts of that atmoſphere. For ſince the denſity or ſpecific gravity of the ſolar atmoſphere is greateſt at the ſun, the rarefyed ſtream, as it riſes from the ſun, will come into ſuch parts of the atmoſphere, as have a leſs denſity or leſs ſpecific gravity. Now the ſtream riſes in the ſolar atmoſphere, becauſe by being rarefyed it becomes ſpecifically lighter than that part of the atmoſphere where it was when it firſt began to riſe; and conſequently its aſcending velocity, by propoſition 31 of hydroſtatics, will be proportional to the difference between its own ſpecific gravity and the ſpecific gravity of the ſolar atmoſphere. But this difference, and conſequently the aſcending velocity of the ſtream, will decreaſe, as it riſes into the rarer parts of the ſuns atmoſphere, where the ſpecific gravity of the atmoſphere is more nearly the ſame with its own. And as the velocity of the ſtream decreaſes, the velocity of the particles, which are buoyed up by that ſtream, will decreaſe. Therefore if the velocity of deviation is given, the proportion of this velocity to the velocity of aſcent or of CL to CO will encreaſe as the particles riſe, becauſe the velocity of aſcent keeps decreaſing.

Having thus ſhewn that the proportion of the velocity of deviation to the velocity of aſcent encreaſes as the particles riſe from the nucleus, the conſequence we are to draw from thence is, that, as the particles riſe, their deviation from the line of oppoſition will encreaſe. And this conſequence ſeems to be evident: for if the velocity of deviation was nothing in reſpect of the velocity of aſcent, the particles would not deviate at all, but would all of them aſcend in the line of oppoſition. Therefore the greater the velocity of deviation is in reſpect of the velocity of aſcent, ſo much more the particles will deviate. Or otherwiſe: the motion of every particle as it riſes is compounded of two others, and is in the diagonal of a parallelogram, one of whoſe ſides is the velocity of deviation, and the other is the velocity of aſcent. And in every parallelogram the longer one ſide is in proportion to the other, ſo much leſs is the angle, which the diagonal will make with the longer ſide. But when the particles begin to riſe, CL is one ſide of the parallelogram and CO is the other, and if the proportion between theſe two ſides was to continue the ſame during the

whole aſcent of the particles, they would aſcend from the nucleus to the top of the tail in the ſame diagonal CD. But as the particles riſe, the proportion of CL to CO encreaſes, and conſequently, as the particles aſcend, the angle made by the diagonal with CL will decreaſe, that is, the line, in which the particles move, will bend more and more towards CL, and conſequently will bend more and more from CO the line of oppoſition.

From hence then we ſee that the loweſt part of the tail, which is contiguous to the nucleus, or the part, where the particles begin to aſcend, will be near the line of oppoſition; the parts of the tail, which are farther from the nucleus, will be bent farther from the line of oppoſition, and the top of the tail will be bent fartheſt of all. So that if the comet was at G, the tail would be in the ſhape GH, where the part G is near the line of oppoſition GT, and the top H is bent the fartheſt from it: and ſuch a bending of the tail will make it convex towards GT, and concave the other way towards GC. In like manner if the comet was at X, the tail XY would be bent farther from the line of oppoſition XE at Y the top of it than at X the bottom of it, ſo that the convex ſide of it would be towards XE, and the concave ſide towards XP.

369. *When the tail of a comet deviates from the ſuns oppoſition, the convex ſide of it is more diſtinctly terminated, and is brighter than the concave ſide.*

The convex ſide of the tails GH or XY, Plat. XXIX. fig. 6, is next the line of oppoſition GT or XE; and conſequently the concave ſide is next GC or XP the part of the trajectory, through which the nucleus has laſt been paſſing, by propoſition 368. From hence it follows that, as the concave ſide is towards that part of the trajectory, which the nucleus has left, the convex ſide muſt be the foremoſt. But the reſiſtance of the medium, through which the tail paſſes, will bring the particles cloſer together on the foremoſt or convex ſide, than they are on the concave ſide. For this reaſon the convex ſide will be more diſtinctly terminated. And ſince the reflecting particles are cloſeſt together on the convex ſide, there will be more light reflected from any given part of that ſide than from an equal part of the other, that is, the convex ſide will be the brighteſt.

370. *The tails of the comets, though they deviate from the ſuns oppoſition, are always in the plane of the comets orbit.*

The nucleus of the comet and the ſun are both of them in the plane of the comets orbit; and the tails are always in the ſame plane with the nu-

nucleus and with the ſun. If the tail was to aſcend directly from the ſun ſo as not to deviate, it would be in the ſame plane with the nucleus X and with the ſun S, Plat. XXIX. fig. 6. For SXE a line drawn from the ſun and paſſing through the nucleus, would be the direction of the tail, if it did not deviate from the line of oppoſition, by propoſition 366. And ſince all the parts of the ſame right line are in the ſame plane, Euc. b. XI. prop. 1, the line of oppoſition, and the tail, if it was in that line, muſt be in the ſame plane with the nucleus X and with the ſun S. But ſince, by propoſition 367, the tail deviates only becauſe it is left behind in the progreſſive motion of the nucleus, ſuch a deviation, though it changes the line will not change the plane, in which the tail lies. Therefore the tails, when they deviate from the line of oppoſition, are in the ſame plane with the nucleus and with the ſun, that is, they are in the plane of the comets orbit.

371. *Though the tail of a comet deviates from the ſuns oppoſition, yet neither the deviation nor the curvature of the tail will be perceived by us, if the earth is in the plane of the comets orbit.*

When the comet is at X, Plat. XXIX. fig. 6, the tail XY deviates from XE the line of oppoſition, and, by propoſition 368, is bent ſo as to be convex towards that line and concave the other way. But neither the deviation nor curvature will be ſenſible to us if the earth is in the plane of the orbit. If XY is the tail and XE is the line of oppoſition; then the deviation of any part in the tail is the diſtance of that part from the line XE; thus particularly the deviation of the part Y is YE or the diſtance between Y in the tail and E in the line of oppoſition. But if the earth is in the ſame plane with XE the line of oppoſition and with XY the tail, the end of this line YE will be towards the eye of a ſpectator upon the earth; and conſequently, by propoſition 129 of optics, the length of this line or the deviation of this part of the tail will not be ſenſible. The ſame may be ſhewn of every other part of the tail, when the earth is in the ſame plane with XE the line of oppoſition and XY the tail. Now, by propoſition 370, the line of oppoſition and the tail are always in the plane of the comets orbit. Therefore the deviation will not be ſenſible to us, when the earth is in the plane of the comets orbit.

The curvature of the tail will not be perceived, unleſs one part of the tail appears to be farther diſtant from the line of oppoſition than another, by what has been ſaid under propoſition 368. But when the earth is in the plane of the comets orbit, the tail does not appear to deviate, and conſequently one part of it cannot appear to deviate more than another.

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Therefore the curvature of the tail will not be perceived, when the earth is in the plane of the comets orbit.

372. *When a comet is in its perihelion, the tail is the longeſt, it deviates the leaſt from the ſuns oppoſition, and is not bent.*

When a comet is at P, Plat. XXIX. fig. 6, at its leaſt diſtance from the ſun S, it will be more heated than at any other time, and ſince, by propoſition 366, the heat is what makes the ſmoke or tail riſe from it, the greateſt plenty of ſmoke and conſequently the longeſt tail will riſe from it, when it is at its leaſt diſtance from the ſun or in its perihelion.

If a comet was to move down the line TGS directly towards the ſun, the tail would follow it in the line of oppoſition and would not deviate at all. If the particles, which compoſe the tail, had continued in the comets atmoſphere their progreſſive velocity in the direction TG would have been equal to the progreſſive velocity of the nucleus; for the atmoſphere conſtantly ſurrounds the nucleus and keeps pace with it: but the heat of the ſun would raiſe theſe particles in the line of oppoſition GT, by propoſition 366, and conſequently they would continue to move after the nucleus with the difference between their progreſſive velocity and the velocity of aſcent. Thus they would be left behind the comet: but ſtill they would follow it in the ſame right line; becauſe as the progreſſive motion and motion of aſcent are in the ſame right line TG only in contrary directions, the tail, when it moves with the difference of theſe velocities, will not change the line of its motion, but will continue in the line of oppoſition. Therefore if the nucleus was moving in a right line directly towards the ſun, the tail would not deviate from the line of oppoſition.

Upon the ſame ſuppoſition that the nucleus is moving in a right line directly towards the ſun, the reſiſtance of the medium, in which the tail moves, will not make it deviate from the line of oppoſition. This reſiſtance may indeed retard the particles of the tail; but as they are moving in the line of oppoſition, ſuch a retardation will only make them move ſlower in that line, but it could not make them deviate from it.

If a comet was to move up the line SXE directly from the ſun, the tail would go before it in the line of oppoſition, but would not deviate at all. The progreſſive velocity of all the particles in the comets atmoſphere is equal to the progreſſive velocity of the nucleus; or, if the nucleus was moving in the line SXE the atmoſphere, which ſurrounds it, would move the ſame way and at the ſame rate: but if the heat of the ſun was to force any of thoſe particles upwards, ſuch particles would then aſcend from

from the ſun with the ſum of their progreſſive velocity added to their velocity of aſcent. Therefore as the nucleus continues to move only with the progreſſive velocity, and the particles, which are ſo forced upwards and compoſe the tail, move with this velocity added to the velocity of aſcent; the tail will always keep before the nucleus. But ſince the progreſſive motion, by the ſuppoſition, is in XE the line of oppoſition, and the motion of aſcent is in the ſame line, by propoſition 366, it follows that the particles of the tail by theſe two motions together will continue to move in the line of oppoſition without deviating from it.

Upon the ſame ſuppoſition that the nucleus is moving in a right line directly from the ſun, the reſiſtance of the medium, in which the tail moves, will not make it deviate from the line of oppoſition. For ſince, by the ſuppoſition, the tail is moving in the line of oppoſition, the reſiſtance of the medium can only retard it or make it move ſlower in that line, but cannot make it change its direction or deviate from that line.

Thus we have ſhewn upon what ſuppoſitions the tail would not deviate; it would always be in the ſuns oppoſition, if the nucleus was moving in the line of oppoſition either directly towards or directly from the ſun. From hence then it follows that the deviation ought to be the greateſt, when the motion of the nucleus differs the moſt from theſe ſuppoſitions; that is, when the nucleus is moving at right angles to the line of oppoſition, or when it is in its perihelion. For when the nucleus is at P, the direction of its motion is in the tangent P*p* at right angles to SPF the line of oppoſition. But yet when the comet is in its perihelion the tail does not deviate at all. Now if the tail was left behind the nucleus in the progreſſive motion, either becauſe the tail deſcribes a greater orbit than the nucleus, or becauſe it is more retarded by the reſiſtance of the medium, through which it paſſes, than the nucleus is, the tail muſt deviate, by propoſition 367. Conſequently when the comet is in the perihelion, the tail is not left behind the nucleus in the progreſſive motion. But from what we have juſt now ſhewn it follows, that if the tail, when the comet is in its perihelion had any progreſſive motion it muſt be ſlower than the progreſſive motion of the nucleus, and for that reaſon it would be left behind the nucleus, and would deviate. Therefore, ſince the tail does not deviate, when the comet is in its perihelion, we muſt conclude that the tail at that time has no progreſſive motion. But if the tail, when in the perihelion, has no progreſſive motion, it cannot be the ſame column of ſmoke that attended the nucleus before it came to the perihelion, nor the ſame that attends it after it has paſſed the perihelion: for if it was the ſame column, it muſt have a progreſſive motion. Therefore the tail of

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the comet, when in its perihelion, is a column of ſmoke, which riſes inſtantaneouſly from the nucleus: the heat at the perihelion is ſo great, as to diſſipate the ſmoke or tail, which had riſen before the comet came thither, and to raiſe a new column of ſmoke or a new tail inſtantaneouſly: and the tail, which is then raiſed, is again diſſipated in the ſame manner, before the comet paſſes its perihelion, and a different tail is raiſed after it has paſſed the perihelion.

373. *The tail of a comet decreaſes, after it has paſſed the perihelion, and is loſt as the comet moves off to its aphelion; ſo that at the next return it will have no tail or a very ſhort one.*

As the heat decreaſes, a leſs quantity of ſmoke will riſe from the nucleus; and conſequently if the ſmoke does not riſe ſo faſt as that, which has riſen, is diſſipated, the tail muſt decreaſe. As the comet moves off to its aphelion it comes into parts of the ſolar atmoſphere rarer than thoſe parts where it was, when the tail was firſt raiſed. Therefore, by propoſition 116 of hydroſtatics, the reſiſtance of the medium will not be ſufficient to ſupport the particles, which had been raiſed before; ſo that many of them will fall back again to the nucleus by their own weight. For theſe reaſons, when a comet firſt appears to us after its return from the aphelion, it has either no tail at all or a very ſhort one.

CHAP. XIV.

Of the deviſion of time.

BEFORE we go on to explane the phyſical cauſes of the celeſtial motions, it will be proper to ſay ſomething of the uſe, which we make of the motions and appearances of the ſun and moon in dividing our time or diſtinguiſhing it into parts, ſuch as days, weeks, months, years &c.

374. *An artificial day is the time that the ſun appears above the horizon; and a night is the time that the ſun is below the horizon.*

Scarce any obſervation is requiſite to make this diviſion: whoever has ſeen the ſun perform one diurnal revolution, could not fail to divide time into day and night.

375. *A natural day conſiſts of the artificial day and night together, or is the time that the ſun takes up to complete one apparent revolution from eaſt to weſt round the earth.*

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This diviſion of time is as obvious as the former: all nations have made uſe of it. Indeed they differ from one another in regard to the point of time from whence the natural day begins. The jews and the athenians ended one natural day, and conſequently began to reckon the next, from the ſun-ſet. What we call a natural day was called *νυχθήμερον* a night and a day by the greeks: the night ſtands firſt in the compound word, becauſe they reckoned the night the firſt part of the nucthemeron or natural day. The jews, who have few or no compound words, ſometimes call a natural day the evening and the morning. Thus Moſes in the firſt chapter of geneſis explaning the word *day*, ſays that the evening and the morning were the firſt day &c: where by the evening is meant the night, and by the morning is meant the artificial day; and both theſe together make up the natural day. Perhaps they might chuſe this point of time rather than any other; becauſe in this reckoning the natural and artificial day would end together. Or they might originally have another reaſon for fixing upon ſun-ſet as the beginning of their natural day: they made uſe of lunar months; and the moſt obvious time to begin their month would be at the firſt appearance of the new moon: now the firſt appearance of the new moon is at ſun-ſet: and if they reckoned this the beginning of the month, it would of courſe be reckoned the beginning of the natural day, that the month and the natural day might begin together. There is however an inconvenience in beginning the natural day from ſun-ſet: for ſun-ſet is a variable point of time, and the natural days, which begin and end with ſun-ſet, will be of different lengths. At the vernal equinox, for inſtance, the ſun ſets at ſix o'clock; the next day it will not ſet, till a little after ſix, and conſequently, the day after the equinox will be a little longer than the day of the equinox: in like manner, as the ſun ſets later every evening till the ſummer ſolſtice, the length of the natural days, which are not to end till ſun-ſet, will keep encreaſing. From that time till the winter ſolſtice, the ſun ſets earlier every evening; and the natural days, if they are reckoned from ſun-ſet, will ſhorten. From the winter ſolſtice till the vernal equinox and from thence, as before, till the ſummer ſolſtice; the natural days will lengthen. The Italians, who at preſent begin and end their natural day at ſun-ſet, are ſenſible of this inconvenience: they divide their natural day into 24 hours: they reckon it the 24th hour at ſun-ſet; and call it 1 o'clock, when the ſun has been down one hour. But as the time of ſun-ſet is variable, their 24th hour will happen later every day from the winter to the ſummer ſolſtice, and earlier every day from the ſummer to the winter ſolſtice: upon this account they are obliged to alter their clocks and watches every day.

The babylonians, as we learn from Macrobius and A. Gellius, uſed to begin and end their natural day with ſun-riſe. They might perhaps make this the beginning of their natural day, that the artificial and natural day might begin together. There is the ſame inconvenience in beginning to reckon from ſun-riſe as from ſun-ſet: the time of ſun-riſe is variable; the natural day reckoning from ſun-riſe would encreaſe from the winter to the ſummer ſolſtice, and would decreaſe from the ſummer to the winter ſolſtice.

The umbrians, an antient people in Italy, began their natural day from noon, when the ſun is in the meridian; as ſome, though not all, the aſtronomers begin it in their reckonings. Now the time between one noon and the next is 24 hours at all times of the year; if a ſmall allowance, which we may here neglect, is made for the equation, by propoſition 25. So that the umbrians would have all their natural days of the ſame length. But yet for common uſe it would be inconvenient to begin the day at noon: for then the natural day would begin in the middle of the artificial day, which is the time for buſineſs. Whatever was done before noon, would be upon one day of the month, and whatever part of the ſame buſineſs was carryed on after the noon of the ſame artificial day, would be tranſacted upon the next day of the month. Whatever was done after the noon of one artificial day, and was carryed on again, after the interval of night, before the noon of the next artificial day, notwithſtanding the long interval, would be tranſacted upon one and the ſame natural day or upon the ſame day of the month. To avoid ſuch a confuſion, it would be the beſt way to reckon the whole artificial day to the ſame natural day, which cannot be done if we begin the natural day in any part of the artificial day. And we have already ſeen the inconvenience of beginning to reckon the natural day either from the end or the beginning of the artificial day.

The moſt convenient time for beginning the natural day is ſome invariable point, which does not happen within the artificial day. Midnight is ſuch a point. The time of midnight is not within the artificial day, and is as invariable as the time of noon: for as there are 24 hours between the time, when the ſun leaves the meridian of a place, and the time, when it returns to the ſame meridian; ſo likewiſe there are 24 hours between the time, when the ſun leaves the oppoſite meridian, and the time, when it returns to the ſame oppoſite meridian. Midnight is the point of time at which we in England end one natural day and begin the next. This was likewiſe the point of time where the Romans began and ended their natural day. We might prove this by many authorities, that of Varro,

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as cited by A. Gellius, l. 3. c. 2, will be ſufficient. Thoſe, ſays he, who are born within the ſame 24 hours reckoning from one midnight to another, are born in the ſame day. The Romans indeed did not begin to reckon their hours from midnight: for their hours were not intended to be 24th parts of the natural day, but 12th parts of the artificial day and 12th parts of the night.

376. *Unequal hours are twelfth parts of the artificial day, and twelfth parts of the night.*

The Roman hours were of this ſort, as was obſerved, under propoſition 375. Each hour in the artificial day was a 12th part of the artificial day: and for that reaſon the hours in the day-time were reckoned from the beginning of the artificial day, or from ſun-riſe. So that the firſt hour of the day ended, when the ſun had been one hour or $\frac{1}{12}$ of the artificial day above the horizon. At noon half the artificial day was over; but half the artificial day is $\frac{6}{12}$ of the whole; and conſequently noon was the 6th hour of the day. The end of the 12th hour was at ſun-ſet. They divided the night likewiſe into 12 parts, which were called hours of the night. The firſt of them ended, when the ſun had been one hour below the horizon, or when $\frac{1}{12}$ of the night was over. Midnight was called the 6th hour of the night. And the 12th hour of night ended at ſun-riſe. The jews uſed the ſame ſort of hours: as we find in our Saviours parable of the labourers; thoſe who were hired at the firſt hour, worked the whole day; and thoſe, who were hired at the 11th hour, and worked from that time till ſun-ſet, worked only one hour.

Such hours as theſe are of different lengths at different times of the year, and therefore are called unequal hours. A winters day is ſhorter than a ſummers day, and conſequently $\frac{1}{12}$ of a winters day, or a winters hour in the day-time will be ſhorter than $\frac{1}{12}$ of a ſummers day or than a ſummer hour in the day time. To this Plautus alludes, Pſeudol. act V. ſcen. II. I believe, ſays Simo to Pſeudolus whom he finds drunk, that you could drink up the moſt plentiful Maſſic vintage in an hour. You might have ſaid, replys Pſeudolus, in a winters hour.

Theſe unequal hours are longer in the day-time than in the night, when the days are longer than the nights; and are ſhorter in the day-time than in the night, when the days are ſhorter than the nights. The hours of the day and night will be of the ſame length only at the equinoxes, when the days and nights are equal. At that time of the year, $\frac{1}{12}$ part either of the artificial day or of the night is $\frac{1}{24}$ of a natural day.

377. *Equal or equinoctial hours are ſuch as divide a natural day into 24 equal parts.*

At preſent we make uſe of ſuch ſort of hours as theſe. They are called equinoctial hours, becauſe at the equinoxes they are of the ſame length and coincide with the unequal hours formerly in uſe.

Thus the hours, which we uſe, and the hours, which the jews uſed formerly, coincide with one another or are of the ſame length at the equinoxes. The only difference at theſe times of the year would be in the point from whence we begin to reckon. We reckon our twelve hours of the evening from 12 o'clock at noon, and our twelve hours of the morning from 12 o'clock at midnight: the jews reckoned their twelve hours of the evening from ſun-ſet, and their 12 hours of the morning or day from ſun-riſe. Thus St. Mark ſays, chap. xv. v. 33, that at the crucifixion there was darkneſs from the ſixth hour till the ninth, which in our account is from 12 o'clock to 3 in the afternoon. For the crucifixion was at the time of the paſſover, which was always celebrated at the vernal equinox. Therefore at that time their hours were of the ſame length with ours. Their ſixth hour was ſix hours after ſun-riſe, which at the equinox is 12 o'clock with us: and their ninth hour was nine hours after ſun-riſe, which at the equinox is 3 in the afternoon with us. But at any other time of the year, except the autumnal equinox, their hours would not coincide with ours in the ſame manner.

378. *We ſubdivide an hour into ſixty equal parts, which we call minutes, our minutes into ſixty equal parts, which we call ſeconds; and our ſeconds are ſometimes farther ſubdivided into ſixty equal parts, which are called thirds.*

379. *A week is a period of ſeven days.*

The obſervance of a ſabbath, and in conſequence the diviſion of time into weeks or periods of ſeven days, was one of the firſt commands, which God gave to Adam. Mr. Kennicott, in his very ingenious diſſertation concerning the oblations of Cain and Abel, has ſhewn that the ſabbath was inſtituted by the divine authority, and obſerved by the patriarchs before the time of Moſes. This inſtitution was again renewed and the obſervance enforced in the fourth commandment: and by this means the diviſion of time into weeks has been conſtantly preſerved amongſt the Iſraelites, and through their hands has been derived to us. Many learned men have endeavoured to ſhew that the heathens as well as the jews made

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uſe of the ſame diviſion: and we will grant that there are traces of ſome ſuch diviſion to be met with in the writings of the antients. But thoſe, who have diſcovered ſuch traces, have been too haſty in concluding, that ſuch a diviſion of time muſt neceſſarily have ariſen from ſome univerſal tradition: they take it for granted that there is no reaſon in nature for dividing our time into weeks or periods of ſeven days; and conſequently, if all nations have made ſome ſuch diviſion, this univerſal cuſtom, as it muſt have ſome univerſal cauſe, muſt have ariſen at firſt from Gods appointment, and muſt have been derived by tradition from Adam to all his poſterity. But we need not reaſon from the univerſal cuſtom of dividing time into weeks to ſhew that God inſtituted the ſabbath: we have many arguments to prove that he did; and therefore will lay no ſtreſs upon this. It is an argument, which depends upon a falſe principle, and muſt therefore do more harm than good to our religion, if we were to make uſe of it. God undoubtedly inſtituted the ſabbath: but it does not follow from thence, that there is no natural meaſure of our time, which will divide it into weeks. God in the law of Moſes inſtituted the feaſt of the new moon; but yet there is a natural appearance which divides our time into lunar months. It would be as juſt a concluſion, that there is no natural meaſure of time, which could give occaſion to a diviſion of it into lunar months, becauſe God inſtituted the feaſt of the new moon; as it is that there can be no natural meaſure of time, which could give occaſion to a diviſion of it into weeks, becauſe God appointed that the ſabbath ſhould be obſerved every ſeventh day. Perhaps they, who lay any ſtreſs upon the argument, that we are now examining, may ſay that I miſtake them: they do not conclude that there can be no natural meaſure of time, which divides it into weeks, becauſe God inſtituted the ſabbath; but that the univerſal diviſion of time into weeks muſt have been owing to ſome poſitive appointment, delivered down by univerſal tradition, becauſe they know of no natural meaſure, which could give occaſion to ſuch a diviſion. We are agreed then that God inſtituted the ſabbath, whether there is any ſuch natural meaſure or not; and that the authority of the Moſaic hiſtory relating to this inſtitution will not be leſſened, if we ſhould find out that the return of this feaſt, like the return of many others, was after ſuch a period of days, as a natural meaſure of time would give others occaſion to make, who had received no traditional account of the inſtitution of the ſabbath.

A lunar month is a very obvious diviſion of time, and has been univerſally uſed by all nations. The time between one new moon and the next is a period, which is ſet out by a very regular and remarkable ap-

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pearance. The full moon is as remarkable an appearance as the new: and, as this happens in the middle of the lunar month, it would divide the month in halves, or into two fortnights. And as the moons diſc is half enlightened in the middle of each fortnight, theſe two appearances of the moon, which are quite different from any others, would ſubdivide the month into quarters. Our firſt parents indeed knew little or nothing of lunar months, when they firſt obſerved the ſabbath and divided their time into weeks, I agree that they received this diviſion from poſitive appointment; as they received their inſtruction in many other things, which experience and reaſon would point out to their poſterity, from expreſs revelation. But then I deny, that we muſt have recourſe to univerſal tradition to account for the diviſion of time into weeks amongſt their poſterity: ſince the moons different appearances, which divide our time into months, might give occaſion to ſubdivide thoſe months into quarters. A lunar ſynodical month conſiſts of 29 days, 12 hours, 44 minutes, and 3 ſeconds, by propoſition 139, a quarter of this is 7 days, 9 hours, 11 minutes, and about 1 ſecond: ſo that a week is more nearly equal to a quarter of a lunar month than any other period of whole days. Therefore as four very remarkable and different appearances of the moon divide the month into quarters; this would naturally give occaſion to thoſe, who had received no traditional account of the ſabbath, to divide their time into weeks. Varro, as cited by A. Gellius, l. 3. c. 10, in his account of the force and efficacy of the number ſeven, takes notice of a diviſion of time into weeks: a woman, he ſays, who goes her full time, is not delivered till after 40 hebdomadas or weeks: but then, amongſt other inſtances of the vaſt importance of this number, he mentions one, which ſhews us, whence this univerſal diviſion of time either was or might have been derived. The moon completes its circuit in 4 times ſeven days; for in 28 days it returns to its former ſituation: and this opinion, as he adds, had Ariſtides the Samian for its author. He is here a little miſtaken, whether he ſpeaks of the periodical or ſynodical month: 28 days is longer than the former and ſhorter than the latter. But whatever his miſtake might be, it ſhews us that the antients had ſome regard to the moons appearances, when they divided their time into weeks.

Indeed this diviſion of time does not ſeem to have been made uſe of by the Romans for any religious purpoſe. They do not appear to have had any weekly feſtival or ſacrifice; their nundinae, when thoſe who lived in the country met together at Rome, have the neareſt reſemblance to ſuch a periodical feſtival: but theſe, as the name imports, returned every ninth day and not every ſeventh. A week ſeems to have been a period,

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made uſe of only in their common converſation to expreſs ſuch a round number of days, as was nearly an aliquot part of a month: juſt as we in England frequently ſpeak of *ten days*; not becauſe ten days is a period that we make uſe of for any ſacred or civil purpoſe; but becauſe we have frequent occaſion to refer either backwards or forwards to ſome ſhort period of time, and *ten days* is nearly an aliquot part of our months, which conſiſt of about thirty days.

This diviſion of days into weeks was ſettled and became more univerſal when the ſeven days of the week had latin names given to them from the ſeven planets in what we now call the Ptolemaic ſyſtem. This ſyſtem was in uſe in Ciceros time, as we learn from the ſomnium Scipionis; and it was after his time that the days of the week had names given to them. Dio Caſſius, l. 35, ſays, that theſe planetary names were invented by the Egyptians not long before he wrote. If we call ſunday the firſt day of the week, the names ſtand in this order, the firſt or ſunday is the day of the ſun, the ſecond or monday is the day of the moon, tueſday is the day of mars, wedneſday of mercury, thurſday of jupiter, friday of venus, ſaturday of ſaturn. And here perhaps it may be enquired both why the names of the days in the week, which are taken from the planets in the Ptolemaic ſyſtem, ſhould ſtand in a different order from the planets in that ſyſtem; and what rule the Egyptians obſerved in giving theſe planetary names, ſince they were not determined by the order of the planets. The rule, which they obſerved, is planely this, they began with the ſun and reckoning from thence downwards in the ſyſtem they named each day from the fourth planet incluſively. Thus the fourth planet incluſively from the ſun is the moon: the fourth planet incluſively from the moon is mars: the fourth incluſively from mars is mercury: the fourth incluſively from mercury is jupiter: the fourth incluſively from jupiter is venus: and the fourth incluſively from venus is ſaturn. The planetary names of the week ſtand in this order. But having found out the rule itſelf we are next to find out the reaſon of it. A week is a quarter of a lunar month; and conſequently we may reaſonably imagine that the names, at leaſt of ſome days in the week, would have ſome reference to a lunar month. Upon this ſuppoſition let us conſider what names would be moſt likely to be given to the firſt two days of the month, for thoſe are the moſt likely names to be given to the two firſt days of the firſt week in the month, and in conſequence of that to the two firſt days of every other week. Now the firſt day of a lunar month is the interlunium, or the day when the moon is in conjunction with the ſun, and therefore is inviſible: ſo that upon the firſt day of the month, there is

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no luminary but the ſun: for which reaſon that day might be called peculiarly dies ſolis the day of the ſun. Perhaps it might be upon this account that the νεομηνία or feaſt of the new moon at Athens, which was ſacred to all the gods, ſhould yet in a more particular manner be devoted to Apollo or the ſun. Upon the ſecond day of the lunar month, the moons diſc begins to be enlightened; it is dies lunae naſcentis, the day, when the moon begins: and for this reaſon, thoſe, who invented the planetary names, might chuſe to call this day dies lunae the day of the moon. Thus we have found out why the firſt day in the firſt week of a lunar month might be called the day of the ſun, and why the ſecond day of the ſame week might be called the day of the moon. Theſe two planets, as they are placed in the Ptolemaic ſyſtem, are the fourth from one another incluſively: and when there was no particular reaſon for giving the name of any one planet more than of any other to the following days; thoſe, who affixed the names, went on in the ſame order with the reſt of the planets, and taking the fourth planet from the moon incluſively, or mars, gave the name of mars to the day that followed the day of the moon. This order would be ſo much the better, becauſe by going on with it, the laſt day of the week would be ſaturns day, and then taking the fourth planet incluſively from ſaturn the firſt day of the ſecond week in the month would be the ſuns day. Thus the rule of taking the fourth planet incluſively in naming the ſucceſſive days was not principally deſigned. The firſt and ſecond days of the firſt week were called the days of the ſun and the moon for another reaſon: but as theſe planets happened to be the fourth from one another, and it was quite indifferent how they named the other days; they choſe to obſerve the ſame order in naming the reſt, which had been obſerved in naming the two firſt. And when the days of one week had received theſe names, it was natural to obſerve the ſame order in naming the days of all other weeks.

380. *A civil lunar month is a period of days, which is deſigned to be equal to the time of one lunation or of one ſynodical month.*

There is a remarkable change of the moon at the new: and as theſe changes return regularly, they ſerve to divide our time into ſuch periods as we call ſynodical months or lunations. A periodical month is not ſo uſeful a meaſure of time; becauſe the return of the moon to the ſame point in its orbit does not fall under the notice of common people; whereas the changes of the moon can ſcarce fail of being taken notice of by every body. But then though a ſynodical month is a period ſet out by ſuch remarkable limits as the changes of the moon; yet ſome art muſt be uſed to make it fit for common uſe. It is neceſſary that our princi-

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pal periods of time ſhould be nearly commenſurate to one another. Ordinary people know how to compute time by days; and that they may know as readily how to compute time by months, it is requiſite that a month ſhould be a period of time conſiſting of a certain number of whole days. A ſynodical month is not ſuch a period: for it conſiſts of 29 days, 12 hours, 44 minutes, and 3 ſeconds: and conſequently ſome contrivance is requiſite to adjuſt the period of a month in ſuch a manner as to make it conſiſt of a determinate number of days without any odd hours; and yet ſo as to make the month correſpond to the moons changes. A month ſo adapted to common uſe is called a civil month. This might be done by making the months conſiſt of 30 and 29 days alternately. One ſynodical month conſiſts of about 29 $\frac{1}{2}$ days, and two conſiſt of about 59 days. Therefore two civil months one of 30 and the other of 29 days will be nearly equal to two ſynodical months: ſo that, though the firſt of theſe civil months would be too long to correſpond with the moons changes, yet the other would be juſt as much too ſhort: conſequently both together would be equal to two lunations: if the firſt began with the new moon, the ſecond would begin 12 hours after the next new moon; but then it would end with the third new moon, from whence the third month is to begin.

381. *A ſolar month is the time that the ſun takes up to go through a ſign in the zodiac.*

Though the *month* has its name from the *moon*, and was originally a period of time ſet out or limited by ſome change in the moon; yet we call any period of time by the ſame name, if it is nearly equal to the time of a lunation. The ſun goes apparently round the zodiac in ſomething more than twelve lunar months: this was undoubtedly the reaſon why the zodiac was divided into twelve parts or ſigns. Properly ſpeaking no period of time ſhould be called a month, unleſs it is ſet out by ſome change of the moon. The time, that the ſun takes up to go through a ſign in the zodiac, is however called a month, becauſe it is nearly equal to one: but as it is a period of time ſet out by the ſuns motion, it is called by way of diſtinction a ſolar month. Its length is 30 days, 10 hours, 29 minutes, and 5 ſeconds. Such a month as this is unfit for common uſe, becauſe it does not conſiſt of a certain number of whole days: the methods of adapting it to common uſe will be beſt underſtood from the following propoſitions relating to years.

382. *A civil year is a period of days, which is deſigned to be equal to one tropical year.*

A tropical year, by propoſition 63, is the time that the ſun takes up to paſs from any one of the cardinal points and return to the ſame point again: it is the time that paſſes between one vernal equinox and the next; or between one ſummer ſolſtice and the next; or between one autumnal equinox and the next; or between one winter ſolſtice and the next: for we may begin the tropical year from any one of theſe points. Such a year contains 365 days, 5 hours, 49 minutes. The tropical year is better fitted for common uſe than the periodical year, which we have ſeen deſcribed in propoſition 61: the ſolſtices and the equinoxes are remarkable events, which fall under the notice of our ſenſes and under common obſervation: though ordinary people cannot determine exactly the times of the ſolſtices and equinoxes, yet they cannot help obſerving that at one ſeaſon of the year the day is much longer, and at another ſeaſon is much ſhorter than the night; and that at two other ſeaſons the day and night are about equal to one another. Whereas it requires more obſervation and ſome knowledge in aſtronomy to know any thing of the ſuns place in the ecliptic, or of the time that it takes up to revolve once in the ecliptic. But yet, though a tropical year is determined by ſuch limits as common people are acquainted with, ſome art is required to adapt it to common uſe. For ſince in the affairs of common life we compute our time by days, every period, which is deſigned for common uſe, ſhould conſiſt of a certain number of whole days. Therefore a civil year, or a year intended for common uſe, ſhould have theſe two characters, it ſhould be equal to the tropical year, and ſhould conſiſt of whole days. It is impoſſible to make theſe two characters agree to any ſingle period: for any ſingle tropical year does not conſiſt of whole days, it contains more than 365 and fewer than 366. But in a number of years this may be brought about. For if ſome of the civil years in that number are made too ſhort for the tropical year, then by making ſome one year of the number too long, the defect, that there was in the reſt, may be ſupplyed. The manner of ſupplying this defect ſhould be obvious and regular; becauſe as the civil year is intended for the uſe of common people, they muſt have ſuch a rule to go by, as is eaſily underſtood and readily remembered.

383. *A period of twelve lunar months is called a lunar wandering year.*

Twelve lunar months, by propoſition 139, contain 354 days, 8 hours, 48 minutes. This period, becauſe it does not fall far ſhort of a tropical year, and is regulated by the moon, is called a lunar year. But as this period is about 11 days ſhorter than a tropical year, ſuch a lunar year will be finiſhed and begin again about 11 days ſooner than the tropical year.

year. Thus if the beginning of the lunar year happens at any time upon the vernal equinox; the lunar year will be finiſhed in about 354 days; whereas the tropical year contains about 365 days. Therefore the lunar year will begin again 11 days before the next vernal equinox. The lunar year next following this will begin ſtill 11 days earlier in reſpect of the vernal equinox, or 22 days before the vernal equinox. The third lunar year will begin ſtill 11 days earlier or 33 days before the vernal equinox. In this manner the beginning of ſuch a lunar year will keep going backwards in reſpect of the vernal equinox, at the rate of about 11 days every year: till in time the year will begin at the winter ſolſtice. Then the beginning of it will move backwards from the winter ſolſtice, at the rate of about 11 days every year, till it comes to the autumnal equinox. From thence it will move backwards at the ſame rate, till it comes to the ſummer ſolſtice; and at laſt returns to the vernal equinox again. And becauſe the beginning of ſuch a lunar year is not fixed, but wanders through the ſeaſons, the year itſelf is called a wandering one.

The mohammedans make uſe of ſuch a year as this. But as the lunar year conſiſts of 354 days, 8 hours, 48 minutes, and is not a period of whole days; they are obliged to uſe ſome art in adapting ſuch a year to the purpoſes of common life. The 8 hours and 48 minutes, by which one lunar year exceeds 354 days will in 30 years time amount to 11 days. Therefore if the firſt year in a cycle of 30 began from a new moon, and each year conſiſted of only 354 days, the firſt year of the next cycle would begin 11 days before the new: ſince, by reckoning a year to conſiſt of only 354 days, they would in each year loſe 8 hours 48 minutes; and conſequently in 30 years time their year would want 11 days of its true meaſure. To remedy this defect, they make 11 of their years in a cycle of 30 conſiſt of 355 days, and the other 19 of 354. By this addition of 11 days in 30 years, the defect is ſupplyed, and 30 years at this rate of reckoning will amount exactly to 10631 days, which is juſt the number of days contained in 30 lunar years reckoning 354 days, 8 hours, 48 minutes, to every year.

384. *A wandering ſolar year conſiſts of* 365 *days.*

The egyptians ſeem originally to have made uſe of a year, that conſiſted of 12 months each containing 30 days, ſo that in the whole year there would be only $12\times30=360$ days. They afterwards found that 360 days were not equal to a tropical year: and therefore to their 12 months they added 5 days at the end of the year, to make the number of days 365. The fable, in which the egyptians dreſſed up the account of theſe

additional 5 days is told by Plutarch, de iſid. et oſirid. pag. 355. Rhea, they ſay, was with child by Saturn, and Sol prayed that ſhe might not be delivered either in any month or in the year. But Mercury playing at dice with Luna overcame; and took the 72d part of every day from the lunar year, out of which he compoſed 5 days, and added theſe to the year of 360 days, that ſhe might be delivered in them. Whatever this fable may mean in other reſpects, it ſhews us what was originally the length of the civil year in Egypt. Theſe 5 additional days, or days of intercalation, were not diſtributed amongſt the months of the year: the months ſtill conſiſted of 30 days as before; and the 5 days of intercalation were added at the end of the year, after the original 12 months were expired.

This was undoubtedly the reaſon for dividing a circle into 360 equal parts or degrees. The firſt aſtronomers imagined, that the ſun deſcribed its annual circle in 360 days: they therefore divided this circle into as many equal parts, as the ſun in their opinion took up days to deſcribe it. And from this ſtandard they divided all other circles in the ſame manner.

But after 5 days were added to the former 360, and the civil year was thus made to conſiſt of 365 days; it was ſtill ſhorter than the tropical year, by propoſition 63. Such a year would be expired, and would begin again, before the tropical year was expired: therefore the beginning of this egyptian year would go backwards in reſpect of the ſeaſons. A tropical year conſiſts of 365 days, 5 hours, 49 minutes; an egyptian year conſiſts only of 365 days. Therefore if one egyptian year was to begin at the inſtant of the vernal equinox; the next egyptian year would begin 5 hours 49 minutes before the inſtant of the equinox; the third year would begin ſtill 5 hours 49 minutes earlier, in reſpect of the equinox, than the laſt: and thus in a courſe of years the beginning of the egyptian year would wander through all the ſeaſons, and return to the vernal equinox again.

385. *A common julian year conſiſts of* 365 *days, and a biſſextile or leap year conſiſts of* 366 *days.*

We make uſe of this ſort of year in England: it was firſt introduced by Julius Caeſar, when he reformed the Roman calendar. The Roman year underwent ſeveral alterations. Plutarch, in the life of Numa, ſays, that the months made uſe of in the time of Romulus were very confuſed; ſome of them conſiſted of fewer than twenty days, ſome of thirty five, and others of more. Little regard was had to the difference between a lunar and a ſolar year; the only principle, that was attended to in the di-

diviſion of time, was that a year ſhould conſiſt of 360 days. March was the firſt month of the year, and in Plutarchs opinion february was the laſt: though others, he ſays, imagined that january and february were added to the year by Numa, and that Romulus's year had only ten months in it, of which march was the firſt and december the laſt. The name of december ſhews us that it was originally the tenth month in the year, and conſequently that march was once the firſt month. But though december was the tenth month, it is not neceſſary that it ſhould be the laſt; notwithſtanding december was the tenth month, yet there might be twelve months in all. Romulus, perhaps, began his year in march, intending to begin it at the vernal equinox. Numa, when he altered the calendar, changed the point of reckoning, and began his year in january intending to begin it from the winter ſolſtice. This agrees with what Varro ſays, that *a year* means the time which the ſun takes up to go round from one winter ſolſtice to the next; de ling. lat. l. 5. and thus january and february, which were the two laſt months in Romulus's year, became the two firſt in Numas. I ſuppoſe it will be granted, that there was ſuch a month as january in Romulus's year, if we can ſhew that there was ſuch a month as february: for it was never thought that Numa added only one month to the year, if he made any addition at all it was of the two months january and february: ſo that if we can ſhew, that the month of february was in Romulus's year, we may take it for granted that january was there too. February in Numas calendar never conſiſted of ſo few as 23 days, and was the ſecond month of the year. Therefore, if we can ſhew, that it was once a month of 23 days, and was then the laſt month of the year, we may conclude, that it was in the calendar of Romulus. Certainly from Plutarchs account of the irregular lengths of Romulus's months, we are at liberty to ſuppoſe, that february might in his calendar conſiſt of only 23 days. The terminalia were celebrated upon the 23d of february: and they were called by this name according to Varro, de ling. lat. l. 5, becauſe that was reckoned the laſt day of the year. Indeed from Ovid, faſt. l. 2, we learn, that upon this feaſt the Romans ſacrificed to terminus the bounder of their eſtates: but we may depend upon Varros authority, that, whatever other terminus might afterwards be regarded in the celebration of this feaſt, it was originally called terminalia, becauſe it was the laſt day of the year, But if february 23d was once the laſt day of the year, then february muſt at that time be the laſt month of the year, and muſt conſiſt of only 23 days. We ſhall find this opinion farther confirmed, as we go on to examine the changes made in the Roman calendar. When Numa corrected the calendar, beſides changing the begin-

ginning of the year from the vernal equinox to what he thought the winter ſolſtice, it was neceſſary that he ſhould lengthen the year and regulate the months. Firſt therefore he added 5 days to the year, ſo as to make it contain 365 days: theſe days were added at the end of february, which was the end of Romulus's year. And though Numas year ended with december, yet theſe 5 additional days were not joyned to december, but to february: for we ſhall find hereafter that theſe 5 days are moveable, that all intercalations were, and ſtill are, made after the 23d of february and before theſe additional 5 days. Indeed he added another day beſides theſe 5, but not at the end of february: this made his year conſiſt of 366 days: the reaſon for making this addition of a ſingle day, we ſhall ſee preſently.

When we ſay that Numas year conſiſted of 366 days, we muſt take in a cycle of more than one year or twelve months. For when Numa regulated the calendar, he divided the days though not with any exactneſs into 12 lunar months, ſo that the year, unleſs there was an intercalation, ſhould regularly have conſiſted of only 354 days, by propoſition 383. But the ſuperſtition of the times made it neceſſary, that there ſhould be an odd number of days in the year: and for that reaſon he made his common years conſiſt of 355 days, as we learn from Macrobius, l. 1. c. 13. By the addition of this day he had another advantage in reſpect of odd numbers, which was, that all the months contained an odd number of days, except february, which was ſacred to the infernal deities, and therefore was allowed to contain an even number. The order of the months and the number of their days was this: january 29, february 28, march 31, april 29, may 31, june 29, july, then called quintilis, 31, auguſt, then called ſextilis, 29, ſeptember 29, october 31, november 29, december 29. But though he made uſe of lunar months, and though each ſingle year, if there was no intercalation, was a lunar year: yet he took care at proper times to add as many days as were requiſite to make the year up 366 days. Numas year fell 10 days and about 6 hours ſhort of a ſolar year. The year made uſe of by the Greeks conſiſted only of 354 days, and fell 11 days and about 6 hours ſhort of a ſolar year. Theſe 11 days 6 hours amount to 90 days in 8 years time: therefore the Greeks in every 8 years intercalated three months of 30 days each. Numa made an intercalation equal to this, though his year was a day longer than theirs: and if theſe intercalations had been conſtantly made, then in 24 years time the year would have been 24 days too long. But to avoid this, it was contrived, that 90 days ſhould be intercalated in the firſt and ſecond cycle of 8 years; but then in the third cycle of 8 years the intercalation was ſo managed as

to contain but 90—24=66 days: and thus in 24 years he threw out thoſe 24 days, by which his year would otherwiſe have exceeded the proper meaſure. This is what Livy means, when he ſays, l. 1. c. 19, that every twenty fourth year, by the management of the intercalations, the days of the year were brought to correſpond with the ſuns motion. In the firſt two cycles of 8 years, the intercalation was made in this manner: in the ſecond and ſixth years there was a month of 22 days intercalated, and in the fourth and eighth years a month of 23 days. In the third cycle of 8 years the intercalations muſt have been otherwiſe made: for intercalations made at this rate would add 22+23+22+23=90 days in eight years; whereas in the third cycle of 8 years only 66 days were to be added: but it does not appear what rule was followed in this intercalation. The intercalary month was called merkedinus: and the intercalation was not made at the end of december, which was the end of Numas year, but after the terminalia or 23d of february, which had before been the end of Romulus's year. So that the 5 additional days, which followed the 23d of february, were moved out of their place to make way for this intercalation.

Such a year as this, though it was rather too perplexed for common uſe, would have agreed well enough with the ſuns motion, if the intercalations had been made regularly. But the management of this affair was left to the pontifices; and they ſometimes intercalated more and ſometimes fewer days than they ought. If they had a mind to favour a magiſtrate, or to give an advantage to a farmer of the publick revenues, they made the year longer than ordinary by an undue intercalation: and on the other hand if they diſliked a magiſtrate, or were willing to do a diſſervice to a publican, they made no intercalation, when they ought to have done it. But diſlike had been of more weight than good will, for more days had been unduly omitted than had been unduly added, ſo that in Julius Caeſars time the year, for want of proper intercalations, was become too ſhort; the beginning of the civil year had gone backwards in reſpect of the ſeaſons; and the firſt or calends of january were at the ſame part of the tropical year where we now place the 23d of october.

Before we explane the manner, in which Caeſar reformed the calendar, it will be neceſſary to ſay ſomething of the Roman civil months. They were divided into three parts; the calends, the nones, and the ides. January, in Numas year, contained 29 days, as we have already obſerved. The firſt day was called the calends; the ſecond day was the fourth of, or before, the nones, the third was the third of the nones, the fourth was the day before the nones, and the fifth was the nones. Then

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the ſixth was the eighth of the ides, the ſeventh was ſeventh of the ides, the eighth was the ſixth of the ides; and ſo on till the 13th, which was called the ides. The fourteenth of january was called the 17th before the calends of february, the 16th was called the 16th before the calends of february; and ſo on, till the 29th which was the day before the calends of february. The reſt of the months were divided much in the ſame manner, except march, may, july, october, and february. Only four days of january from the ſecond to the fifth incluſively were named from the nones; ſo that january is ſaid to have four nones: but ſix days in march, may, july, and october from the ſecond to the ſeventh incluſively were named from the nones, ſo that theſe months are ſaid to have ſix nones. All the other months had four nones as january had. Each month had eight days that were named from the ides: the firſt of theſe was the day after the nones, and it was called the eighth of, or before, the ides; the laſt of them was the day of the ides, which was always the ninth day incluſively from the nones. Now the nones of january, february, april, june, auguſt, ſeptember, november, and december were upon the fifth day of the month, and conſequently the ides, which were the ninth day from the nones incluſively, muſt be the 13th day of the month. But the nones of march, may, july, and october were upon the ſeventh day of the month, and conſequently their ides muſt be upon the 15th. From the day after the ides of any month, except february, to the firſt day of the next month incluſively were 17 days: the four months, whoſe ides were upon the 14th day, contained in all 31 days: therefore there remained 16 days after the ides, the firſt of theſe 16 days, or the day after the ides, was called the 17th of, or before, the calends of the month following; becauſe from that day to the calends of the month following there were 17 days, if the calends of the month following are included in the reckoning. Thus the day after the ides of may was called the 17th of, or before, the calends of june. All the months, whoſe nones were upon the 13th day, except february, contained only 29 days: ſo that in theſe likewiſe as well as in the other four months, there were 16 days after the ides: and by an incluſive reckoning the firſt of theſe 16 days was called the 17th of, or before, the calends of the month following, as we have juſt now ſeen in the inſtance of january. February contained only 28 days: it had four nones, like january, and eight ides, like all the other months: its ides were upon the 13th day of the month: and conſequently after the ides there would remain only 15 days in the month: the firſt of theſe by an incluſive reckoning was called the 16th of, or before, the calends of march.

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The account of theſe names which were given to the three principal days, from whence the three parts of the months had their names, is this. Upon the firſt day of the month, one of the pontifices, when he had diſcovered the new moon, did *calare* or give notice of it: from whence this day was called the calends. Some imagine that the ides had their name ἀπὸ τῦ εἴδους, becauſe about that time of the month the moon did plenam ſpeciem monſtrare or was at the full. The nones ſeem to have been ſo called, becauſe they were dies nonus the ninth day incluſively from the ides.

But to return to Caeſars reformation of the calendar. The beginning of the civil year by the omiſſion of intercalations had gone backwards in reſpect of the ſeaſons. Therefore when, as pontifex maximus, he undertook to remedy this, his firſt buſineſs was to add all the days, which had been omitted, in order to bring the beginning of the year near to the winter ſolſtice. And to ſpeak more diſtinctly upon this ſubject we muſt obſerve, that this great intercalation was made in the 46th year before Chriſt. In the year before this, or in the 47th year before Chriſt, the former omiſſions had amounted to 80 days, ſo that the calends of january fell at the ſame ſeaſon with our 13th of october. The year 46, before Chriſt, like all the other Roman years conſiſted of only 335 days, and therefore was 10 days ſhorter than the tropical year. Conſequently in this year there were 80+10=90 days to be intercalated, in order to reduce the calends of january to their proper place. Caeſar therefore intercalated the month merkedinus of 23 days after the 23d of february, and likewiſe in the ſame year made another intercalation of two months, which together made 67 days. Thus he added 23+67=90 days to the year of 355 days: by which means this year conſiſted of 90+355=445 days. This is called, by Macrobius, annus ultimus confuſionis, the laſt year of confuſion, becauſe it put an end to the former confuſion in the lunar year.

It was in the year before this year of confuſion that Caeſar, as Hirtius ſays, Bell. Afric. 2, took ſhip for Africa on the 6th of the calends of january, which is the 27th of our december. But Cicero, in his ſecond book concerning divination, chap. 24, affirms that he ſailed ante brumam, before the winter ſolſtice. Now Pliny informs us, Nat. Hiſt. 18, 25, that the antients placed the winter ſolſtice upon the 25th of our december. Cicero may therefore appear inconſiſtent with Hirtius; ſince the former affirms that Caeſar ſailed before the winter ſolſtice, and the latter that he ſailed upon the 6th of the calends of january. But here we ſhould remember, that this expedition into Africa happened in the year before the calendar was reformed: and in that year the 6th of the calends of january

fell 80 days ſooner than they do now, that is the 6th of the calends of january fell at the ſame ſeaſon of the year where our 8th of october now falls: ſo that Caeſar in that year might ſail upon the 6th of the calends of january and yet ſail before the winter ſolſtice. The reader is obliged for this account to a perſon of moſt excellent learning, who printed a diſſertation upon this ſubject in the year 1722. But as he did not chuſe to put his name to that diſſertation, I do not think myſelf at liberty to mention it.

Caeſar was aſſiſted in reforming the calendar by Soſigenes an alexandrian aſtronomer; who not only taught him to correct the errors, that were paſſed, but likewiſe to prevent any conſiderable ones for the future. A common year, according to Numas calendar, conſiſted of 355 days. Caeſar added 10 days to the common year, and made it conſiſt of 365 days. He added 2 days to january, 1 to april, 1 to june, 2 to auguſt, 1 to ſeptember, 1 to november, and 2 to december. This made the months of the length, that we find them at preſent. Theſe days were added at the ends of the months: ſo that the nones and ides continued as they were in Numas calendar. The only alteration made by theſe additions was, that the months, where theſe days were added, have more calends than they had before. But then a year of 365 days is ſhorter than the tropical year, by propoſition 63; Caeſar ſuppoſed it to be 6 hours ſhorter. Theſe 6 hours in 4 years time amount to a day: and therefore in every fourth julian year a day is added or intercalated, or every fourth year conſiſts of 366 days. Theſe fourth years, which conſiſt of 366 days, we call biſſextile or leap-years. The intercalation of a day, by the julian inſtitution, is made in the ſame place where the additional month or merkedinus uſed to be intercalated in Numas year, that is, immediately after the terminalia or 23d of february. And the year is called biſſextile ob bis ſext: calend. mart. becauſe in that year there are two 6th of the calends of march or two 24ths of february. By this additional day february every fourth year contains 29 days. We call ſuch a year leap-year, becauſe each day of the year leaps or paſſes over a day of the week after the intercalation is made. Thus ſuppoſe the 25th of march to fall upon a monday one year; then in the year following, if it was a common year the 25th of march would fall upon a tueſday: but if that following year was a leap-year the 25th of march would then be upon a wedneſday: ſo that the 25th of march would by this means paſs over tueſday or would not happen upon it.

The uſe of the julian year began with the 45th year before Chriſt, but a miſtake was immediately made in the time of intercalating; for the

the pontifices intercalated a day at the beginning of every fourth year; whereas they ſhould not have intercalated till the end of it: by this means they added a day for every three years, whereas they ought to have added only a day for every four years. This error continued for 36 years: ſo that they had intercalated 12 days in theſe 36 years; though only 9 days ought to have been intercalated. There were therefore 3 days to be thrown out of the calendar, which was done by making no intercalation for 12 years: for in 12 years, there ſhould regularly be 3 leap-years or 3 days intercalated: therefore by omitting to make any intercalation for 12 years, they threw out thoſe 3 days, which their former error had added. Thus it was 48 years after the julian year was contrived, before it was received and eſtabliſhed ſo as to go on in the ſame manner that Caeſar deſigned it ſhould.

386. *A julian year is longer than a tropical year.*

The method of intercalation in the julian year, ſuppoſes every year to conſiſt of 365 days, 6 hours: for every common year contains 365 days, and the 6 hours amount to 1 day in four years time, when an intercalation is made; and that year, when it is made, contains 366 days. But a tropical year conſiſts of only 365 days, 5 hours, 49 minutes. Therefore by the julian method of intercalation the civil year is made 11 minutes longer than the tropical year.

In 131 years time 11 minutes for each year amount to 1 day. Therefore in 131 years time the julian year becomes 1 day longer than the tropical year. Now as we have ſeen in propoſition 384, if the civil year is too ſhort, the beginning of it will go backwards in reſpect of the ſeaſons; ſo if the civil year is too long, the ſeaſons will go backwards in reſpect of the beginning of the year. Thus ſuppoſe we were to begin our civil year upon the 21ſt of march, and that the vernal equinox was to happen upon the 21ſt of march; in 131 years time our year becomes a day too long for the tropical year, and conſequently the vernal equinox will come round a day ſooner than our year: ſo that as our year by the ſuppoſition is not to begin till the 21ſt of march, the vernal equinox will happen upon the 20th or 1 day before the vernal equinox. In 131 years more the vernal equinox will happen upon the 19th of march: and thus the vernal equinox or any other ſeaſon will go backwards in reſpect of the civil year, at the rate of 1 day in 131 years.

387. *The gregorian method of intercalation makes the civil year very nearly equal to the tropical year.*

At the time of the council of Nice, in the year after Chriſt 325, the vernal equinox happened upon the 21ſt of march. In the year 1582 pope Gregory XIII. found that it had gone backwards 10 days and happened upon the 11th of march. From hence it appeared that the julian year, by which they then reckoned, had been 10 days too long between the time of the council of Nice and the year 1582. Gregory therefore determined to throw theſe 10 days out of the calendar: which he did by ordering, that what would otherwiſe have been the 5th of october ſhould be reckoned the 15th; and thus what would otherwiſe have been the 11th of march became the 21ſt. So that the equinox, which, if this had not been done, would have happened upon the 11th of march, was reduced to the 21ſt. Wherever his authority was admitted this change was made in the account of time. But in England, where the pope had no authority, we ſtill adhered to our old account. This made the firſt difference between our account of time, which we call the old ſtyle, and the foreign account, which we call the new ſtyle. The firſt difference between old ſtyle and new ſtyle was 10 days: we called the day of the vernal equinox the 11th of march, and the foreigners called it the 21ſt. Where the reader ſhould obſerve that the foreigners do not ſuppoſe the vernal equinox to happen at one time and we at another; we placed the vernal equinox upon our 11th of march, and they, not upon our 21ſt, but upon their 21ſt, which is the ſame day with our 11th. We both placed the vernal equinox upon the ſame day, only we called this day the 11th, and they called it the 21ſt of march.

When Gregory had thus corrected the error, which had ariſen before his time, he contrived ſuch a method of intercalation, as will prevent the like error for the future. The julian year is 1 day too long in 131, and conſequently is 3 days too long in $3\times131=393$ years time. Therefore, to bring it to a proper length, 3 days are to be thrown out of the julian account in 393 years. For the ſake of a round number, which will be more eaſily remembered, and is fitter for common uſe, let us ſay 400 inſtead of 393 years. By the gregorian method of intercalation 3 days of the julian year are omitted in 400 years. The method is this: in the julian account every 4th year is a leap-year: and conſequently every year, which is any multiple of 4 is likewiſe a leap-year. Therefore every 100th year is a leap-year; becauſe 100 is a multiple of 4, for $25\times4=100$. But in a cycle of 400 years 3 days of this account are to be omitted: and this omiſſion for common uſe is to be under ſuch a regulation as may eaſily be remembered. Now if in a cycle of 400 years the 100th the 200th and the 300th years are made common ones, then 3 days will be left out

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of the julian account: for in the julian account each of theſe would be leap-years, that is, there would be a day intercalated in each of them, which is omitted by making them common years. If the 400th year was made a common year, then in the cycle of 400 years 4 days would be left out of the julian account: whereas in 400 years only 3 days ought to be left out. Therefore in the gregorian as well as in the julian account, every 400th year is a leap-year: Thus in a cycle of 400 years the gregorian intercalation by making each 100th year, except the laſt, a common one, leaves out 3 days, which in the julian account would have been intercalated. Therefore by this method 400 gregorian years are 3 days ſhorter than 400 julian years. But ſince in 400 years the julian year is 3 days too long; the gregorian method of intercalation will make the civil year of a proper length.

Now if every 400th year in the gregorian account is a leap-year, as well as in the julian account; it follows that every 100th year which is a multiple of 400 will be a leap-year with other nations, who reckon by new ſtyle, as well as with us, who reckon by old ſtyle. Thus 1600 is a multiple of 400, for $400\times4=1600$. But every 100th year, which is not a multiple of 400, is a leap-year in old, but a common year in new ſtyle. Thus 1700 is not a multiple of 400, for there is no number, which multiplyed into 400, will produce 1700. The account of new ſtyle began in the year 1582: and at that time, as has been obſerved already, there was a difference of 10 days between old and new ſtyle. The year 1600 did not encreaſe this difference; for the year 1600 is a leap-year both in the julian and in the gregorian account. Therefore the difference continued to be 10 days from the year 1582 to the year 1700: and then the difference became 11 days. For the year 1700 was a leap-year with us, who uſe the old-ſtyle; but was a common year with the foreigners, who uſe new ſtyle: ſo that we intercalated a day, where they intercalated none: and conſequently this difference of 1 day added to the former of 10 days made their account differ 11 days from ours. This difference will continue till the year 1800. The year 1800, becauſe it is a 100th year, will be a leap-year with us; but, becauſe it is no multiple of 400, it will be no leap-year with thoſe, who uſe new ſtyle. Conſequently as at that time we ſhall intercalate a day, when they make no intercalation, this will make another days difference between old ſtyle and new ſtyle; ſo that then they will differ 12 days. For the ſame reaſons this difference will be 13 days in the year 1900. But the year 2000, becauſe it is a 100th year will be a leap-year with us, and becauſe $2000=400\times5$ or is a multiple of 400, it will likewiſe be a leap-year with them. Therefore the year 2000 will not encreaſe the difference between old ſtyle and new ſtyle.

383. *The*

388. *The cycle of the moon or Metos cycle is a period of* 19 *julian years, in which time the new moons and full moons return to the ſame day of the julian year.*

We have ſhewn in propoſition 383, that 12 lunar months make a wandering lunar year conſiſting of 354 days: and we have ſhewn in propoſition 385, that the beginning of the lunar year may be fixed, in reſpect of the ſeaſons, by intercalating a month of 22 days every ſecond year and a month of 23 days every third year. But there is another method of fixing the beginning of the lunar year; and that is by intercalating 7 months in 19 years: of which months 6 are to conſiſt of 30 days and 1 of 29 days. One lunar year of 354 days is about 11 days ſhorter than a tropical year. Therefore 19 lunar years are $19\times11=209$ days ſhorter than 19 tropical years. Conſequently if 209 days are added to 19 lunar years, the ſum will be equal to 19 tropical years. Now $6\times30=180$; or 6 months of 30 days each make 180 days; and if to theſe we add 1 month of 29 days the whole will be $180+29=209$. Therefore, if in a courſe of 19 lunar years, 6 months of 30 days and 1 month of 29 days are intercalated, the 19 lunar years with theſe intercalations will be equal to 19 tropical years. By this means at the end of this cycle of 19 years the lunar year will begin with the tropical year. Theſe months are intercalated, whenever the difference between the lunar and tropical years amounts to a month, or more than a month. Thus in the firſt year of the cycle the difference is 11 days: in the ſecond year of the cycle the difference is 11 days more, or 22 days in all: in the third year the difference is 11 days more, or 33 days in all; but as this difference amounts to more than a month, in this third year of the cycle a month of 30 days is added to the lunar year; which leaves the difference between the lunar and ſolar or tropical year only $33-30=3$ days. In the fourth year of the cycle the difference will be encreaſed 11 days, that is, it will then be $3+11=14$ days: in the fifth year it will be $14+11=25$ days: in the ſixth year it will be $25+11=36$ days; but then again in this year a month of 30 days is added to the lunar year, which leaves the difference $36-30=6$ days. If the 7 months are intercalated by this rule, the intercalations will be made in the 3d, 6th, 9th, 11th, 14th, 17th, and 19th years of the cycle. Only in the 19th year the difference amounts to no more than 29 days, and for that reaſon the month of intercalation conſiſts only of 29 days inſtead of 30.

The difference between the ſolar and lunar years in each year of this cycle after the proper intercalations have been made are called the lunar epacts.

epacts. Thus in the firſt year of the cycle the epacts are 11, in the ſecond year they are 22, in the third year they are 3, in the fourth year they are 14, and ſo on to the 19th or laſt year of the cycle, when the epacts are 29; and by intercalating theſe 29 days at the end of the 19th year, the difference between the ſolar and lunar years becomes nothing.

In 19 lunar years conſiſting of 12 lunar months each, there are 19×12 =228 lunar months, if to theſe we add the 7 months that are intercalated, there are then 228+7=235 ſuch months in all. Now as 235 lunar months are equal to 19 ſolar years, it follows, that whatever day of the ſolar year a new moon fell upon in the firſt year of a cycle of 19 years, a new moon will fall again upon the ſame day in the firſt year of the next cycle of 19 years; that is, after a period of 19 years the new moons return to the ſame place in the ſolar year. Meto, who invented this cycle, ſeems to have imagined that 235 lunar months were exactly equal to 19 ſolar years, ſo that the new moons after ſuch a period would return not only to the ſame day but to the ſame hour and minute of the day. In this however he was miſtaken: for in 19 ſolar years there are 6939 days 18 hours, whereas in 235 lunar months there are but 6939 days, 16 hours, 32 minutes, 28 ſeconds. Therefore 19 ſolar years are 1 hour, 27 minutes, 32 ſeconds, or about an hour and a half longer than 235 lunar months. For this reaſon 19 ſolar years will come round 1 $\frac{1}{2}$ hour ſooner than 19 lunar years with their proper intercalations, or 1 $\frac{1}{2}$ hour ſooner than 235 lunar months. Thus ſuppoſe a new moon was to happen jan. 20th, at 10 o'clock, in 19 years time a new moon will happen again jan. 20th, though not at 10 o'clock, but 1 $\frac{1}{2}$ hour ſooner, or at $\frac{1}{2}$ hour after 8: in 19 years more a new moon will happen again jan. 20th, though not at $\frac{1}{2}$ hour after 8, but 1 $\frac{1}{2}$ hour ſooner, or at 6 o'clock. Now if the new moon happens 1 $\frac{1}{2}$ hour ſooner in every cycle of 19 years, then, ſince 16×1 $\frac{1}{2}$ hours =24 hours or one whole day, it follows that after 16 cycles or 16×19 =304 years the new moons will happen a whole day ſooner. Thus if, as before, a new moon happens jan. 20th at 10 o'clock, in 304 years time the new moon will happen jan. 19th at 10 o'clock: in 304 years more it will happen jan. 18th at 10 o'clock: and ſo on.

When the uſe of this cycle was firſt introduced, the new moons in the firſt year of the cycle happened upon jan. 23d. feb. 21ſt. march 23d &c. For this reaſon in the firſt column of the calendar the figure 1 is placed oppoſite to theſe days, and to all the other days, when a new moon happened in the 1 year of the cycle. The new moons happened in the ſecond year of the cycle upon jan. 12th. feb. 10th. mar. 12th. apr. 10th &c. Therefore 2 is placed in the firſt column of the calendar oppoſite to theſe days, and

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to all the other days when the new moons happened in the ſecond year of the cycle. For the ſame reaſon 3 ſtands oppoſite to jan. 1ſt. and 31ſt. to march 1ſt, and 31ſt; to apr. 29th; &c: becauſe in the third year of the cycle the new moons happened upon thoſe days. The ſame method is obſerved through the whole cycle; the number, which denotes the year of the cycle, is placed oppoſite to the days, upon which the new moons happened in that year. This number is called the prime, becauſe it ſtands in the firſt column of the calendar: and it is called the golden number, either becauſe it was written in golden characters, or upon account of the excellent uſe it was thought to be of in ſettling the time of the new moons.

The jewiſh paſſover was celebrated upon the fourteenth day of the firſt month. Their month was a lunar month, and the fourteenth day was reckoned the full moon. They began their year about the vernal equinox. So that their paſſover was upon the full moon next after the vernal equinox. The time of celebrating eaſter was ſettled in the council of Nice, in the year 325. It was deſigned that eaſter ſhould always be upon a ſunday, as near the time of the jewiſh paſſover as could be, but not upon the ſame day. For this reaſon the following rule was made for determining the time of eaſter: it was always to be obſerved upon the ſunday after the full moon, which happened next after the 21ſt of march, which at that time was the day of the equinox: but if the full moon fell upon a ſunday, then eaſter was to be the ſunday following. At that time Metos cycle was ſuppoſed to be exact, the authors of this rule not knowing the defects of it imagined, that the new moons and conſequently the full moons might always be found by Metos cycle. But Metos cycle errs 1 day in 304 years: and conſequently it errs 4 whole days ſince the time of the council of Nice. Therefore the new moons, and likewiſe the full moons happen 4 whole days ſooner now than they did, when that rule for finding eaſter was made and the golden number was fixed in the calendar. However in order to determine the day of the paſchal full moon, we do not take the day when the moon is at the full in the heavens, but go upon the ſame ſuppoſition, which they made, who inſtituted the rule; we take the day, when the moon would have been at the full, if Metos cycle had been true, which is 4 whole days later than the full moon in the heavens. Upon this account thoſe, who know the rule for determining the time of eaſter, but who are not aware how we apply this rule, are apt to think that eaſter is obſerved upon a wrong day. Thus in the year 1748, the new moon was upon mar. 18th, and the full of that moon was upon apr. 2d. The next day after this full moon or apr. 3d was ſunday; and conſequently, if in applying the rule we had been guided

by

by the full moon in the heavens, apr. 3d ſhould have been eaſter ſunday. But in applying the rule we neglect the full moon in the heavens, and conſider, when it would have happened if Metos cycle had been exact, as they ſuppoſed it to be, who made the rule. Now in the year 1748 the golden number was 1, which in the calendar ſtands oppoſite to mar. 23d: ſo that, if we determine the new moon by the golden number, mar. 23d was the day of the new moon, which is 4 whole days later than mar. 18th, when the new moon really happened. In the ſame year the full moon next after mar. 21ſt fell upon apr. 2d, but the full moon, if Metos cycle had been exact, would have fallen 4 whole days later or upon apr. 7th. Therefore, this cycle being ſuppoſed true in the application of the rule, eaſter was the ſunday following apr. 7th, that is, it was apr. 10th.

There is another difficulty in applying this rule, which has been leſs taken notice of than the former. Since eaſter is to be the ſunday after the full moon, which happens next after mar. 21ſt, and if the full moon falls upon a ſunday, then eaſter is to be the ſunday following; it ſeems to be impoſſible that eaſter ſhould ever be upon mar. 22d. For if the paſchal full moon is to be after mar. 21ſt, it cannot be ſooner than the 22d, and conſequently the ſunday after this full moon cannot be ſooner than the 23d: and if the 22d, which is the day of this full moon, ſhould be ſunday, then eaſter would not be upon mar. 22d, but upon mar. 29th, by the latter part of the rule. And yet by looking over the table for finding eaſter we may obſerve that when 16 is the golden number and D the ſunday-letter, eaſter will fall mar. 22d. This difficulty muſt be explaned by ſettling the meaning of the words *next after*. The rule does not ſay ſimply either that the full moon next, or that the full moon after, but the full moon *next after* mar. 21ſt is the paſchal full moon. If it had ſaid the full moon next mar. 21ſt, then a full moon before mar. 21ſt might have been the paſchal full moon. Or if it had ſaid the full moon after mar. 21ſt, then the rule would have deviated from the reaſon, upon which it was founded: for they, who made it, deſigned that eaſter ſhould be the ſunday after the jewiſh paſſover; and if a full moon happens mar. 21ſt or upon the day of the vernal equinox, this would be the paſſover, and conſequently by the reaſon of the rule the ſunday following ſhould be eaſter ſunday. Thus by the reaſon of the rule it appears, that if the full moon happens mar. 21ſt, and the day following, or mar. 22d, is a ſunday, then mar. 22d is to be eaſter. Therefore by the reaſon of the rule we are led to underſtand the words *next after* ſo as to include mar. 21ſt, the full moon which is next to mar. 21ſt, provided it happens either upon or after and not before mar. 21ſt.

We are farther to obſerve here, that by counting backwards we ſhould find that the year of Chriſts birth was the 2d year of the lunar cycle.

389. *The cycle of the ſun, or rather of the ſunday-letter, is a period of 28 years, in which time the ſundays and all the other days in the year return to the ſame letter.*

In the calendar we uſe the ſeven firſt letters of the alphabet A, B, C, D, E, F, G, to mark the days of each week in the year. A is placed in the calendar oppoſite to jan. 1ſt. B oppoſite to jan. 2d. C to jan. 5th: and ſo on to G, which ſtands oppoſite to jan. 7th. Then the ſame letters begin again, A ſtands oppoſite to jan. 8th. B to jan. 9th, &c. Theſe letters are repeated in the ſame manner to every week of the year; that is, they are repeated 52 times in the whole calendar, for there are 52 weeks in a year. Indeed there are in a year 52 weeks and one day; ſo that after theſe letters have been repeated 52 times, they begin again for the laſt day of the year; and A, which is placed oppoſite to the firſt day of the year or jan. 1ſt, is likewiſe placed oppoſite to the laſt day of the year or dec. 31ſt.

If the year is a common one, and not a leap-year, the ſame letter, which ſtands oppoſite to the firſt ſunday in january, will mark out the ſundays or will be the ſunday-letter through the whole year. Thus ſuppoſe jan. 7th to be a ſunday; jan. 7th is marked G in the calendar; and every day, which is marked G, will be a ſunday that year. But in a biſſextile or leap-year, the ſunday-letter changes at the 6th of the calends of march, which is february 24th. If for inſtance G is the ſunday-letter at the beginning of january, then F, which is the next letter before G, muſt be the letter that marks the ſaturdays in that year: and conſequently february 24th, which is marked F in the calendar, will be upon a ſaturday. But in a leap-year, by propoſition 385, there are two 24ths of february; and both of them are marked F in the calendar or almanack for that year: the firſt of them is a ſaturday in courſe, but then the next of them muſt be a ſunday. Therefore at the time of intercalation F ſtands oppoſite to ſunday; and then, as the letters go on again in their order, F will continue to be the ſunday-letter for the reſt of the year.

The ſunday-letter changes every year, upon account of the odd day there is in every year: this change is made in the inverted order of the letters, in the order G, F, E, D, C, B, A. If there were juſt 52 weeks in a year, the letters would be repeated each year 52 times in the calendar, and then would begin again with the firſt day of the next year. But there is in a common year 355 days or 52 weeks and 1 day. So that A, which ſtands oppoſite to jan. 1ſt, ſtands oppoſite to dec. 31ſt. Therefore what-

whatever day of the week jan. 1ſt falls upon, dec. 31ſt will fall upon the ſame day of the week; becauſe they are both marked with the ſame letter A. This will occaſion a change of the ſunday-letter every year. For ſuppoſe that the year begins with monday, then, becauſe jan. 1ſt is monday, jan. 7th will be ſunday, and G will be the ſunday-letter. But as jan. 1ſt is monday, by the ſuppoſition, dec. 31ſt will be monday too; and conſequently the next year will begin with tueſday. Then as jan. 1ſt marked A is tueſday, jan. 6th marked F will be ſunday. Therefore in this next year F will be the ſunday-letter. Now as, in the year ſuppoſed, jan. 1ſt is tueſday, dec. 31ſt will likewiſe be tueſday: therefore the year following will begin with wedneſday: and as jan. 1ſt marked A is wedneſday, jan. 5th marked E muſt be ſunday, and conſequently E will be the ſunday-letter. If all our years conſiſted of 365 days, ſuch a change as this would be made every year; and the ſeven letters would each of them be the ſunday-letter in its turn; ſo that the cycle of the ſunday-letter would be completed in ſeven years time. But in every leap-year, that is, in every fourth year, there is a double change of the ſunday-letter, one at the beginning of the year, and the other at the day of intercalation, as has been ſhewn already. Since then the regular change of the ſunday-letter, which would otherwiſe be completed in 7 years, is thus interrupted once in 4 years, the whole change will not be completed, ſo that the ſunday-letters may again return in the ſame order, in fewer than $4\times7=28$ years.

If we were to reckon backwards by cycles of the ſun, or cycles of 28 years each, we ſhould find that the birth of Chriſt happened in the 10th year of this cycle.

390. *The indiction is a cycle of* 15 *years.*

This cycle has no dependence upon the motion of the heavenly bodies. The indiction is generally looked upon to have been originally certain annual taxes, the tariffs whereof were publiſhed every year. But why they are ſo called, why they are confined to a cycle of 15 years, when and on what occaſion they were inſtituted, is not known. Chriſt was born in the 5th year of this cycle.

391. *The julian period conſiſts of* 7980 *julian years: or it is the product of the three cycles of the moon, ſun, and indiction multiplyed into each other.*

The cycle of the moon contains 19 years, the cycle of the ſun 28 years, and the cycle of the indiction 15 years. If theſe cycles were ever to begin together it would be $19\times28\times15=7980$ years before they could

begin together again. This period was invented by Joſeph Scaliger: it is called the julian period, becauſe it conſiſts of julian years. We will here conſider ſome of its uſes and how it is applyed in chronology.

Each year in this period is diſtinguiſhed from all the reſt by cycles peculiar to itſelf. Thus in the firſt year of this period, the cycle of the moon, or golden number, was 1, the cycle of the ſun was 1, and the cycle of the indiction 1. Now other years of the julian period might be the 1ſt year of one or two of theſe cycles; but it is only the 1ſt year of it, that can be the 1ſt year of all the three cycles. The year after Chriſt 1749, is the 2d of the cycle of the moon, the 22d of the cycle of the ſun, and the 12th of the indiction: theſe three characters agree to no other year of the julian period but the year 6462. Therefore the year after Chriſt 1749 is in the julian period the year 6462. In the year of Chriſts birth, the cycle of the moon was 2, of the ſun 10, of the indiction 4: theſe three characters agree to no other year of the julian period beſides the year 4714. Therefore the year, when Chriſt was born, was in the julian period the year 4714. The common account of the time when the world was created is, that it was 3950 years before Chriſt. Now Chriſt was born in the year of the julian period 4714: and 4714 —3950=764. Therefore the world was created in the year 765 of the julian period: and conſequently this period muſt begin before the creation of the world; that is, from the preſent poſition of the cycles of the moon, ſun, and indiction, we ſhould find that theſe three cycles could not have begun in one and the ſame year, unleſs we reckon backwards 764 years before the creation of the world. From hence then we ſee that the firſt 764 years of this period are imaginary. But the period itſelf is of ſuch a length that all the remarkable events which have happened from the creation to this time, may be referred to ſome year of it. The creation was in the year of this period 765: and the preſent year 1749 is the year 6462. And as the period contains 7980 years, there are ſtill to come 7980—6462=1518 years of it: ſo that all the remarkable events, which ſhall happen for 1518 years to come, may by future chronologers be referred to ſome year of the julian period.

392. *An aera or epoch is a point of time, from which we begin to reckon in computing a ſeries of years.*

We commonly for this purpoſe chuſe a point of time, when ſome remarkable event happened. Computations of years are ſometimes made from the aera of the creation, thus we ſay that Chriſt was born, according to the common account, in the year of the world 3950. The aera

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of Nabonaſſar is the beginning of Nabonaſſars reign; from this point of time the Chaldaeans and Egyptians reckon their years. This aera begins 747 years before Chriſt, in the year 3967 of the julian period. The Greeks reekoned their years by olympiads: each olympiad contained four years: and the reckoning began from the time, when Iphitus inſtituted or rather reſtored the olympic games, in the year 3939 of the julian period, 776 years before Chriſt. The aera of building the city was what the Romans uſed: the time when the city of Rome was built, from whence this reckoning begins, was, according to Varros account, in the year of the julian period 3961, and before Chriſt 753 years; but according to the faſti capitolini it was a year later.

The aera, which we make uſe of, begins from the incarnation: and that, which other chriſtian nations make uſe of, begins from the birth of Chriſt. We in our way of reckoning ſuppoſe that it was 1748 years on the 25th of laſt december ſince Chriſt was born; and conſequently that on the 25th of this march it was 1749 years ſince his incarnation. Though we begin our civil year from jan. 1ſt, or from the beginning of the month, which follows the winter ſolſtice, yet we begin our eccleſiaſtical year and change our date mar. 25th, or from the day, where the vernal equinox fell at the time when the calendar was reformed, and the julian year was firſt brought into uſe. Thoſe of other nations, who uſe the chriſtian aera, ſuppoſe Chriſt was born a year ſooner, or that on the 25th of laſt december it was 1749 years ſince he was born: and they change their date jan. 1ſt. So that for one quarter of a year from jan. 1ſt, to mar. 25th. our date differs from theirs. But though we differ a year from them in the account of the time when Chriſt was born, yet for the other three quarters of the year our dates agree. Becauſe, though we both reckon by the chriſtian aera, and ſhould have differed a year in our dates, if we had begun this aera from the ſame point of time; yet as they begin to reckon from the birth of Chriſt, and we from the incarnation, which is three quarters of a year ſooner, our date differs from theirs only for one quarter of the year. We have indeed a method of writing our date, which prevents any miſtake that might ariſe from the difference between theſe two accounts: from jan. 1ſt. to mar. 25th of this preſent year, it is uſual with us in England to write our date $174\frac{8}{9}$, by which we mean, that with us it is the year 1748, but with other nations, who uſe the chriſtian aera, it is the year 1749.

CHAP.

CHAP. XV.

Of the cauſes, which produce the motions of the planets.

393. *The primary planets are retained in their orbits by a force which urges them towards the center of the ſun.*

IF we are aſked from what cauſe the motion of the planets began; we ſhould anſwer that it was the omnipotent Author of all things, who firſt moved them: for true philoſophy, inſtead of excluding Him from the work of creation and providence, teaches us to trace out his power and wiſdom in the production of the world, and in the laws, by which it is governed. To his immediate act we refer the motion of the planets; and to a law of his appointing, we refer their motion in curves, which return into themſelves. It is our buſineſs at preſent to trace out this law, and to determine what quality there is in the ſun and planets, which makes the planets revolve in orbits round the ſun.

The primary planets, by the firſt law of motion, propoſition 11 of mechanics, when they are once moved, ſhould continually move on uniformly in right lines. We are therefore to enquire what force it is, which draws them from ſuch right lines, and makes them deſcribe orbits round the ſun. In this propoſition we are to determine where this force acts, or towards what point it urges the planets. Now it is found by obſervation, that each of the primary planets, as they revolve in their reſpective orbits, deſcribe by a radius drawn to the ſun equal areas in equal times. Therefore ſince, by propoſition 228, the ſun is at reſt within the orbits of the primary planets, it follows, from propoſition 160 of mechanics, that the centripetal force, which retains them in their orbits, and prevents them from running out in right lines, urges or impells them towards the ſun.

394. *The centripetal force, which retains the primary planets in their reſpective orbits, is, at different diſtances from the ſun, inverſely as the ſquares of thoſe diſtances.*

Kepler obſerved that the ſquares of the periodical times of the primary planets, are as the cubes of their middle diſtances from the ſun. Thus ſaturns periodical time is to jupiters periodical time about as 30 to 12, by propoſitions 113, 114: the ſquares of which numbers are 900 and 144. The diſtance of ſaturn from the ſun is to the diſtance of jupiter from the ſun, about as 9 to 5, by the ſame propoſitions: the cubes of which numbers are 729 and 125. And 900 is to 144 nearly as 729 to 125. We have here

here taken neither the periodical times nor diſtances with exactneſs: the numbers, as they are ſhort and near the truth, will ſerve to explane the meaning of Keplers analogy. But the proportion would have been more exact if the preciſe numbers had been made uſe of. In like manner the periodical time of the earth is to the periodical time of mercury in a ſomething greater proportion than that of 4 to 1: the ſquares of their periodical times are nearly as 17 to 1. The diſtance of the earth from the ſun is to the diſtance of mercury from the ſun in a ſomething greater proportion than that of 10 to 4: the cubes of their diſtances are nearly to one another as 1000 to 59. And 17 is to 1 nearly as 1000 to 59. This is what we mean when we ſay that the ſquares of the periodical times of the primary planets are as the cubes of their diſtances.

But as this proportion is found to obtain in all the primary planets, it follows, by propoſition 171 of mechanics, that, at different diſtances from the ſun, the centripetal force, by which they are retained in their reſpective orbits, is inverſely as the ſquares of thoſe diſtances. And by ſuch a centripetal force as this, they are made to deſcribe elliptical orbits round the ſun placed in one of the focuſes; ſee propoſition 7, and propoſition 172 of mechanics.

395. *The ſatellites of jupiter and of ſaturn are retained in their orbits by a force, which urges them towards the center of their reſpective primary.*

The ſatellites of jupiter by a radius drawn to the center of jupiter deſcribe equal areas in equal times. Now jupiter and its ſatellites move together round the ſun, that is, this motion is common to both of them: and conſequently the motion of the ſatellites in reſpect of jupiter is the ſame as if neither of them moved round the ſun; or the ſame as if the ſatellites revolved round jupiter at reſt. But ſince jupiter in reſpect of the ſatellites is relatively at reſt within their orbits, and they by a radius drawn to the center of jupiter deſcribe equal areas in equal times; it follows, from propoſition 160 of mechanics, that the force, which retains theſe ſatellites in their reſpective orbits, conſtantly urges or impells them towards the center of jupiter.

In the ſame manner we may prove, that the ſatellites of ſaturn are retained in their orbits by a force, which urges them towards the center of ſaturn.

396. *The centripetal force, which retains the ſatellites of jupiter or of ſaturn in their orbits, is, at different diſtances from the centers of jupiter or of ſaturn, inverſely as the ſquares of thoſe diſtances.*

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This follows from propoſition 171 of mechanics: for the ſquares of the periodical times of jupiters ſatellites are found by obſervation to be as the cubes of their diſtances from jupiter: and the ſquares of the periodical times of ſaturns ſatellites are likewiſe found to be as the cubes of their diſtances from ſaturn.

397. *The moon is retained in its orbit by a force, which urges it towards the earths center.*

This follows from propoſition 160 of mechanics: for the moon by a radius drawn to the earths center deſcribes equal areas in equal times round the earth. Indeed the earth is not at reſt in the moons orbit: for the moon and earth together revolve round the ſun. But then, ſince this motion round the ſun is common both to the moon and earth, the motion of the moon in reſpect of the earth is the ſame as if neither of them moved round the ſun, or the ſame as if the moon revolved round the earth at reſt.

398. *The force, which retains the moon in its orbit, is, at different diſtances from the earth, inverſely as the ſquares of thoſe diſtances.*

This follows from propoſition 172 of mechanics: for the moon deſcribes an immoveable ellipſis round the earth, which is placed in one of the focuſes.

Indeed the moons orbit is a moveable ellipſis: but the motion of it ariſes from the ſuns action upon the moon. At preſent we neglect the force, with which the ſun diſturbs the moons motion: and when this force is neglected, or if no ſuch force acted, the moons orbit would be an immoveable ellipſis; that is, if the moon was acted upon by no other force but that, which by propoſition 397, urges it towards the earths center, it would deſcribe an immoveable ellipſis round the earth placed in one of the focuſes. Therefore, by propoſition 172 of mechanics, the centripetal force, which urges the moon towards the earth and retains it in its orbit, is, at different diſtances from the earth, inverſely as the ſquares of thoſe diſtances.

399. *The mooon is retained in its orbit by the force of gravity,*

We have proved in propoſition 105 of mechanics, that a heavy body, if it was as far off from the earth as the moon is, would deſcend by the force of gravity through 1 rod or pole in a minute of time. And here we will prove farther, in order to prepare the way for the demonſtration of this propoſition, that the centripetal force, which urges the moon towards

wards the earth, makes it fall towards the earth through 1 pole in a minute of time.

Let AEMR, Plat. XXIX. fig. 7, be the earth, PLV part of the moons orbit, LC an arc, which the moon deſcribes in its orbit in a minutes time. The moon deſcribes its whole orbit in 27 days, 7 hours, 43 minutes, by propoſition 138. But if 27 days, 7 hours, 43 minutes are reduced to minutes, they will make 39343; that is, the moon deſcribes its whole orbit in 39343 minutes. From hence we may determine the length of the arc LC, which the moon deſcribes in a minutes time. For if it deſcribes the whole orbit or 360°. in 39343 minutes; then in 1 minute it muſt deſcribe the $\frac{1}{39343}$ part of the whole orbit or of 360°. But the $\frac{1}{39343}$ part of 360°. is 33″. Therefore the arc LC, which the moon deſcribes in 1 minute of time, is an arc of 33″. If the moon, when it ſets out from L, was not urged towards the earth by any force at all, it would move along the right line LB, by propoſitions 11, 153, of mechanics. Therefore, if, inſtead of moving in the tangent LB, it moves in the arc LC, there muſt be a centripetal force or force urging it towards T the earths center, which draws it from the tangent LB, and makes it deſcribe the arc LC: ſo that in a minutes time, or whilſt it is deſcribing this arc LC, it is drawn from the tangent, or falls towards the earth from the tangent, through the ſpace BC = LD. Or otherwiſe: When the moon ſets out from L; it is acted upon by two forces; by a projectile force in the tangent LB, and by a centripetal force in the direction LT. Theſe two forces make it deſcribe the arc LC in a minute: and conſequently by propoſition 16 of mechanics, we may conſider LC as the diagonal of a parallelogram; and then LB in the direction of the projectile force is one ſide of this parallelogram, and LD in the direction of the centripetal force is the other ſide. But the moon in a minute deſcribes the diagonal LC by the joynt action of theſe two forces. Therefore by the projectile force alone it would deſcribe the ſide LB in a minute, and by the centripetal force alone it would deſcribe LD in a minute. From hence then it appears that LD or its equal BC, is the ſpace through which the force, that retains the moon in its orbit, would make the moon fall towards T the earths center in 1 minutes time. Now LD is the verſed ſine of the arc LC: and LC is an arc of 33″, as has been proved already. Therefore the force, which urges the moon towards the earths center, is ſuch an one, as would make the moon fall in a minutes time through the verſed ſine of an arc of 33″. We muſt next enquire what is the length of LD the verſed ſine of an arc of 33″. in the moons orbit. AEMR the earths circumference contains 123249600

Paris feet, according to Picarts menſuration. The earths ſemidiameter AT is ſomething leſs than a 6th part of its circumference and contains 19615800 feet. If AT the earths ſemidiameter, or 19615800 feet, is multiplyed by 60, this will give us the length of LT a ſemidiameter of the moons orbit 1176948000 feet. Now LT is to LD as the radius to the verſed ſine of 33″, or about as 1176948000 to 15 $\frac{1}{12}$. Therefore, ſince the length of LT is 1176948000 feet, the length of LD is 15 $\frac{1}{12}$ Paris feet, which is nearly equal to 16 $\frac{1}{12}$ Engliſh feet or to 1 pole. But LD is the ſpace through which the moons centripetal force would make it fall towards the earths center in a minute. Therefore, if the moon was to fall towards the earth, the centripetal force, which urges it towards the earth, would make it fall 1 pole in the firſt minute of its deſcent.

Thus we have ſhewn that the moons centripetal force is equal to the force of gravity. For if the moon was falling towards the earth, the centripetal force would make it fall 1 pole in the firſt minute of its deſcent: and if a heavy body was as far off from the earth as the moon is, the force of gravity would likewiſe make ſuch a heavy body fall 1 pole in the firſt minute of its deſcent. We are next to prove that this centripetal force is the force of gravity.

It is one rule of true philoſophy, that whatever we find to be a quality of all ſuch bodies, as we can make experiments upon, that is to be looked upon as a quality of all bodies whatever. But all bodies that we can make experiments upon have gravity. Therefore all bodies whatever, the moon amongſt the reſt, have gravity. Now the moons centripetal force is equal to the force of gravity; and conſequently the force of gravity is ſufficient to produce the effect, which this centripetal force produces, or is ſufficient to retain the moon in its orbit. But it is another rule in true philoſophy, that whatever cauſe exiſts in nature and is ſufficient to produce any effect, that is to be looked upon as the true cauſe of this effect. Gravity is a quality, which exiſts in nature, and has been proved to be a quality, which belongs to the moon: and gravity has likewiſe been ſhewn to be a cauſe ſufficient to retain the moon in its orbit. Therefore gravity is the cauſe, which produces this effect, or gravity is the centripetal force, which retains the moon in its orbit.

Or otherwiſe. The moon has gravity, or the moon gravitates towards the earth as other heavy bodies do. And the force of gravity would make the moon fall towards the earth through 1 pole in a minute. But the centripetal force, which retains the moon in its orbit, would make the moon fall towards the earth through 1 pole in a minute. Therefore there is no other centripetal force, that acts upon the moon to retain it in its orbit,

but

but the force of gravity. The reaſon for this concluſion is obvious. Becauſe, if the moons gravity would make it fall through 1 pole in a minute, then its gravity together with any other centripetal force would make it fall through more than 1 pole in a minute. But as the moon revolves in its orbit, it falls from the tangent towards the earths center through no more than 1 pole in a minute, as has been proved. Therefore, as it revolves in its orbit, there is no other centripetal force acts upon it, beſides the force of gravity: and conſequently gravity is the centripetal force, which retains it in its orbit. That the force of gravity, which at different diſtances from the earth is inverſely as the ſquares of thoſe diſtances, may make the moon deſcribe an elliptical orbit having the earth in one of its focuſes has been ſhewn in propoſition 172 of mechanics.

400. *The ſatellites of jupiter and of ſaturn are retained in their orbits by the force of gravity.*

The ſatellites of jupiter are retained in their orbits by a force, which conſtantly urges them towards the center of jupiter, by propoſition 395; juſt as the moon, by propoſition 397, is retained in its orbit by a force, which urges it towards the center of the earth. The force, which retains the ſatellites of jupiter in their orbits, is, at different diſtances from jupiter, inverſely as the ſquares of thoſe diſtances, by propoſition 396; juſt as the moon, by propoſition 398, is retained in its orbit by a force, which at different diſtances from the earths center is inverſely as the ſquares of thoſe diſtances. Thus in reſpect of jupiter the force, which retains the ſatellites in their orbits, acts in the ſame manner and by the ſame law, as the force, which retains the moon in its orbit acts in reſpect of the earth: ſo that the revolution of the ſatellites round jupiter is in all particulars exactly ſimilar to the revolution of the moon round the earth. But it is a rule of true philoſophy that all effects, which are of the ſame ſort, are produced by the ſame cauſes: and the moon, by propoſition 399, is retained in its orbit by its gravitation towards the earth. Therefore the ſatellites of jupiter are likewiſe retained in their orbits by their gravitation towards jupiter.

We may prove in the ſame manner that the ſatellites of ſaturn are retained in their orbits by their gravitation towards ſaturn.

401. *The primary planets are retained in their orbits by the force of gravity.*

As the moon, by propoſitions 397, 398, is retained in its orbit by a force, which conſtantly urges it towards the earth; and as this force, at different

diſtances from the earth, is inverſely as the ſquares of the diſtances; ſo likewiſe the primary planets, by propoſitions 393, 394, are retained in their orbits by a force, which conſtantly urges them towards the ſun, and this force, at different diſtances from the ſun, is inverſely as the ſquares of the diſtances. Now ſince the revolution of the primary planets round the ſun is an effect of the ſame ſort with the revolution of the moon round the earth; and ſince the centripetal force, which retains the primary planets in their orbits, acts, in reſpect of the ſun, in the ſame manner and by the ſame laws, as the force, which retains the moon in its orbit, acts in reſpect of the earth, the conſequence is, that the primary planets are retained in their orbits by their gravitation towards the ſun. For, by propoſition 399, the moon is retained in its orbit by the force of gravity; and it is a rule of true philoſophy, that all effects, which are of the ſame ſort, are produced by the ſame cauſe.

402. *As the ſun and any planet gravitate mutually towards one another, the common center of gravity of theſe two bodies will continue at reſt: and both the ſun and the planet will revolve round that common center of gravity.*

We have hitherto ſuppoſed that the ſun is immoveable, but now we muſt correct this ſuppoſition, and muſt ſhew, that as the planets are retained in their orbits by their gravitation towards the ſun, the ſun cannot be immoveable.

The Planet P, Plat. XXIX. fig. 8, gravitates towards the ſun S, by propoſition 401: but ſince the planet gravitates towards or is attracted by the ſun, it follows, by propoſition 73 of mechanics, that the ſun likewiſe gravitates towards or is attracted by the planet.

Imagine a right line PS to be drawn from the planet P to the ſun S: then the common center of gravity is ſome point C in this line, which divides the line PS or the diſtance of P from S in the inverſe proportion of their quantities of matter, or if PC is as much greater than SC, as the quantity of matter in P is leſs than the quantity of matter in S, then C is the common center of gravity of the planet P and the ſun S: as appears from what has been ſaid in propoſition 50 of mechanics. The propoſition now before us affirms, that if the planet P revolves in an orbit, and is retained in this orbit by its gravitation towards the ſun S; then the ſun S muſt likewiſe revolve, becauſe C the common center of gravity will continue at reſt.

If neither of the two bodies P and S revolved in any orbit at all, then by their mutual gravitation they would move towards each other, and would

would meet at C their common center of gravity; ſo that during the approach of theſe two bodies C their common center of gravity would reſt, by propoſition 74 of mechanics. Let us then ſuppoſe that the body P has a projectile force given to it in the direction PT, and that this projectile force and its gravitation towards S makes it deſcribe an orbit round S, by propoſition 152 of mechanics: ſuch a projectile force will prevent the body P from approaching to S, though it gravitates towards S. But if S has not as great a projectile force given to it at the ſame time in the oppoſite direction SR, then becauſe S continues to gravitate towards P, and there is no force, which can prevent its approaching to P, it follows that S will approach to P, or as P revolves round S the mutual gravitation of theſe two bodies will diminiſh the diſtance SP. Now we have ſhewn already that C the common center of gravity always divides this diſtance SP in the inverſe proportion of S to P, or that SC is always as much leſs than PC as the quantity of matter in S is greater than the quantity of matter in P: conſequently ſince the quantity of matter in S and in P is always the ſame, SC and PC always bear the ſame proportion to one another. But as S approaches to P, SC decreaſes. Therefore PC muſt decreaſe in the ſame proportion. But PC can decreaſe no otherwiſe than either by the approach of P to C or by the approach of C to P. But the projectile force prevents P from approaching to C. Therefore C muſt approach to P. Thus we have ſhewn that, if P has a projectile force given to it and is made to revolve, unleſs S has an equal projectile force given to it at the ſame time, the mutual gravitation of theſe two bodies towards one another will put C, which is their common center of gravity, in motion: But, by propoſition 74 of mechanics, the mutual action of two bodies upon one another cannot put their common center of gravity in motion. Therefore as the planet P begins to move in the direction PT, the ſun S will likewiſe begin to move in the direction SR: and C their common center of gravity will continue at reſt. And as they tend mutually towards each other or towards C their common center of gravity, their motions will not continue to be performed in the right lines PT, SR; but the planet P will revolve round C in an orbit of which *Pp* is a part, and the ſun S will revolve round C in an orbit of which *Ss* is a part.

403. *The ſun and any planet, whilſt they mutually gravitate towards each other, deſcribe ſimilar figures round their common center of gravity and round each other.*

If S, Plat. XXIX. fig. 9, is the ſun, P any planet, and C their common center of gravity, we have proved, in propoſition 402, that, as S

and

and P mutually gravitate towards each other, they will revolve round C their common center of gravity. But to a ſpectator at the ſun S, who imagines his own place to be at reſt, the planet P will appear to revolve round S: and to a ſpectator at P, who imagines his own place to be at reſt, the ſun S will appear to revolve round P. Thus if P is the earth, and a ſpectator is at the ſun S, as he does not attend to the motion of the ſun round C the common center of gravity, but imagines his own place to be at reſt, the whole apparent motion will be the ſame to him, as if S actually reſted and P revolved round it: or if the ſpectator is at P the earth, as he does not attend to the motion of the earth, but imagines it to be at reſt, the whole apparent motion will be the ſame to him, as if P the earth actually reſted and S the ſun revolved round it, by propoſition 26. The ſame would be equally true, if P was any other planet inſtead of the earth.

Now in the propoſition we affirm, firſt, that if C is the common center of gravity of the ſun S and any planet P, whatever orbit the planet P deſcribes round C this common center of gravity, the orbit, which the ſun S deſcribes round C, will be ſimilar to the orbit of the planet: ſecondly the orbit, which the planet P appears to deſcribe round the ſun S conſidered, as at reſt, and likewiſe the orbit, which the ſun S appears to deſcribe round the planet P conſidered as at reſt, will be ſimilar to the orbits, which both the ſun and the planet deſcribe round C.

The common center of gravity C of the ſun S and any planet P is always in a right line drawn from the ſun to the planet, as appears from what has been ſaid under propoſition 402. So that if the planet moves from P to *p*, then the line *p*C continued muſt paſs through the ſun, and conſequently the ſun muſt have moved from S to *s*. Thus P*p* and S*s* are the arcs, which the planet and the ſun deſcribe in their reſpective orbits in the ſame time: or PC*p* and SC*s* are the areas deſcribed in the ſame time by a radius drawn reſpectively from the planet P and the ſun S to C their common center of gravity. Theſe two areas PC*p* and SC*s* are ſimilar. For ſince, by what has been ſaid, PS and *ps* are right lines, the vertical angles PC*p* and SC*s* are equal. Euc. b. I. prop. 15. And C the common center of gravity divides each of theſe lines in the ſame proportion; becauſe, by what has been ſaid under propoſition 402, it divides each of them in the inverſe proportion of the quantities of matter in P and S. Conſequently SC is to PC, as *s*C to *p*C. Thus we have proved that, in the two figures SC*s* and PC*p*, the angles at C are equal, and the ſides SC, PC, and *s*C, *p*C, which include thoſe equal angles, are proportional; and from hence it follows that the two figures are ſimilar. Euc. b. VI. prop. 5.

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In the ſame manner that we have proved theſe parts of the two orbits to be ſimilar, we might likewiſe prove all the other parts of each to be ſimilar: and conſequently the whole orbit, which S the ſun deſcribes round C the common center of gravity, is ſimilar to the whole orbit, which P the planet deſcribes round the ſame center.

Secondly, the orbit, which the planet P apparently deſcribes round the ſun S conſidered as at reſt, will be ſimilar to the orbit, which the ſame planet P deſcribes in the mean time round C their common center of gravity. When the planet has revolved quite round the center of gravity, the ſun will likewiſe have revolved quite round the ſame center: but if a ſpectator had been at the ſun, as he would imagine his own place to be at reſt, this revolution of the planet would appear to him as if it had been performed round the ſun: that is, when P has completed its whole orbit round C, or when the angular motion of P round C has been equal to 360°. it will appear to a ſpectator at S to have completed its orbit round S, or its apparent angular motion round S will likewiſe be equal to 360°. In like manner when P has revolved half round C, or when its angular motion round C has been equal to 180°, its apparent angular motion round S conſidered as at reſt will have been equal to 180°: and ſo univerſally, the angular motion of the planet P round C the common center of gravity, in any given time, will be equal to its apparent angular motion, in the ſame time round S, as this motion is ſeen by a ſpectator at S, who imagines his own place to reſt. Thus ſuppoſe the planet P, in any given time has moved round C from P to *p*, the ſun S in the mean time will have moved from S to *s*, by propoſition 402. In the mean time a radius drawn from the planet to the center of gravity C has been carryed from PC to *p*C, and the angle P*C*p contained between the radius in theſe two different ſituations is the meaſure of the planets angular motion. The diſtance of the planet from the ſun at the beginning of this motion is PS, and at the end of it is *ps*. But if a ſpectator had been at the ſun S during the time of this motion, he would have imagined his own place or the ſun to be at reſt; and the apparent angular motion of P round S thus conſidered as at reſt would have been equal to P*C*p. Now ſince the diſtance of the planet from the ſun at the beginning of the motion is PS, and at the end of it is *ps*; let S*t* be taken equal to *ps*, and make the angle PS*t* equal to the angle P*C*p; and then, whilſt P by a radius drawn to C is deſcribing the figure P*C*p, it will by a radius drawn to S apparently deſcribe the figure PS*t*. And we are to prove that theſe two figures P*C*p and PS*t* are ſimilar. C the common center of gravity always divides the diſtance between the ſun and the planet in the inverſe proportion of their quantities of matter;

ter: therefore, whether the ſun is at S and the planet at P, or the ſun is at *s* and the planet at *p*, C divides PS in one caſe and *ps* in the other caſe in the ſame proportion; ſo that SC is to PC, as *s*C to *p*C. From hence it folows that SC+PC is to PC, as *s*C+*p*C is to *p*C. Euc. b.V. prop. 18. But SC+PC=PS and *s*C+*p*C=*sp*=S*t*. Thefore PS is to PC, as S*t* is to *p*C. Thus in the two figures PS*t* and PC*p* the angles at S and C have been proved to be equal, and the ſides PS, S*t*, and PC, C*p* including theſe equal angles are proportional, therefore the two figures are ſimilar. Euc. b.VI. prop. 5. What has been here ſhewn of one part of theſe orbits, which P deſcribes round C and S, might be ſhewn of every other part. Therefore the whole orbit, which any planet appears to deſcribe round the ſun conſidered as at reſt, is ſimilar to the whole orbit, which the ſame planet deſcribes round the center of gravity. And ſince the orbit, which the ſun deſcribes round the common center of gravity, has been proved to be ſimilar to the orbit, which the planet deſcribes round the ſame center; it follows that the orbit, which any planet appears to deſcribe round the ſun conſidered as at reſt, will be ſimilar to the orbit, which the ſun in the mean time deſcribes round the common center of gravity.

If the ſpectator is at the planet, whilſt the planet P and the ſun S revolve round their common center of gravity, he imagines his own place to be at reſt; and the whole motion will appear to belong to the ſun: in this caſe we might prove in the ſame manner, that the orbit, which the ſun S appears to deſcribe round the planet P, is ſimilar to either of the orbits, which the planet and ſun deſcribe round their common center of gravity.

From hence it follows, that if the ſuns apparent motion ſeen from the earth or from any other planet is in an ellipſis having the earth in one of its focuſes, the earths apparent motion ſeen from the ſun will be in a ſimilar ellipſis having the ſun in one of its focuſes: and if the ſun and earth or ſun and any planet mutually gravitate towards each other; they will both deſcribe ſimilar elliptical orbits having their common center of gravity in one of the focuſes.

404. *The common center of gravity of the ſun and all the planets is at reſt, and is the center of the ſolar ſyſtem.*

Whatever is the center of the ſolar ſyſtem, it is agreed by all that this center is at reſt: thus the defenders of Ptolemys or Tychos ſyſtems maintain that the earth, which they ſuppoſe to be the center of the ſolar ſyſtem, is at reſt: and the defenders of Copernicus's ſyſtem maintain that

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the ſun, which they ſuppoſe to be the center of the ſolar ſyſtem, is at reſt. We have ſhewn indeed, in propoſition 228, that the earth moves, and are to ſhew here that the ſun moves: ſo that we do not refer to Ptolemys opinion to ſhew that the earth is the center of the ſyſtem, or to Copernicus's to ſhew that the ſun is the center of it: the uſe, which we are here to make of their opinions is this; though aſtronomers have differed about the center of the ſolar ſyſtem in other reſpects, ſome ſuppoſing the earth and others ſuppoſing the ſun to be that center, yet they have all agreed in this, that the center of the ſyſtem is at reſt. We may therefore take this for granted, as it is a point in which all aſtronomers are agreed. Now the planets are made to revolve in their orbits by their gravitation towards the ſun, as has been proved in propoſition 401. And ſince all gravitation is mutual, by propoſition 73 of mechanics, it follows, that as the planets gravitate towards the ſun, ſo likewiſe the ſun gravitates towards the planets. In the mutual gravitation of the ſun and any one planet, and conſequently in the mutual gravitation of the ſun and all the planets, the ſun as well as the planets will revolve round their common center of gravity, by propoſition 403. From hence we may conclude that neither the ſun, nor the earth, nor any other planet can be looked upon as the center of the ſolar ſyſtem: becauſe the center of the ſyſtem is ſome point at reſt within the ſyſtem; but the ſun, and the earth, and all the other planets are in motion. But this motion of the ſun and the planets and their mutual gravitation towards one another will not put their common center of gravity in motion, by propoſition 74 of mechanics. Therefore the common center of gravity of the ſun and all the planets is to be looked upon as the center of the ſolar ſyſtem.

We have indeed hitherto conſidered the Copernican ſyſtem as the true ſyſtem of the world: and ſo we may ſtill without any great error. For though the Copernican ſyſtem ſuppoſes the ſun to be at reſt in the center of the ſolar ſyſtem, and we have now ſhewn that the ſun is not at reſt, but revolves round the common center of gravity; yet the quantity of matter in the ſun is ſo much greater than in all the other planets taken together, that their common center of gravity will never be far from the center of the ſun.

405. *The quantities of matter in ſuch bodies of the ſolar ſyſtem, as have other bodies revolving round them, may be compared with each other: for the quantities of matter in the central bodies are in a ratio compounded of the cubes of the diſtances of the revolving bodies directly, and of the ſquares of their periodical times inverſely.*

The ſun, the earth, jupiter, and ſaturn, have other bodies revolving round them: the primary planets revolve round the ſun; the moon revolves round the earth; jupiter and ſaturn have their reſpective ſatellites. And the ſun, the earth, jupiter, and ſaturn are here called central bodies. The quantities of matter in any two or in all of theſe central bodies may be compared with one another, ſo as to determine how much the quantity of matter in any one of them is greater or leſs than in any other of them. In particular, the quantity of matter contained in the ſun may be compared with the quantity of matter contained in the earth, by the help of any primary planet, of venus for inſtance, which revolves round the ſun, and of the moon, which revolves round the earth. For the quantity of matter contained in the ſun is to the quantity of matter contained in the earth, directly as the cube of venus's diſtance from the ſun to the cube of the moons diſtance from the earth, and inverſely as the ſquare of venus's periodical time to the ſquare of the moons periodical time.

Call venus's diſtance from the ſun D, and the moons diſtance from the earth d: call venus's periodical time P, and the moons periodical time p. Then the cubes of theſe diſtances are DDD and ddd; and the ſquares of theſe periodical times are PP and pp. All fractions are directly as their numerators and inverſely as their denominators. Therefore $\frac{DDD}{PP}$ and $\frac{ddd}{pp}$ are directly as DDD to ddd and inverſely as PP to pp. Now we will prove that the quantity of matter in the ſun is to the quantity of matter in the earth as $\frac{DDD}{PP}$ to $\frac{ddd}{pp}$: and, if we can prove this, it will follow that the quantity of matter in the ſun muſt be to the quantity of matter in the earth as DDD to ddd directly and as PP to pp inverſely, that is as the cube of venus's diſtance from the ſun to the cube of the moons diſtance from the earth directly, and as the ſquare of venus's periodical time to the ſquare of the moons periodical time inverſely.

If there was any planet, which moved round the ſun, and was juſt as far from the ſun as the moon is from the earth; then, becauſe the diſtances from the ſun and earth are reſpectively equal, this planets gravitation towards the ſun would be juſt as much greater than the moons gravitation towards the earth, as the planet is attracted more by the ſun than the moon is by the earth, or as the quantity of matter in the ſun is greater than the quantity of matter in the earth. The diſtance of this imaginary planet from the ſun is d, becauſe its diſtance is ſuppoſed equal to the moons diſtance, and conſequently the cube of its diſtance is ddd: call its periodical time L, and then the ſquare of its periodical time will be LL. Now

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the imaginary planet and the moon revolve in equal orbits, by the ſuppoſition, one round the ſun, the other round the earth: therefore their centripetal forces, or their gravitations, or the quantities of matter in the ſun and earth, muſt be inverſely as the ſquares of their periodical times, or as $\frac{1}{LL}$ to $\frac{1}{pp}$, by propoſition 168 of mechanics. Now if the numerators of theſe fractions are reſpectively multiplyed by ddd, the proportion, which they bear to each other, will not be altered. Therefore $\frac{ddd}{LL}$, is to $\frac{ddd}{pp}$, as $\frac{1}{LL}$, to $\frac{1}{pp}$, or as the quantity of matter in the ſun to the quantity of matter in the earth. The two parts, of which the ſecond fraction conſiſts, are known: for ddd is the cube of the moons diſtance from the earth, and pp is the ſquare of the moons periodical time. The numerator of the firſt fraction is likewiſe known, for it is the cube of the imaginary planets diſtance from the ſun, which by the ſuppoſition is equal to the cube of the moons diſtance from the earth: but then the denominator LL of this fraction is unknown; for it is the ſquare of the periodical time of ſuch a planet; and as there is no ſuch planet, we cannot at once determine what would be the ſquare of its periodical time. And as the value of the denominator is unknown, the value of the fraction likewiſe is unknown. But though we have here expreſſed the quantity of matter in the ſun by a fraction, the value of which is unknown, yet by the help of this we may find out another, that is known and that is equal to it. This imaginary planet, by the ſuppoſition, is retained in its orbit by its gravitation towards the ſun, and as the force of gravity at different diſtances from the ſun is inverſely as the ſquares of thoſe diſtances, this imaginary planet, if there was ſuch an one, would, by propoſition 171 of mechanics, obſerve the ſame analogy, which all the other primary planets obſerve, the cube of its diſtance, from the ſun would be to the cube of any other primary planets diſtance as the ſquare of its periodical time to the ſquare of the periodical time of that other primary planet. Thus if we compare it with venus for inſtance ddd would be to DDD as LL to PP. But when there are four quantities and the firſt is to the ſecond as the third to the fourth, the firſt divided by the third is equal to the ſecond divided by the fourth. Therefore $\frac{ddd}{LL} = \frac{DDD}{PP}$. But we have proved that the quantity of matter in the ſun is to the quantity of matter in the earth as $\frac{ddd}{LL}$ to $\frac{ddd}{pp}$; and we have now proved that $\frac{ddd}{LL} = \frac{DDD}{PP}$. Therefore the quantity of matter in the ſun is to the quantity of matter in the earth as $\frac{DDD}{PP}$ to $\frac{ddd}{pp}$. Now as all frac-

tions expreſs a quotient ariſing from a diviſion of the numerator by the denominator; it follows, that if the cube of venus's diſtance from the ſun is divided by the ſquare of its periodical time, and the cube of the moons diſtance from the earth is divided by the ſquare of its periodical time, the quotients will be to one another as the quantity of matter contained in the ſun to the quantity of matter contained in the earth.

By the ſame method we might prove that the quantity of matter in the ſun is to the quantity of matter in jupiter or in ſaturn, as the cube of any primary planet from the ſun to the cube of any ſatellites diſtance from jupiter or ſaturn directly, and as the ſquare of that primary planets periodical time to the ſquare of the ſatellites periodical time inverſely. And thus univerſally in all bodies in the ſolar ſyſtem, which have other bodies revolving round them; if the cubes of the diſtances of the revolving bodies are reſpectively divided by the ſquares of their periodical times, the quotients will be as the quantities of matter contained in the central bodies.

The quantities of matter in the ſun, jupiter, ſaturn, and the earth are found to be reſpectively, as 1, $\frac{1}{1067}$, $\frac{1}{3021}$, $\frac{1}{169282}$. Now ſince the quantity of matter in the ſun is to the quantity of matter in jupiter as 1 to $\frac{1}{1067}$ or as $\frac{1}{1}$ to $\frac{1}{1067}$; and ſince theſe fractions are as their denominators inverted, it follows that the quantity of matter in the ſun is to the quantity of matter in jupiter as 1067 to 1. And conſequently the common center of gravity of the ſun and jupiter will be 1067 times farther from the center of jupiter than it is from the center of the ſun. Or the diſtance of jupiters center from the common center of gravity is to the diſtance of the ſuns center from the common center of gravity, as 1067 to 1. Therefore if the diſtance of jupiter from the ſun was to a ſemidiameter of the ſun as 1067 to 1; the diſtance of the common center of gravity from the ſuns center would be equal to a ſemidiameter of the ſun. But the diſtance of jupiter from the ſun is ſomething more than 1067 ſemidiameters of the ſun. Therefore the diſtance of the common center of gravity from the ſuns center is ſomething more than a ſemidiameter of the ſun; ſo that this common center of gravity is a little above the ſuns ſurface.

In like manner we might ſhew that the common center of gravity of the ſun and ſaturn is a little below the ſuns ſurface. And by the ſame method of reaſoning it may be proved, that if all the planets were on the ſame ſide of the ſun, the common center of gravity of the ſun and all the planets

nets in this ſituation would be only at the diſtance of about 1 diameter of the ſun from the ſuns center or of 1 ſemidiameter from its ſurface. From hence then it appears, that though, by propoſition 403, the ſun is moveable, and cannot for that reaſon be looked upon as the center of the ſolar ſyſtem; yet it is never far from the common center of gravity, which is the center of the ſyſtem.

406. *The moon and the earth deſcribe ſimilar orbits round their common center of gravity, whilſt they revolve together round the ſun.*

This propoſition is proved in the ſame manner with propoſitions 402, 403, if S, Plat. XXIX. fig. 8, is the earth and P the moon. For ſince, by propoſition 399, P is retained in its orbit by its gravitation towards S, and the gravitation of S and P towards each other is mutual; therefore, if S is the earth and P the moon, the caſe is here the ſame as in the foregoing propoſitions, where we called S the ſun and P one of the planets.

In the ſame manner we might ſhew, that as jupiter or ſaturn and their reſpective ſatellites revolve together round the ſun; each of theſe planets and its ſatellites deſcribe ſimilar orbits round their common center of gravity.

CHAP. XVI.

Of the lunar inequalities.

THAT we may treat more diſtinctly of the irregularities in the motion of the moon, we will divide them into four ſorts. The firſt ſort of inequalities are ſuch as would happen, though the moons orbit was circular, and the earth was placed in the center of it. The ſecond ſort depend upon the eccentricity of the earths orbit. The third ſort are ſuch as can happen only in an elliptical orbit having the earth in one of its focuſes. The fourth ſort are called inequalities in latitude, and are ſuch as do not depend at all upon the figure of the moons orbit, but happen becauſe the moons orbit is inclined to the plane of the ecliptic. In order to explane theſe irregularities, we muſt firſt explane the cauſes that produce them.

407. *When the moon is in quadrature, its centripetal force is encreaſed by its gravitation towards the ſun.*

The centripetal force, which retains the moon in its orbit, is its gravitation towards the earth, by propoſition 399. But ſince the earth and moon revolve together round the ſun, they are retained in this orbit,

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which they thus deſcribe together, by their gravitation towards the ſun. Therefore the moon gravitates towards the ſun as well as towards the earth. The moons centripetal force is encreaſed by its gravitation towards the ſun, when the moon is in quadrature. Let T, Plat. XXIX. fig. 10, be the earth, S the ſun, ETP an arc in the earths orbit, QLCQ the moons orbit. When the moon is either at L or at Q it is in quadrature: and at that time its gravitation towards the ſun S, or the ſuns attraction, will encreaſe the moons centripetal force, or will make it tend towards the earth in the direction LT more than its own gravity alone would make it tend thither.

The ſun attracts both the earth and the moon. And when the moon is at L or in quadrature, TS the diſtance of the earth from the ſun is nearly equal to LS the diſtance of the moon from the ſun: and conſequently they will both be equally attracted. If the earth was attracted in the line TS and the moon in the line LB parallel to TS, ſuch an attraction would never impell the moon towards the earth or encreaſe its gravity. But the moon is attracted in the line LS, which is oblique to TS: and this oblique attraction may be reſolved into two others, by propoſition 20 of mechanics, of which LB will be one and LT the other. Now that part of the ſuns attraction, which is reſolved into LT, is directed from the moon towards the earth: and conſequently this part of the ſuns attraction will impell the moon towards the earth or will encreaſe its centripetal force.

Or otherwiſe. Suppoſe that both the earth and the moon were to loſe their projectile force, and were to fall by their own gravity towards the ſun: then the earth would deſcend in the line TS; and if the moon was to deſcend in the line LB parallel to TS, though they were to deſcend for ever, yet from the nature of parallel lines, the moon as it deſcended would not approach nearer to the earth. Therefore upon ſuppoſition that the ſun attracts the earth and the moon in parallel lines, this attraction would not impell the moon towards the earth: becauſe if they were made to deſcend by this attraction, the moon in their deſcent would not approach to the earth. But, whilſt the earth is attracted in the line TS, the moon is attracted in the line LS, which is oblique to TS, and thoſe two lines meet at S. Therefore if, by their own gravity or the ſuns attraction, they were both to fall towards the ſun; the moon would meet the earth at S; and conſequently, becauſe the force, with which the ſun attracts the moon in the line LS, is oblique to the force, with which it attracts the earth in the line TS, this attraction muſt impell the moon towards the earth: for if they were made to deſcend by ſuch an attraction, the moon

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in their deſcent would keep approaching to the earth, and would meet it at the ſun. But the ſun attracts the moon in the ſame oblique direction LS, whether, as we before ſuppoſed, the moon and the earth by loſing their projectile force fall towards the ſun, or whether by means of a projectile force they are kept from falling thither, and are made to revolve together round the ſun in the orbit ETP. Therefore, whilſt the moon and the earth are revolving round the ſun, though their projectile force keeps them from falling towards the ſun, and ſo keeps them from coming together; yet as the ſun attracts the earth in the line TS and the moon in the line LS oblique to TS, this attraction when the moon is at L in quadrature, will impell the moon towards the earth, or will encreaſe the moons centripetal force and will make it tend more towards the earth than the force of gravity alone would make it tend thither. The ſame might be ſhewn if the moon was at Q in the other quadrature.

408. *When the moon is in the ſyzygies, its centripetal force is diminiſhed by its gravitation towards the ſun.*

When the moon is at C, Plat. XXIX. fig. 10, it is in conjunction with the ſun S; and when it is at O, it is in oppoſition. In either of theſe ſituations, which are called the ſyzygies, the ſuns attraction diminiſhes the moons centripetal force, or makes the moon gravitate leſs towards the earth than it otherwiſe would do.

Firſt, let the moon be at C or in conjunction; then if we were to ſuppoſe the moon and earth to loſe their projectile force and to fall by their own gravity towards the ſun; it is plane that the moon would fall faſter than the earth: for the ſuns attraction is inverſely as the ſquares of the diſtances from it, by propoſition 22 of mechanics; and the moon in conjunction is nearer to the ſun than the earth is: therefore the moon will be more attracted than the earth, and for that reaſon would fall faſter than the earth. In conſequence of this the diſtance between the moon and the earth would encreaſe, as they fell. But a force, which would encreaſe their diſtance from one another, muſt make them tend leſs towards one another. And the attraction of the ſun would encreaſe their diſtance from one another. Therefore the ſuns attraction would diminiſh their tendency towards one another. This would be the caſe if they were both to loſe their projectile force and were to fall towards the ſun. Now all that the projectile force does is to keep them from falling. The ſuns attraction acts upon them in the ſame manner, whilſt they are revolving together in the orbit ETP, as it would if they were falling towards the ſun: and conſequently this attraction diminiſhes their tendency towards one another.

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But if this attraction of the ſun diminiſhes the moons tendency towards the earth, it diminiſhes the moons centripetal force. Nay indeed we may ſay, that, whilſt the earth is revolving round the ſun in its orbit ETP, and the moon is revolving along with it; both the moon and the earth are made conſtantly to fall towards the ſun, by the ſun's attraction For the earth, and the moons orbit with it, would run off in a tangent to the orbit ETP, if the ſuns attraction did not prevent it: conſequently this attraction makes the earth and the moon conſtantly fall towards the ſun from the tangent. Secondly, when the moon is at O in oppoſition, it is farther from the ſun S than the earth T is. Therefore the earth will be more attracted by the ſun than the moon is. From hence it follows, that if the earth and moon were to loſe their projectile force, and were to fall towards the ſun, the greater attraction would make the earth fall faſter than the moon: and by this means, in their fall, the diſtance between them would encreaſe. Now as this attraction would encreaſe their diſtance, it muſt diminiſh their mutual tendency towards other. But this attraction acts upon them in the ſame manner, though they have a projectile force, and revolve together round the ſun, as it would do, if they had no projectile force and were falling towards it. Therefore the ſun's attraction diminiſhes the mutual tendency of the moon and earth towards one another: and if their mutual tendency towards one another is diminiſhed, the moons centripetal force muſt be made leſs. Indeed, properly ſpeaking, it is the earths tendency or gravitation towards the moon, which is diminiſhed in this caſe. But ſince all gravitation is mutual, the earths gravitation towards the moon cannot be diminiſhed without diminiſhing the moons gravitation towards the earth at the ſame time. If the reader underſtands, that, upon ſuppoſition, the earth and the moon were falling towards the ſun, their mutual tendency towards each other would be diminiſhed; and yet is at a loſs to conceive how the ſame effect will be produced by the ſuns attraction, whilſt they are moving round it without falling towards it; he may recollect what has been ſaid already, that the earth, and the moon in its orbit along with the earth, would run off from the orbit ETP, if they did not gravitate towards the ſun: and conſequently this gravitation or the ſuns attraction makes them conſtantly fall from the tangent into the orbit. So that we may ſay, the earth and moon, as they move together round the ſun, are conſtantly falling towards it.

It is generally thought very eaſy to conceive that the moons centripetal force will be diminiſhed by its gravitation towards the ſun, when

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it is at C in conjunction. Becauſe as its centripetal force or gravitation towards the earth is in the direction CT, and its gravitation towards the ſun is in the oppoſite direction CS; it ſeems obvious that this oppoſite gravitation towards the ſun will diminiſh its gravitation towards the earth. But then it appears difficult to imagine, why the moons gravitation towards the ſun ſhould diminiſh its gravitation towards the earth, when the moon is at O in oppoſition. For as its gravitation towards the earth is in the direction OT, and its gravitation towards the ſun is in the ſame direction OS; it ſeems moſt likely that this gravitation towards the earth in the direction OT ſhould be encreaſed by a gravitation towards the ſun in the ſame direction OS.

Now there is juſt the ſame difficulty, though leſs taken notice of, when the moon is in conjunction: we will therefore ſhew that there is the ſame difficulty, and by clearing it up in this caſe we ſhall be led to the true method of clearing it up in the other.

But firſt we muſt premiſe, that whatever encreaſes or diminiſhes the earths gravitation towards the moon will encreaſe or diminiſh the moons gravitation towards the earth. This follows from propoſition 73 of mechanics: for ſince all gravitation is mutual, whatever change is made in the gravitation of one body towards another, the ſame alteration will be made at the ſame time in the gravitation of the latter body towards the former. Suppoſe, for inſtance, the quantity of matter contained in the earth was by any means to become twice what it is at preſent; the earth would then gravitate towards the moon with twice the force that it does at preſent, by propoſition 32 of mechanics; and conſequently the moon would gravitate towards the earth with twice the force that it does at preſent, by propoſition 73 of mechanics. Or ſuppoſe that the quantity of matter in the earth was by any means to become only half what it is at preſent; the earth would then gravitate towards the moon with only half its preſent force, and conſequently the moons gravitation towards the earth would be only half its preſent gravitation.

When the moon is at C in conjunction, the earth gravitates towards the moon in the direction TC, and towards the ſun in the ſame direction TS. Conſequently in this ſituation the earths gravitation towards the ſun muſt encreaſe its gravitation towards the moon, becauſe both theſe gravitations are in the ſame direction. But whatever encreaſes the earths gravitation towards the moon will, as we have juſt ſhewn, encreaſe the moons gravitation towards the earth. Therefore the moons centripetal force, when in conjunction, ought to be encreaſed by the ſuns action. But to clear up this difficulty we ſhould obſerve, that the moon gravi-

tates towards the earth in the direction CT, and towards the ſun in the contrary direction CS: this gravitation towards the ſun will diminiſh the moons gravitation towards the earth. From hence it appears that the ſuns action encreaſes the moons centripetal force upon one account and diminiſhes it upon another account: the ſuns action encreaſes the moons centripetal force, becauſe it encreaſes the earths tendency towards the moon; but then the ſame action diminiſhes the moons centripetal force, becauſe it diminiſhes the moons tendency towards the earth. If the encreaſe and decreaſe were equal, the ſuns action, upon the whole, would not make any alteration in the moons centripetal force. Let us therefore enquire whether the encreaſe or decreaſe is the greater. The ſuns action upon the moon at C is greater than its action upon the earth at T, as much greater as the ſquare of CS is leſs than the ſquare of TS: becauſe the ſuns action upon the moon C and upon the earth T is inverſely as the ſquares of their diſtances. Now the moons centripetal force is diminiſhed by the ſuns action upon the moon C, and is encreaſed by its action upon the earth T. But the ſuns action upon the moon C is greater than its action upon the earth T. Therefore the moons centripetal force is more diminiſhed than encreaſed by the ſuns action, or is made leſs upon the whole, when the moon is at C in conjunction with the ſun.

The ſame method of reaſoning will clear up the difficulty when the moon is at O in oppoſition. The moon at O gravitates towards the earth in the direction OT and towards the ſun in the ſame direction OS: conſequently the moons gravitation towards the ſun ought to encreaſe its gravitation towards the earth. But then at the ſame time the earth gravitates towards the moon in the direction TO, and towards the ſun in the contrary direction TS: conſequently, as theſe gravitations are in contrary directions, the gravitation of the earth towards the ſun S will diminiſh its gravitation towards the moon O. Now as all gravitation is mutual, whatever diminiſhes the earths gravitation towards the moon will at the ſame time diminiſh the moons gravitation towards the earth. But the action of the ſun diminiſhes the earths gravitation towards the moon in oppoſition. Therefore the action of the ſun will at the ſame time diminiſh the moons gravitation towards the earth. Thus we have ſeen, that, when the moon is in oppoſition, the ſuns action upon the moon encreaſes the moons gravitation towards the earth or encreaſes the moons centripetal force: but at the ſame time the ſuns action upon the earth diminiſhes the moons gravitation towards the earth or diminiſhes the moons centripetal force. If the moons centripetal force was encreaſed as much upon one account as it is diminiſhed upon the other account, the action of the

ſun

ſun would upon the whole make no alteration in that centripetal force. But we ſhall find that the centripetal force of the moon, when in oppoſition, is more diminiſhed by the ſuns action upon the earth than it is encreaſed by the ſuns action upon the moon. For ſince the earth is nearer to the ſun than the moon is, when in oppoſition; the ſun will act more forcibly upon the earth than upon the moon, the ſuns action upon the earth is as much greater than its action upon the moon as the ſquare of TS is leſs than the ſquare of OS, by propoſition 22 of mechanics. But it is the ſuns action upon the earth, which diminiſhes the moons centripetal force in oppoſition; and it is the ſuns action upon the moon at the ſame time, which encreaſes its centripetal force. Therefore, as the former action is greater than the latter, the moons centripetal force, when in oppoſition, will upon the whole be made leſs by the ſuns action.

409. *That part of the ſuns action, which encreaſes the moons tendency towards the earth, is called the addititious force.*

410. *That part of the ſuns action, which diminiſhes the moons tendency towards the earth, is called the ablatitious force.*

411. *When the moon is in quadrature the ablatitious force is nothing.*

The ablatitious force ariſes from the different gravitations of the earth and moon towards the ſun, upon account of their different diſtances from it, by propoſition 408. But when the moon is at L or Q, Plat. XXIX. fig. 10, in quadrature, the moon and earth are at the ſame or very nearly the ſame diſtance from the ſun; and upon that account their accelerating force of gravity towards the ſun will be equal. Therefore there will be no ablatitious force.

412. *When the moon is in ſyzygy the addititious force is nothing.*

If the moon and earth gravitated towards the ſun in parallel directions, there would be no addititious force, as has been ſhewn under propoſition 407. But when the moon is in conjunction at C or in oppoſition at O, Plat. XXIX. fig. 10, the earth T and the moon gravitate towards the ſun in the ſame or parallel directions. Therefore, when the moon is in conjunction or in oppoſition, there is no addititious force.

413. *When the moon is any where between quadrature and ſyzygy, it is acted upon both by the ablatitious and the addititious force.*

Let the moon be at A, Plat. XXX. fig. 1, between the quadrature L and the conjunction C. The moon is then nearer to the ſun S than the

earth T is, and conſequently will be more attracted by the ſun than the earth is, or the moons accelerating force of gravity towards the ſun will be greater than the earths, by propoſition 22 of mechanics. Let us enquire how much the force, with which the moon gravitates towards the ſun, exceeds the force, with which the moon gravitates thither. The diſtance of the earth from the ſun is TS: therefore take RS equal to TS, then RS is the earths diſtance and AS the moons diſtance from the ſun. The force of gravity towards the ſun is inverſely as the ſquares of the diſtances: or ſince AS is the moons diſtance and RS the earths diſtance, the moons gravitation towards the ſun is to the earths gravitation towards the ſun, as the ſquare of AS to the ſquare of RS inverted, that is as the ſquare of RS to the ſquare of AS, or as much as the ſquare of RS exceeds the ſquare of AS ſo much the moons gravitation towards the ſun exceeds the earths gravitation thither. Therefore, if we make BS to RS, as the ſquare of RS to the ſquare of AS, then, by this conſtruction, the moons gravitation towards the ſun is to the earths gravitation thither, as BS to RS, or as BS to TS, becauſe TS is equal to RS. Now BS exceeds TS; therefore the moon is attracted more by the ſun or gravitates more towards the ſun than the earth does: but the moons centripetal force is diminiſhed by this means, as appears from what has been ſaid under propoſition 408. Therefore when the moon is at A, its centripetal force is diminiſhed by the ſuns action, or the ſun acts upon the moon by an ablatitious force. BS is likewiſe oblique to TS, or the direction, in which the moon gravitates towards the ſun, is oblique to direction, in which the earth gravitates towards it. Therefore, by what has been ſhewn under propoſition 407, the ſuns attraction, which cauſes the moons gravitation towards it, will encreaſe the moons tendency towards the earth, that is, the ſun acts upon the moon at A ſo as to produce an addititicious force.

We may prove this more particularly from the ſame conſtruction, where the moons gravitation is to the earths as BS to TS, and where the moon, becauſe it is at A, is attracted in the direction BS, and the earth in the direction TS. Draw the line AT from the moon to the earth: in this direction the moon tends to the earth by the force of gravity, whilſt it tends towards the ſun likewiſe by the force of gravity in the direction BS. Now ſince BS is an oblique force in reſpect of TS, it may be reſolved into two forces, one of which ſhall be parallel to AT, and the other parallel to TS, by propoſition 20 of mechanics. Theſe two forces are BM and BG. Thus when the moon is at A, and the ſuns oblique attraction is reſolved in this manner, we ſhall find that this attraction produces an addititicious and an ablatitious force. The ſuns attraction acts upon the

moon

moon at A in the direction BM, which is parallel to AT; but AT is the direction, in which the moon tends towards the earth: therefore the force BM will encreaſe the moons tendency in the directions AT, or BM is an addititious force. The moons gravitation towards the ſun in the direction AS, when the moon is at A, is to the earths gravitation towards the ſun as BS to TS, by the conſtruction; and when this oblique gravitation is reſolved, the moons gravitation towards the ſun in the direction BG parallel to TS is to the earths gravitation towards the ſun, as BG to TS, by propoſition 20 of mechanics, or as MS to TS ſince BG and TS are equal, becauſe they are oppoſite ſides of a parallelogram. Euc. b. I. prop. 34. Therefore the moons gravitation towards the ſun exceeds the earths gravitation thither in the ſame or in a parallel direction, as much as MS exceeds TS. But if the ſun attracts the moon more than the earth in the ſame or in a parallel direction, this will diminiſh the moons centripetal force. The difference between theſe attractions is MT the difference between MS and TS. Therefore this difference MT diminiſhes the moons centripetal force, or MT is an ablatitious force.

Let the moon be at D, Plat. XXX. fig. 2, between the quadrature L and the oppoſition O; then likewiſe the ſuns action will produce an addititious and an ablatitious force. The diſtance of the earth from the ſun is TS, the diſtance of the moon from the ſun is DS. Take RS equal to TS; and then the earths gravitation towards the ſun will be to the moons gravitation towards the ſun as the ſquare of RS to the ſquare of PS inverted, by propoſition 22 of mechanics, that is as the ſquare of DS to the ſquare of RS. But if we make as D the ſquare of DS to the ſquare of RS, ſo RS to BS, then RS or TS, which is equal to RS, will repreſent the earths gravitation towards the ſun and BS will repreſent the moons gravitation towards the ſun. Now ſince BS is oblique to TS, or ſince the direction, in which the ſun attracts the moon, is oblique to the direction in which it attracts the earth; this oblique action of the ſun will encreaſe the moons tendency towards the earth or will encreaſe the moons centripetal force, by what has been ſaid under propoſition 407, that is, the ſuns action, when the moon is at D, produces an addititious force. BS likewiſe is leſs than TS; therefore the moon gravitates leſs towards the ſun than the earth gravitates towards it; or the earth is more attracted than the moon. This, by what has been ſaid under propoſition 408, will diminiſh the earths tendency towards the moon, in conſequence of which the moons centripetal force will be diminiſhed. Therefore in this reſpect the ſuns action produces an ablatitious force, when the moon is at D.

If the conſtruction remains the ſame in all other reſpects and we draw DT from the moon to the earth, we may ſhew more particularly that

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the moon at D, beſides its gravitation towards the earth in the direction DT, is acted upon by an addititious and by an ablatitious force in conſequence of its gravitation towards the ſun. The moon gravitates towards the ſun in the direction BS and the earth in the direction TS. This oblique force BS may be reſolved into two others BM and BG, one parallel to DT, the other parallel to TS, by propoſition 20 of mechanics. The ſun therefore, when the moon is at D, acts upon it in a direction BM parallel to DT: and as this part of the ſuns action is in the ſame direction with the moons gravitation towards the earth, it will encreaſe the moons centripetal force, or BM is an addititious force. The moons oblique gravitation towards the ſun, by this conſtruction, is to the earths gravitation towards it, as BS to TS: and when this oblique gravitation is reſolved, as above, the moons gravitation towards the ſun is to the earths gravitation towards it in parallel directions as BG to TS, or as MS to TS, becauſe BG and MS are equal. From hence then it appears that the earths gravitation towards the ſun exceeds the moons gravitation towards it in the ſame or in a parallel direction, as much as TS exceeds MS. But if the earth gravitates more towards the ſun than the moon does, this will diminiſh the earths tendency towards the moon, which in conſequence diminiſhes the moons centripetal force or its tendency towards the earth, by propoſition 408. Therefore when the moon is at D, the ſun diminiſhes the moons centripetal force, or the ſuns action produces an ablatitious force, which ablatitious force is TM or the difference between TS the earths gravitation and MS the moons gravitation towards the ſun in the ſame or in a parallel direction.

What we have ſhewn of the moon, when at A or at D, fig. 1, 2, the ſame we might ſhew of it in the ſame manner, when at *a* or *d*, or when it is any where elſe between either quarter Q, L, and either ſyzygy C, O. Therefore, when the moon is any where between quadrature and ſyzygy, the ſuns action produces both an addititious and an ablatitious force.

414. *The addititious force is always directed towards the earths center.*

This follows from the nature of the addititious force; ſince it is a force, which encreaſes the moons tendency towards the earths center.

415. *The ablatitious force is directed towards the ſun, when the moon is in that half of its orbit, which is neareſt to the ſun; and is directed from the ſun, when the moon is in the other half of its orbit, which is fartheſt from the ſun.*

We here conſider the quadratures Q and L, Plat. XXX. fig. 1, 2, as dividing the orbit in halves: QCL is the half that is neareſt to the ſun;

and

and QOL is the half that is fartheſt from the ſun. At C in the middle of that half, which is neareſt to the ſun, the moon is in conjunction: and in the middle of the other half at O it is in oppoſition. When the moon is paſſing from the quadrature Q to the quadrature L through C the conjunction, that is, when it is in that half of its orbit, which is neareſt to the ſun, the ablatitious force acts in a direction *af* or AF towards the ſun S. This is almoſt ſelf-evident. For ſince the ſun produces this ablatitious force by attracting the moon towards it more than it attracts the earth, this ablatitious force muſt neceſſarily be directed towards the ſun. When the moon is paſſing from the quadrature L to the quadrature Q, through the oppoſition O, or when it is in that half of its orbit, which is fartheſt from the ſun, the ablatitious force acts in ſome direction DF, or *df* from the ſun. There may be ſome difficulty in conceiving this: becauſe it may be imagined that, if the ſuns attraction diminiſhes the moons gravity, when in that half of the orbit, then as the ſun always attracts towards itſelf, the force, which diminiſhes the moons gravity muſt be directed towards the ſun. But here we muſt remember, what has been ſhewn in propoſitions 408, 413, that when the moon is in oppoſition or in that half of the orbit where it is farther from the ſun than the earth is, the ſun diminiſhes the moons centripetal force, not by acting immediately upon the moon, but by acting immediately upon the earth. The ſun makes the earth gravitate leſs towards the moon, and the effect of this is that the moon will gravitate leſs towards the earth. Thus properly ſpeaking the ablatitious force makes the moon tend leſs towards the earth: and ſince when the moon is in that half of its orbit LOQ, which is fartheſt from the ſun, the earth and the ſun are on the ſame ſide of it, as it tends leſs towards the earth it muſt likewiſe tend leſs towards the ſun. So that if we conſider the effect, which the ablatitious force then has upon the moon, it will be the ſame as if the moon was drawn from the ſun in ſome ſuch direction as DF, or *df*.

416. *The ablatitious force in ſyzygy is double the addititious force in quadrature: and the ablatitious force in conjunction is ſomething greater than in oppoſition.*

When the moon is in quadrature at L, Plat. XXIX. fig. 10, the oblique action of the ſun, in the direction LS, may be reſolved into two parts, of which one is LB parallel and equal to TS, and the other is LT. This part LT acts in the ſame direction with the moons gravitation towards the earth, and is the addititious force, by propoſition 407. If LS is put to repreſent the whole force, with which the moon gravitates towards

wards the ſun, then by reſolving this force into the two parts LT and LB, the part LT or the addititious force is to the moons whole gravitation towards the ſun as LT to LS, But the earth at T and the moon at L are at nearly the ſame diſtance from the ſun and conſequently gravitate equally towards the ſun. Therefore the addititious force is to the earths gravitation towards the ſun as LT to LS, or as LT to TS. Since LS and TS are equal. Thus we have ſhewn that the addititious force is to the earths gravitation towards the ſun as a ſemidiameter of the lunar orbit to a ſemidiameter of the earths orbit; for LT is a ſemidiameter of the lunar orbit and TS is a ſemidiameter of the earths orbit.

Now in order to prove that the ablatitious force is double the addititious force, we muſt ſhew that the ablatitious force is to the earths gravitation towards the ſun, as a diameter of the lunar orbit to a ſemidiameter of the earths orbit or as 2LT to TS. For ſince the earths gravitation towards the ſun is a given quantity, if the ablatitious force bears twice the proportion to this given quantity that the addititious force bears to it, the conſequence is that the ablatitious force muſt be twice the addititious force.

Let the moon be at C in conjunction, then the earths diſtance from the ſun is TS and the moons diſtance from the ſun is CS, the gravitation of the moon towards the ſun is to the gravitation of the earth towards the ſun, inverſely as the ſquare of TS to the ſquare of CS, that is, as theſe ſquares inverted or as the ſquare of CS to the ſquare of TS, by propoſition 22 of mechanics. Therefore if we can find two lines, which are to one another as the ſquare of CS to the ſquare of TS, then theſe lines will likewiſe be to one another as the earths gravitation towards the ſun to the moons gravitation thither: ſo that, if one of theſe lines is put to repreſent the force, with which the ſun attracts the moon, the other will repreſent the force, with which the ſun attracts the earth. We will therefore endeavour to find two ſuch lines. The two lines are CS and OS; for CS is to OS as the ſquare of CS to the ſquare of TS. This we prove in the following manner.

The three quantities CS, TS, OS, are in arithmetical progreſſion, their common difference is TC or OT=TC: CS is the firſt of theſe quantities, CS+TC=TS is the ſecond, and TS+OT=OS is the third: where becauſe TC and OT are equal, as being each of them ſemidiameters of the lunar orbit, it is plane that the third quantity exceeds the ſecond juſt as much as the ſecond exceeds the firſt, or OT the difference between the third and ſecond is equal to TC the difference between the ſecond and firſt. This is what we mean by ſaying that theſe three quantities are in

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arithmetical progreſſion. Perhaps this will appear planer in numbers, and we will take ſuch as, by propoſition 347, are near the truth. If CS=342, then TS=343, and OS=344: theſe three numbers 342, 343, 344, are in arithmetical progreſſion and their common difference is 1: the third exceeds the ſecond juſt as much as the ſecond exceeds the firſt: 1 is the difference between the third and ſecond; and 1 is likewiſe the difference between the ſecond and firſt. We are to obſerve farther of theſe three quantities, that in each of them the common difference is very ſmall in reſpect of the whole; TC or OT a ſemidiameter of the lunar orbit bears a ſmall proportion to TS, which is a ſemidiameter of the earths orbit, or to CS or to OS. As in the three numbers 342, 343, 344, 1 the common difference bears a ſmall proportion to, or is very ſmall in reſpect of, any of theſe numbers. Now where three quantities are in arithmetical progreſſion, and the common difference is very ſmall in reſpect of any of thoſe quantities, they are nearly in geometrical progreſſion; or the firſt is nearly to the ſecond, as the ſecond is to the third; CS is to TS, nearly as TS is to OS. Therefore CS is to OS, nearly as the ſquare of CS is to the ſquare of TS; the firſt is to the third, nearly as the ſquare of the firſt to the ſquare of the ſecond. Euc. b.V. def. 10. Thus in the three numbers 342, 343, 344, the firſt is to the third, nearly as the ſquare of the firſt to the ſquare of the ſecond. 342 is to 344, nearly as 342×342=116964 is to 343×343=117649. We have here ſaid, that if three quantities are in arithmetical progreſſion, and the common difference is very ſmall in reſpect of the whole; then thoſe quantities are nearly in geometrical progreſſion; the firſt is to the third, nearly as the ſquare of the firſt to the ſquare of the ſecond. This is expreſſed otherwiſe, in the late profeſſor Saunderſons uſeful treatiſe of algebra. If there be two quantities, whoſe difference is very ſmall in compariſon of the quantities themſelves; then if one of the quantities is encreaſed ſo as to make the difference double, the two quantities after this alteration will be nearly in a duplicate ratio of that they were in before ſuch alteration was made. Thus in the two numbers 342, 343, the difference 1 is very ſmall in compariſon of the numbers themſelves: if we encreaſe 343 by adding 1 to it, this will make the difference double what it was before: and the numbers after this alteration are 342, 344. But the numbers 342, 344, are nearly in a duplicate ratio of 342 to 343, or 342 is to 344 nearly as the ſquare of 342 to the ſquare of 343; or 342 is to 344 nearly as 116964 to 117649. So likewiſe in the two quantities CS and TS the difference TC is very ſmall in compariſon of the quantities themſelves: and if we encreaſe TS by adding to it OT=TC, the quantities will then be CS and OS; and theſe quanti-

ties after this alteration will be nearly in the duplicate ratio of that they were in before ſuch alteration was made; that is CS is to OS nearly as the ſquare of CS to the ſquare of TS.

Since then the earths gravitation towards the ſun is to the moons gravitation towards the ſun, as the ſquare of CS to the ſquare of TS, as was obſerved before; it follows that the earths gravitation towards the ſun is to the moons gravitation towards the ſun as CS is to OS nearly. The difference between the gravitation of the earth and moon towards the ſun, or OS—CS=OC is the ablatitious force, by propoſition 408. Therefore the ablatitious force is to the earths gravitation towards the ſun as OC to CS, or nearly as OC to TS, becauſe CS and TS are nearly equal. But OC is a diameter of the lunar orbit, and TS is a ſemidiameter of the earths orbit. Therefore the ablatitious force is to the earths gravitation towards the ſun as a diameter of the lunar orbit to a ſemidiameter of the earths orbit. From hence it follows that the ablatitious force in conjunction is double the addititious force in quadrature: becauſe the addititious force in quadrature is to the earths gravitation towards the ſun only as LT to TS, or only as a ſemidiameter of the lunar orbit to a ſemidiameter of the earths orbit.

When the moon is in oppoſition at O, its diſtance from the ſun is OS and the earths diſtance from the ſun is TS: their reſpective gravitations towards the ſun will be as the ſquares of their diſtances inverted, by propoſition 22 of mechanics, that is the moons gravitation towards the ſun will be as much leſs than the earths gravitation towards the ſun, as the ſquare of TS is leſs than the ſquare of OS: ſo that if we can find two lines which are to one another as the ſquare of TS to the ſquare of OS, theſe two lines will be to one another as the moons gravitation towards the ſun is to the earths gravitation towards the ſun. Make OM=TO. Then the three quantities, TS, OS, and MS, are in arithmetical progreſſion; MS exceeds OS juſt as much as OS exceeds TS; the difference between MS and OS is MO, and the difference between OS and TS is TO=MO. Therefore MO or its equal TO is the common difference. Now TO the common difference is very ſmall in compariſon of TS the earths diſtance from the ſun, and conſequently is very ſmall in reſpect of OS or MS. Therefore the three quantities TS, OS, and MS, are nearly in geometrical progreſſion; ſo that the firſt is to the third, as the ſquare of the firſt to the ſquare of the ſecond. TS is to MS, nearly as the ſquare of TS to the ſquare of OS. Therefore TS is to MS as the moons gravitation towards the ſun is to the earths gravitation towards the ſun; or the ſun S attracts the moon at O as much leſs than it attracts the earth at T as TS is leſs than MS. The

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difference of theſe attractions or MT is the ablatitious force. Therefore the ablatitious force is to the earths gravitation towards the ſun, as MT is to MS; or as MT to TS nearly, becauſe MS is nearly equal to TS. Now MT is equal to 2OT, or to a diameter of the lunar orbit, by the conſtruction: and conſequently the ablatitious force in oppoſition is to the earths gravitation towards the ſun, as a diameter of the lunar orbit to TS a ſemidiameter of the earths orbit. But the addititious force in quadrature is to the earths gravitation towards the ſun, only as LT a ſemidiameter of the lunar orbit to TS a ſemidiameter of the earths orbit. Therefore the ablatitious force in oppoſition is double the addititious force in quadrature.

In proving that the ablatitious force in conjunction and oppoſition is double the addititious force in quadrature, we have likewiſe proved the latter part of the propoſition, that the ablatitious force is ſomething greater in conjunction than in oppoſition. For we have ſhewn that the ablatitious force in conjunction is to the earths gravitation towards the ſun as OC to CS; and that the ablatitious force in oppoſition is to the earths gravitation towards the ſun as MT=OC to MS. But CS is leſs than MS. Therefore OC bears a greater proportion to CS than to MS, or the ablatitious force in conjunction bears a greater proportion to the earths gravitation towards the ſun, than the ablatitious force in oppoſition bears to it. Euc. b.V. prop. 8. Now the earths gravitation is the ſame whether the moon is in conjunction or in oppoſition. Therefore ſince the ablatitious force in conjunction bears a greater proportion to the earths gravitation, than the ablatitious force in oppoſition bears to the ſame quantity, it follows that the ablatitious force in conjunction is ſomething greater than the ablatitious force in oppoſition. Euc. b.V. prop. 10.

417. *The addititious force is directly as the moons diſtance from the earth, and inverſely as the cube of the earths diſtance from the ſun.*

Firſt; ſuppoſe the earths diſtance from the ſun or TS, Plat. XXIX. fig. 10, to be given: then the addititious force will encreaſe or decreaſe in the ſame proportion that LT encreaſes or decreaſes; that is, the addititious force is directly as LT the moons diſtance from the earth. TS the earths diſtance from the ſun is here ſuppoſed to be given; and conſequently the earths gravitation towards the ſun is given, becauſe this gravitation at an equal diſtance will always be the ſame. But if TS is given, then as LT encreaſes it will bear a greater proportion to TS, and as LT decreaſes it will bear a leſs proportion to TS. Euc. b.V. prop. 8. Now the proportion, which LT bears to TS, is the proportion, which the addititious force bears to the earths gravitation towards the ſun, as has been

proved in propoſition 415. Therefore as LT encreaſes the additious force will bear a greater proportion to the earths gravitation towards the ſun; and as LT decreaſes the addititious force will bear a leſs proportion to that gravitation. But ſince TS is given, the earths gravitation towards the ſun is given. Therefore as the addititious force bears a greater proportion to this given gravitation it muſt encreaſe; and as the addititious force bears a leſs proportion to the ſame gravitation it muſt decreaſe. Euc. b.V. prop. 10. And conſequently the addititious force encreaſes as LT encreaſes, and decreaſes as LT decreaſes; that is, the addititious force, when TS is given, is directly as LT the moons diſtance from the earth.

Secondly; ſuppoſe LT the moons diſtance from the earth to be given, then the addititious force will be inverſely as the cube of TS the earths diſtance from the ſun. The addititious force is to the earths gravitation towards the ſun, as LT to TS, as has been ſhewn in propoſition 415. Therefore if it was poſſible for the earths gravitation towards the ſun to encreaſe, whilſt LT and TS continue of the ſame length, then the addititious force muſt neceſſarily encreaſe in the ſame proportion with that gravitation. For ſince the addititious force is to the earths gravitation as LT to TS, and ſince by the ſuppoſition LT and TS are invariable quantities, or the proportion of LT to TS is given; it follows that the proportion of the addititious force to the earths gravitation is given too; and conſequently, if the earths gravitation encreaſes the addititious force muſt encreaſe, for otherwiſe the proportion between them would vary. And for the ſame reaſons, if the earths gravitation was to decreaſe, the addititious force muſt decreaſe in the ſame proportion. Therefore we may ſay, that the addititious force is directly as the earths gravitation towards the ſun. Indeed when the earths gravitation is varyed it is impoſſible that the ſuppoſition we have made ſhould be true: becauſe the earth gravitates more or leſs, from being nearer to or farther from the ſun. Therefore TS cannot continue of the ſame length, whilſt that gravitation varys. However the ſuppoſition has ſerved to lead us to this concluſion, that by whatever means the earths gravitation towards the ſun varys, the addititious force will vary in the ſame proportion. Now the earths gravitation towards the ſun is inverſely as the ſquare of its diſtance from the ſun. Therefore the addititious force is inverſely as the ſquare of the earths diſtance from the ſun. But this is not all. Suppoſe it was poſſible for the earths gravitation towards the ſun to continue the ſame, whilſt TS the earths diſtance from the ſun varys: then as TS lengthens, LT will bear a leſs proportion to it, and as TS ſhortens, LT will bear a greater proportion to it. Euc. b.V. prop. 8. But the addititious force is to the earths

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gravitation as LT to TS, by propoſition 415. Therefore as TS lengthens, the addititious force bears a leſs proportion to the earths gravitation, and as TS ſhortens, the addititious force bears a greater proportion to the earths gravitation. But the earths gravitation is here ſuppoſed to be a given quantity. Therefore the addititious force muſt decreaſe, when it bears a leſs proportion to this given quantity, and muſt encreaſe when it bears a greater proportion to the ſame quantity. Conſequently the addititious force decreaſes as TS lengthens, and encreaſes as TS ſhortens; that is, the addititious force is inverſely as TS. Thus we have ſeen that the addititious force is inverſely as the ſquare of TS, becauſe the earths gravitation towards the ſun varys in this proportion; and that the addititious force is likewiſe inverſely as TS, becauſe the proportion of LT to TS is inverſely as TS. Therefore upon both accounts together the addititious force is inverſely as the ſquare of TS and inverſely as TS: and theſe proportions compounded give the inverſe proportion of the cube of TS; ſo that the addititious force is inverſely as the cube of TS. Thus call TS x, then the additious force is as $\frac{1}{xx}$ upon one account, and as $\frac{1}{x}$ upon the other account. Therefore upon both accounts together it is as $\frac{1}{xx} \times \frac{1}{x} = \frac{1}{xxx}$, or as the reciprocal of the cube of x, that is, reciprocally as the cube of TS the earths diſtance from the ſun, becauſe we made $x = TS$.

418. *The ablatitious force is directly as the moons diſtance from the earth, and inverſely as the cube of the earths diſtance from the ſun.*

The double of any quantity is varyed in the ſame proportion with the quantity itſelf. Now the ablatitious force is double the additious force, by propoſition 416: and the addititious force is directly as the moons diſtance from the earth and inverſely as the cube of the earths diſtance from the ſun, by propoſition 417. Therefore the ablatitious force is likewiſe directly as LT, Plat. XXIX. fig. 10, the moons diſtance from the earth and inverſely as the cube of TS the earths diſtance from the ſun.

We have now explaned the manner, in which the ſun acts upon the moon to diſturb its regular motion round the earth. The moons motion is diſturbed, or its inequalities are produced, partly by the addititious and partly by the ablatitious force. We are next to enquire what inequalities are produced by theſe cauſes. And firſt we will ſuppoſe that the plane of the moons orbit coincides with the plane of the ecliptic, that it is a circle, and that the earth is placed in its center. The inequalities, which would be produced upon this ſuppoſition, belong to the firſt claſs of the four

into

into which are divided all the inequalities at the beginning of this chapter.

419. *The moon is accelerated by the ablatitious force as it paſſes from quadrature to ſyzygy, and is retarded by the ſame force as it paſſes from ſyzygy to quadrature.*

Since, by propoſitions 407, 408, the addititious force ariſes from the oblique direction of the moons gravitation towards the ſun, and the ablatitious force from the difference between the earths and moons gravitation towards the ſun; it follows that if the gravitations of the earth and moon towards the ſun were in parallel directions and were equal to one another, there would be no addititious and no ablatitious force at all; that is, the ſuns attraction of the earth and moon, as far as they are both attracted equally and in parallel lines, will not diſturb their motion at all. If they were thus attracted, they would move round one another in the ſame manner, whilſt they both gravitate towards the ſun, as if neither of them gravitated thither at all. Therefore this part of the ſuns attraction will produce no inequalities in the moons motion.

The moon by a radius drawn to the earths center deſcribes equal areas in equal times, by propoſition 159 of mechanics; becauſe the force of gravity, which retains it in its orbit, impells it towards the earth. But the addititious force impells it likewiſe towards the earths center, by propoſition 414. Therefore when the moons centripetal force is made up of its gravity towards the earth and the addititious force together, this compound centripetal force will impell the moon towards the earths center: and conſequently the moon by this compound centripetal force will ſtill continue by a radius drawn to the earth to deſcribe equal areas in equal times, by propoſition 159 of mechanics. But if the moon continues to deſcribe equal areas in equal times, notwithſtanding the addititious force, then this force can neither accelerate nor retard it: for if it was accelerated, it muſt then deſcribe greater areas than uſual in a given time; and if it was retarded, it muſt then deſcribe leſs areas than uſual in the ſame time.

But the ablatitious force does not act towards the earths center: when the moon is in that half of its orbit, which is neareſt to the ſun, this force acts towards the ſun, by propoſition 415. Now when the moon is in this half of its orbit QCL, Plat. XXX. fig. 1, and is moving from the quadrature Q to the conjunction C, it approaches the ſun; and as the ablatitious force acts towards the ſun, it will in part conſpire with the moons motion, and therefore will accelerate it. But as the moon in the ſame half of its orbit is moving from the conjunction C to the quadrature

L,

L, it departs from the ſun, and the ablatitious force ſtill acting towards the ſun is in part contrary to the moons motion, and therefore will retard it. When the moon is in the other half of its orbit, LOQ, fig. 2, which is fartheſt from the ſun, the ablatitious force acts from the ſun, by propoſition 415. But as the moon is moving in this half of its orbit from the quadrature L to the oppoſition O, it departs from the ſun. Therefore the ablatitious force in part conſpiring with the moons motion will accelerate it. As the moon moves from the oppoſition O to the quadrature Q, it approaches to the ſun; and conſequently the ablatitious force acting from the ſun will in part be contrary to its motion, and for that reaſon will retard it. This is the caſe when the moon is any where between quadrature and ſyzygy, or between ſyzygy and quadrature.

Indeed as the ablatitious force, when the moon is either in conjunction C or in oppoſition O, acts directly from the earth; the remainder of the moons gravitation, when this force is taken from it, will impell the moon towards the earth: and as this remainder is then the moons centripetal force; the moon in either ſyzygy will, by its centripetal force, be impelled towards the earths center, and conſequently in oppoſition and conjunction, as well as in quadrature it will deſcribe equal areas in equal times round the earth, by propoſition 159 of mechanics.

420. *The moons velocity is the greateſt in ſyzygy, and the leaſt in quadrature.*

This follows from propoſition 419. For as the moon is conſtantly accelerated as it moves from quadrature to ſyzygy, and is conſtantly retarded as it moves from ſyzygy to quadrature, its velocity muſt be the greateſt in ſyzygy, where its acceleration ends, and the leaſt in quadrature, where its retardation ends.

421. *The curvature of the moons orbit is the greateſt in quadrature and the leaſt in ſyzygy.*

The more the moons courſe is bent from a right line, the greater is the curvature of its orbit; and the leſs its courſe is bent from a right line the leſs is the curvature of its orbit. Now, if the centripetal force is given, the moons courſe will be moſt bent from a right line, where its velocity is leaſt: becauſe in proportion as its velocity is leſs, its moment will be leſs; and the leſs moment it is moving with, the more a given centripetal force will change its direction and bend its courſe from a right line. For a like reaſon where its velocity is the greateſt, its moment will be the greateſt and a given centripetal force will bend its courſe leſs from a right line.

line. But the velocity is leaſt in quadrature, and greateſt in ſyzygy, by propoſition 420. Therefore a given centripetal force will make its orbit more curve in quadrature and leſs curve in ſyzygy.

But this is not all. For if the moons velocity was the ſame both in quadrature and in ſyzygy; yet the centripetal force is greater in quadrature, where the addititious force encreaſes it, and leſs in ſyzygy, where the ablatitious force diminiſhes it, by propoſitions 407, 408. Now if the moon was moving with a given velocity, a greater centripetal force will bend its courſe more from a right line, or will make its orbit more curve; and a leſs centripetal force will bend its courſe leſs from a right line, or will make its orbit leſs curve. Therefore the curvature of the orbit will be greateſt in quadrature and leaſt in ſyzygy.

Thus we ſee that there are two reaſons why the curvature of the orbit ſhould be greateſt in quadrature and leaſt in ſyzygy: firſt, becauſe the moons velocity is leaſt in quadrature and greateſt in ſyzygy: and ſecondly, becauſe the centripetal force is greateſt in quadrature and leaſt in ſyzygy.

422. *The moon is farther from the earth in quadrature than it is in ſyzygy.*

If the moon by the force of gravity alone was to deſcribe a circular orbit having the earth in its center; the ſuns diſturbing force would change the figure of the orbit: for the moon acted upon by the ſun S, Plat. XXX. fig. 3, would be farther from the earth in the quadratures Q and L, and nearer to it in the ſyzygies C and O: ſo that the orbit would put on an elliptical form, QOLC having the earth in its center, the longer axis QL in the quadratures, and the ſhorter axis OC in the ſyzygies. This may appear a ſort of paradox; becauſe it will be thought unlikely that the moon ſhould be fartheſt from the earth in the quadratures, where, by propoſition 407, its centripetal force is the greateſt, and neareſt to the earth in the ſyzygies, where its centripetal force, by propoſition 408, is the leaſt. But to explane this ſeeming paradox it ſhould be obſerved, that the diminution of the moons centripetal force in the ſyzygies is the reaſon why it is fartheſt from the earth in the quadratures: and the encreaſe of the centripetal force in the quadratures is the reaſon why it is neareſt to the earth in the ſyzygies. Thus when the moon is in the conjunction at C, it is leſs impelled towards the earth than the force of gravity would impell it thither, by propoſition 408: and conſequently it will run out farther from the earth, than it would have done, if it had been acted upon by the force of gravity alone. So that when it arrives at the next quadrature L, LT its diſtance from the earth will be greater, than it would have been, if its motion

tion had not been disturbed by the sun. But when the moon is at the quadrature L, it is more impelled towards the earth, than the force of gravity alone would impell it thither, by proposition 407: and consequently its course will be more bent towards the earth, or it will keep approaching nearer to the earth, than if the force of gravity alone had acted upon it. So that when it arrives at O the opposition, OT its distance from the earth will be less, than if the sun had not disturbed its motion. In like manner, as the moon passes from the opposition O to the next quadrature Q, the ablatitious force, which diminishes its tendency towards the earth, will let it run farther off from the earth than it would have done otherwise: and this will make QT its distance from the earth in the quadrature Q greater, than it would have been, if there had been no such ablatitious force. And as the moon passes from the quadrature Q to the conjunction C, the addititious force, which encreases its tendency towards the earth, will make it keep approaching nearer to the earth, than it would have done otherwise: and consequently CT its distance from the earth in the conjunction C will be less, than it would have been, if there had beed no such addititious force.

These are the principal inequalities, which the sun would produce in the moons motion, if the orbit was circular and the earth was in its center. Indeed the orbit is elliptical and the earth is placed in one of the focuses: so that these inequalities will not happen exactly in such a manner as we have been describing. But still in an elliptical eccentric orbit; these inequalities are to be allowed for.

Thus in an elliptical orbit having the earth in one of its focuses, the moons velocity will be the greatest in perigee and the least in apogee. Suppose therefore, that in any conjunction it should be in apogee and that in any quadrature it should be in perigee: the moons velocity in such a conjunction will be less than in such a quadrature. But still the inequality described, in proposition 420, must be allowed for: the moons velocity in that conjunction, though it is not greater than in the quadrature, will be greater, than it would have been, if the sun had not disturbed its motion. And though its velocity in such a quadrature is not less than in the conjunction, yet it will be, less than it would have been, if there had been no such inequality.

In a circular orbit the moons inequality, described in proposition 421, would make it approach to the earth in conjunction or opposition, and depart from it in quadrature. But suppose the moon in an elliptical orbit to be in apogee at conjunction and in perigee at quadrature: then it will be farther from the earth at conjunction than at quadrature: yet

by allowing for the inequality it will be nearer to the earth in that conjunction and farther from the earth at that quadrature, than it would have been, if there had been no ſuch inequality.

Thus all the inequalities hitherto deſcribed are to be allowed for in an elliptical orbit: though they will not take place in the manner we have deſcribed them, unleſs the moons orbit was circular and the earth was in its center; yet with this reſtriction, if all other circumſtances are alike, they will take place in an elliptical orbit having the earth in one of its focuſes.

The next ſort of inequalities to be explaned are thoſe, which ariſe from the eccentricity of the earths orbit. The earth revolves in an ellipſis and the ſun is in one of the focuſes. Upon this account, the earth and the moon are ſometimes nearer to the ſun and ſometimes farther from it, as in propoſition 57.

423. *The moons orbit is dilated, when the earth is in perihelion, and is contracted, when the earth is in aphelion.*

The ſuns attraction changes the moons centripetal force: the ablatitious force diminiſhes and the addititious force encreaſes the moons tendency towards the earth, by propoſitions 408, 407. But the ablatitious force is greater than the addititious force, by propoſition 416. Therefore by theſe forces taken together, or by the ſuns attraction, the moon is made to tend leſs towards the earth, than it would do if there was no other force acted upon it beſides its own gravity. Theſe diſturbing forces encreaſe in the earths approach to the ſun, and are the greateſt when it is in perihelion; they decreaſe in the earths departure from the ſun, and are the leaſt, when it is in aphelion, by prop. 417, 418. But ſince by the action of theſe forces taken together the moons tendency towards the earth is diminiſhed, it follows, that when the earth is in perihelion, the moon is leaſt impelled towards it, and when it is in aphelion, the moon is moſt impelled towards it. Now ſince the force, by which the moon is attracted towards the earth, decreaſes as the earth paſſes from aphelion to perihelion, the moon will keep receding farther from the earth, or the ſemidiameter of its orbit will encreaſe. On the contrary ſince the force, by which the moon is attracted towards the earth, encreaſes as the earth moves from aphelion to perihelion, the moon will keep approaching to the earth, or the ſemidiameter of its orbit will decreaſe. Thus the ſemidiameter of the lunar orbit and conſequently the orbit itſelf, will be greateſt when the earth is in perihelion, and leaſt when it is in aphelion.

424.

424. *The moons periodical time, or a periodical month, is longer in winter than it is in ſummer.*

In our winter the earth is in perihelion, by propoſition 58; and the lunar orbit is dilated, by propoſition 423. In our ſummer the earth is in aphelion, by propoſition 58; and the lunar orbit is contracted, by propoſition 423. The moon takes up more time to deſcribe the dilated or larger orbit, than it does to deſcribe the contracted or ſmaller orbit; that is, the moons periodical time in the larger orbit, when the earth is in perihelion, will be longer, than its peroidical time in the contracted or ſmaller orbit, when the earth is in aphelion.

If LOQC, Plat. XXIX. fig. 10, is the moons orbit. Let us firſt ſuppoſe that the moons centripetal force could be encreaſed and diminiſhed without changing the length of the ſemidiameter LT. Upon this ſuppoſition, the ſemidiameter of the moons orbit is given, whatever its centripetal force is. But, by propoſition 168 of mechanics, the centripetal force, when the ſemidiameter is given, is inverſely as the ſquare of the periodical time; or the ſquare of the periodical time is inverſely as the centripetal force. Now the moons centripetal force decreaſes, whilſt the earth deſcends from aphelion to perihelion, by propoſition 423; and conſequently, if the radius LT was given the ſquare of its periodical time would encreaſe in the ſame proportion that its centripetal force is diminiſhed. On the contrary the moons centripetal force encreaſes, whilſt the earth aſcends from perihelion to aphelion, by propoſition 423; and conſequently, if the radius LT was given, the ſquare of its periodical time would decreaſe in the ſame proportion, that its centripetal force is encreaſed.

Secondly, let us ſuppoſe that LT is varyed, ſo as to be ſometimes greater and ſometimes leſs; and that no other alteration was made in the moons centripetal force or gravitation towards the earth, than what ariſes from an encreaſe or decreaſe of its diſtance LT. Now the moon in theſe changes of LT revolves ſometimes in a larger orbit at a greater diſtance from the earth, and ſometimes in a ſmaller orbit at a leſs diſtance. And the force of gravity, by which according to the ſuppoſition it is impelled towards the earth, is inverſely as the ſquares of the diſtances. Therefore, by propoſition 171 of mechanics, the ſquare of the moons periodical time will vary as the cube of LT its diſtance from the earth varys.

When the earth is paſſing from aphelion to perihelion the moons centripetal force is diminiſhed upon a double account; firſt the ſuns diſturbing forces are encreaſed, by propoſitions 417, 418: and we have ſhewn, in

propoſition 423, that by theſe diſturbing forces taken together the moons centripetal force is diminiſhed, that is, at a given diſtance from the earth it would tend leſs towards the earth; and conſequently as theſe diſturbing forces encreaſe, the moons centripetal force will be diminiſhed. But the ſquare of the moons periodical time is inverſely as its centripetal force. Therefore in the approach of the earth and moon to the ſun, whilſt the earth is paſſing from aphelion to perihelion, the ſquare of the moons periodical time will encreaſe in the ſame proportion that its centripetal force is diminiſhed. But ſecondly, as the earth and moon approach the ſun, the moons diſtance from the earth encreaſes, as we have ſhewn in propoſition 423: and though no other alteration was made in the moons centripetal force, yet the change of diſtance would alter it; and in this variation of the moons centripetal force, the ſquare of its periodical time will encreaſe as the cube of its diſtance from the earth encreaſes. Therefore upon both theſe accounts taken together, as the earth and moon along with it paſs from aphelion to perihelion, the ſquare of the moons periodical time will encreaſe in the inverſe proportion of its diminiſhed centripetal force, and in the direct proportion of the cube of its encreaſed diſtance from the earth. In like manner, whilſt the earth and moon along with it paſs from perihelion to aphelion, the ſquare of the moons periodical time will decreaſe upon a double account; it will decreaſe in the inverſe proportion of the encreaſed centripetal force, and in the direct proportion of the cube of encreaſed diſtance from the earth.

Thus we have ſeen that the ſquare of the moons periodical time encreaſes, whilſt the earth paſſes from aphelion to perihelion, and decreaſes, whilſt the earth paſſes from perihelion to aphelion. So that the ſquare of the periodical time and conſequently the periodical time itſelf will be greateſt when the earth is in perihelion, and leaſt when it is in aphelion.

But if this is the proportion, in which the ſquare of the moons periodical time is varyed, if it is inverſely as the moons centripetal force and directly as the cube of its diſtance from the earth; we may from hence deduce the proportion in which the periodical time itſelf is varyed. But in order to do that, it will firſt be neceſſary to explane what we mean by a ſeſquiplicate or ſeſquialteral ratio.

If there are four quantities, which we will call X, x, Z, z, and the ſquare of the firſt is to the ſquare of the ſecond as the cube of the third is to the cube of the fourth, ſo that XX is to xx as ZZZ to zzz, then the firſt is to the ſecond in a ſeſquiplicate ratio of the third to the fourth, X is to x in a ſeſquiplicate or ſeſquialteral ratio of Z to z. The ratio of XX

to

to xx is the duplicate or double ratio of X to x. And the ratio of ZZZ to zzz is the triplicate or triple ratio of Z to z. Therefore ſince XX is to xx, as ZZZ to zzz, or ſince the ratio of XX to xx is the ſame as, or equal to, the ratio of ZZZ to zzz, it is plane that double the ratio of X to x is equal to triple the ratio of Z to z, or that twice the ratio of X to x is equal to thrice the ratio of Z to z. Now if twice the ratio of X to x is equal to thrice the ratio of Z to z, it follows that once the ratio of X to x is equal to once and a half the ratio of Z to z. And, becauſe the ratio of X to x contains the ratio of Z to z once and a half, X is ſaid to be to x in a ſeſquialteral or ſeſquiplicate ratio of Z to z. We may apply this to a particular inſtance. The ſquares of the periodical times of the primary planets are as the cubes of their middle diſtances from the ſun. But where the ſquares of any quantities are proportional to the cubes of any other quantities, then the former quantities are in a ſeſquiplicate ratio of the latter. Therefore the periodical times of the primary planets are in a ſeſquiplicate ratio of their middle diſtances from the ſun.

Let us now enquire in what proportion the periodical time of the moon is varyed, according as the earth is in perihelion or aphelion. We have proved that the moons centripetal force, or the earths attraction, at a given diſtance of the moon from the earth, will be leſs in perihelion than in aphelion; and that, in conſequence of this, the ſquare of the moons periodical time is inverſely as that centripetal force. Therefore the periodical time will be inverſely as the ſquare root or in an inverſe ſubduplicate ratio, of the earths attractive power or the moons centripetal force at a given diſtance. We have proved likewiſe that the moons diſtance from the earth is greater in perihelion than in aphelion; and that, in conſequence of this the ſquare of the moons periodical time is directly as the cube of its diſtance. Therefore the periodical time will be in a direct ſeſquiplicate ratio of the diſtance.

From hence it follows that upon the whole the moons periodical time in perihelion and aphelion will be varyed in the inverſe ſubduplicate ratio of the earths attractive force, and in a direct ſeſquiplicate ratio of the moons diſtance from the earth.

We are now come to the third ſort of inequalities in the moons motion. The moons orbit is an ellipſis having the earth in one of its focuſes. And there are ſome of the lunar inequalities, which ariſe from this particular form of the orbit and this particular poſition of the earth.

425. *The moons orbit is a moveable ellipſis: its longer axis revolves the ſame way that the moon does, when the moon is in ſyzygy, and revolves the contrary way, when the moon is in quadrature.*

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If the moons gravitation towards the earth was not affected by the suns action, the moon would describe an immoveable ellipsis, by proposition 172 of mechanics: for the force of gravity, at different distances from the earth, is inversely as the squares of the distances. But when the moon is in syzygy, the sun diminishes the moons gravitation; so that its centripetal force is then the remainder of its gravitation after the ablatitious force is taken from it, by proposition 408. Now the moon, as it revolves round the earth in an elliptical orbit, is sometimes at a greater and sometimes at a less distance from the earth, and, by proposition 23 of mechanics, its gravitation at these different distances will be inversely as the squares of the distances. Let the distance of the moon from the earth, as it moves from perigee to apogee encrease in the proportion of the numbers 3, 4, 5, the squares of these distances are 9, 16, 25, these squares inverted are 25, 16, 9. So that as the distances encrease in the proportion of 3, 4, 5, the moons gravitation will decrease in the proportion of 25, 16, 9. But whilst the moon departs from the earth the ablatitious force encreases in the same proportion with the distances, by proposition 418. Therefore at the distances 3, 4, 5, the ablatitious force will encrease in the proportion of 3, 4, 5. Now if from the numbers 25, 16, 9, which express the moons gravitation we subtract respectively the numbers 3, 4, 5, which express the ablatitious force; the remainders 22, 12, 4, will be proportional to the moons centripetal force at the distances 3, 4, 5. But the numbers 22, 12, 4, decrease faster than the numbers 25, 16, 9. Therefore the moons centripetal force decreases faster in the moons departure than in the inverse ratio of the squares of the distances. And universally if in the moons departure we subtract the ablatitious force, which is directly as the distances, from the force of gravity, which is inversely as the squares of the distances, the remainder or the moons centripetal force will decrease faster than the squares of the distances inverted, or faster than in an inverse duplicate ratio of the distances. Thus we have proved that the moons centripetal force, when it is in syzygy, decreases faster than the force of gravity, as the moon departs from the earth: and, in proposition 175 of mechanics, we have shewn that any force, which decreases faster in the moons departure from the earth, will encrease faster in its approach to the earth. Therefore when the moon is in syzygy the ablatitious force makes the moons centripetal force decrease and encrease faster than the force of gravity, or faster than in an inverse duplicate ratio of the moons distances. But such a centripetal force will make the moon describe a moveable ellipsis, whose longer axis will revolve the same way with the moon, by proposition 175 of mechanics. Therefore when the moon is

in

in ſyzygy its orbit is ſuch an ellipſis, the line of its apſes or its longer axis revolves at that time from weſt to eaſt.

When the moon is in quadrature, its centripetal force is encreaſed by the ſuns action, by propoſition 407: for the centripetal force is then the moons gravitation and the addititious force together. The force of gravity at different diſtances from the earth is inverſely as the ſquares of the diſtances: at the diſtances 3, 4, 5, the force of gravity decreaſes in the proportion of 25, 16, 9. But the addititious force at theſe diſtances 3, 4, 5, encreaſes in the proportion of 3, 4, 5, that is, it encreaſes in the ſame proportion with the diſtance, by prop. 417. Now if to the numbers 25, 16, 9, which expreſs the force of gravity, we reſpectively add the numbers 3, 4, 5, which expreſs the addititious force, the ſums 28, 20, 14, will expreſs the proportion, in which the moons centripetal force decreaſes, as the moon in quadrature departs from the earth. But theſe numbers 28, 20, 14, decreaſe ſlower than 25, 16, 9. Therefore the moons centripetal force decreaſes ſlower than the force of gravity, or ſlower than in the inverſe ratio of the ſquares of the diſtances. And ſo univerſally, if in the moons departure from the earth we add the addititious force, which is directly as the diſtances, to the force of gravity, which is inverſely as the ſquares of the diſtances, the ſum or the moons centripetal force will decreaſe ſlower than the ſquares of the diſtances inverted, or ſlower than in an inverſe duplicate ratio of the diſtances. Thus we have proved that the moons centripetal force, when it is in quadrature, decreaſes ſlower than the force of gravity, as the moon departs from the earth: and, in propoſition 176 of mechanics, we have ſhewn, that any force, which decreaſes ſlower in the moons departure from the earth, will encreaſe ſlower in its approach to the earth. Therefore when the moon is in quadrature, the addititious force makes the moons centripetal force decreaſe and encreaſe ſlower than in an inverſe duplicate ratio of the moons diſtances. But ſuch a centripetal force will make the moon deſcribe a moveable ellipſis, whoſe longer axis will move the contrary way to the moon, by propoſition 176 of mechanics. Therefore when the moon is in quadrature, its orbit is ſuch an ellipſis, the line of its apſes or longer axis revolves at that time from eaſt to weſt.

426. *The line of the moons apſes goes forward more, when the moon is in ſyzygy, than it goes backward, when the moon is in quadrature.*

The line of the apſes is made to go forward by the ablatitious force, when the moon is in ſyzygy; and it is made to go backward by the addititious force when the moon is in quadrature, by propoſition 425. But the

the ablatitious force is greater than the addititious force, by propoſition 416. Therefore, in like circumſtances, the line of the apſes in one revolution of the moon will go forward more than it goes backward, or will move farther in the ſame direction that the moon moves, than in the contrary direction.

427. *The progreſs of the line of the apſes in one revolution of the moon is the greateſt, when the line of the apſes paſſes through the ſyzygies.*

Without conſidering the motion of the line of the apſes deſcribed in the foregoing propoſitions, as the earth and moon move round the ſun, this line will ſometimes paſs through the ſyzygies and ſometimes through the quadratures. Let S be the ſun, Plat. XXX. fig. 4, AQLB the lunar orbit, then AB the line of the apſes paſſes through the ſyzygies, or the moon will be in this line at B, when it is in conjunction, and at A, when in oppoſition. In a quarter of a years time the earth and the moons orbit with it will have gone a quarter round the ſun, and will be at ORPV, where OP the line of the apſes paſſes through the quadratures, for the moon will be in this line at O, when it is in one quadrature, and at P, when it is in the other quadrature. In a quarter of a year more the earth and moons orbit will be at GMHN, and GH the line of the apſes will paſs through the ſyzygies. In a quarter more, the earth and moons orbit will be at CEDF, and CD the line of the apſes will paſs through the quadratures. And when the year is completed, the earth and moons orbit will be returned to their firſt ſituation AQBL. Thus in the courſe of a year the line of the apſes paſſes twice through the ſyzygies and twice through the quadratures. This ariſes wholly from the motion of the earth and moons orbit round the ſun, by which motion ſometimes one part of the orbit and ſometimes another is towards the ſun. The lower apſis B is next the ſun at one time of the year, and the higher apſis G is next it at the oppoſite ſeaſon. The ſide V is next the ſun at one ſeaſon, and the oppoſite ſide E is next it at the oppoſite ſeaſon. This change in the ſituation of the line of the apſes in reſpect of the ſun is made without any motion of that line: for in this change of ſituation this line is always parallel to itſelf, whether it is at AB, OP, GH, or CD: ſo that in all theſe different ſituations it points towards the ſame part of the heavens: the moon ſeen from the earth T will appear in the ſame part of the heavens, when it is in its higher apſis, at A, O, G, or C; therefore this higher apſis in this change of ſituation in reſpect of the ſun does not move in reſpect of the fixed ſtars: but the orbit is an immoveable ellipſis, notwithſtanding the line of the apſes is ſometimes in ſyzygy and ſometimes in quadrature.

But

But though the orbit might be an immoveable ellipſis, notwithſtanding the line of the apſes thus changes its ſituation in reſpect of the ſun; yet in propoſition 425, we have ſhewn that by the ſuns action this orbit is made a moveable ellipſis; the line of the apſes moves forward, when the moon is in ſyzygy, and moves backward when the moon is in quadrature: and, by propoſition 426, the progreſs of the line of the apſes in like circumſtances exceeds the regreſs in one revolution of the moon: ſo that the line of the apſes, and conſequently either the greateſt or leaſt diſtance, moves in conſequentia. By this means, and not by the line of the apſes being ſometimes in ſyzygy and ſometimes in quadrature, it happens, that if the moon at any time appears in a certain part of the heavens, when it is at its greateſt diſtance from the earth, it will appear in a part of the heavens more to the eaſt, when it comes to its greateſt diſtance the next month. But this is the caſe only in like circumſtances; the progreſs of the line of the apſes is not the ſame in every revolution of the moon: this progreſs is greater or leſs according to the different ſituation of the line of the apſes in reſpect of the ſun. In this propoſition we are to ſhew that the progreſs of the line of the apſes will be the greateſt in one revolution of the moon, when that line paſſes through the ſyzygies, that is, when the moons orbit is in the ſituation AQBL or GMHN.

The line of the apſes moves forwards when the moon is in ſyzygy, by propoſition 425; becauſe the centripetal force, which retains the moon in its orbit, decreaſes in the departure of the moon from the earth, and encreaſes, in the moons approach to the earth, faſter than the force of gravity, or faſter than in the inverſe duplicate ratio of the diſtances. If the moons centripetal force depended only upon the force of gravity, that is, if it was always in an inverſe duplicate ratio of the moons diſtance from the earth; the lunar orbit would be an immoveable ellipſis, by propoſitiom 172 of mechanics. But, when the moon is in ſyzygy, the centripetal force decreaſes and encreaſes faſter than in the inverſe duplicate ratio of the diſtances: and this makes the line of the apſes revolve in conſequentia, by propoſition 425. Thus by the ablatitious force the moons centripetal force is made to vary by a different law from the force of gravity: and the effect of its varying by a different law is, that the line of the apſes goes forward. But if the diſtance of the moon in conjunction and oppoſition is the ſame; the centripetal force in conjunction and oppoſition will be equal. If BT was equal to AT, by whatever law the centripetal force may vary as the diſtances vary, yet, upon this ſuppoſition, the moons diſtance from the earth being the ſame

at B and at A, the centripetal force will be the ſame at B and at A, or will not have varyed at all. Now the progreſs of the line of the apſes is owing to the law, by which the centripetal force varys, being different from the law, by which the force of gravity varys. But if the centripetal force does not vary at all, it cannot vary by a different law from the force of gravity: and conſequently there can be no progreſs of the apſes. Therefore if BT was equal to AT, or if the moon was at the ſame diſtance from the earth in conjunction and oppoſition, there could be no progreſs of the apſes. The progreſs of the line of the apſes will be the greateſt, when the centripetal force varys the moſt by a different law from the force of gravity. If it varys at all, it will vary by a different law, by propoſition 425. And conſequently the more it varys, the more it will vary by a different law from the force of gravity. Therefore the more the centripetal force in conjunction differs from the centripetal force in oppoſition, the greater will be the progreſs of the line of the apſes. But the centripetal force in conjunction will differ the moſt from the centripetal force in oppoſition, when the moons diſtance from the earth in conjunction differs the moſt from its diſtance in oppoſition: becauſe if thoſe diſtances were equal, the centripetal force would be the ſame in both ſyzygies. But the moons diſtance from the earth in conjunction will differ the moſt from its diſtance in oppoſition, when in one ſyzygy it is at its leaſt diſtance BT, and in the other ſyzygy at its greateſt diſtance AT. Now the line of the apſes AB paſſes through the greateſt and leaſt diſtance. Therefore the moons diſtance from the earth in one ſyzygy will differ the moſt from its diſtance in the other ſyzygy, when the line of the apſes paſſes through the ſyzygies. Conſequently, in this ſituation of the line of the apſes, the centripetal force in conjunction will differ the moſt from the centripetal force in oppoſition: and from hence it follows, as we have juſt ſhewn, that the progreſs of the line of the apſes, when the moon is in ſyzygy will be the greateſt.

When the line of the apſes is in ſyzygy and the moon is in quadrature; there is little or no regreſs of the line of the apſes. For the regreſs is owing to the addititious force, by propoſition 425, which makes the moons centripetal force, at different diſtances from the earth, decreaſe and encreaſe ſlower than in the inverſe duplicate ratio of the diſtances. If the centripetal force varyed according to the regular courſe of gravity, or in the inverſe duplicate ratio of the diſtances, there would be no regreſs. Now the moons centripetal force cannot vary by a different law from the force of gravity, when that centripetal force does not vary at all. But the centripetal force does not vary, when the moons diſtance is

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the ſame. Therefore if the moon in each quadrature is at the ſame diſtance from the earth, the centripetal force will not vary at all in the quadratures, and conſequently cannot vary by a different law from the force of gravity. But if AB the line of the apſes is in ſyzygy, the moon, when in one quadrature at Q, and in the other quadrature about L, is nearly at an equal diſtance from the earth T. Therefore, as the variation of the centripetal force in theſe quadratures will be ſmall, it can differ but little from the law by which the force of gravity is varyed: and conſequently there will be but little regreſs of the line of the apſes.

Now ſince if the line of the apſes is in ſyzygy, its progreſs is the greateſt when the moon is in ſyzygy, and its regreſs is little when the moon is in quadrature; it follows that in this revolution of the moon the progreſs exceeds the regreſs as much as it can do. Therefore the line of the apſes in ſuch a revolution will go forward upon the whole more than in any other.

What we have here ſhewn to be the caſe, when the moons orbit is at AQBL is equally true, when the orbit is at GMHN: for in either ſituation the line of the apſes is in ſyzygy.

428. *In one revolution of the moon the line of the moons apſes, if it is in quadrature, will go backward.*

When the moon is in quadrature, the addititious force, by propoſition 425, makes the line of the apſes go backward; becauſe it makes the moons centripetal force encreaſe or decreaſe ſlower than according to the regular courſe of gravity, that is, ſlower than in the inverſe duplicate ratio of the diſtances. If the centripetal force varyed by this law when the moon is in quadrature, there would be no regreſs at all of the line of the apſes: and the more it is varyed by a different law, in conſequence of the addititious force, the greater will be the regreſs of the line of the apſes. But when the moon is in quadrature, if the centripetal force is varyed at all, it will be varyed by a different law. Therefore the more the centripetal force is varyed, when the moon is in quadrature, the more it will be varyed by a different law. And conſequently the more the centripetal force is varyed, when the moon is in quadrature, the greater will be the regreſs of the line of the apſes. Now if OT, Plat. XXX. fig. 4, was equal to PT, the moons diſtance from the earth in one quadrature O would be equal to its diſtance from the earth in the other quadrature P. And by whatever law the centripetal force varys, when the diſtances are unequal, yet when they are equal the centripetal force will not vary at all: and conſequently will not vary by a different law

from the force of gravity. But if OP is the line of the apſes, that is, if the line of the apſes paſſes through the quadratures; then the moons diſtance from the earth at the quadrature O is its greateſt diſtance; and its diſtance at the quadrature P is its leaſt diſtance. Theſe diſtances differ the moſt from one another that any two diſtances can do. Therefore the centripetal force, as the moon paſſes from one quadrature to the other, will vary the moſt that it can do. But the centripetal force varys by ſuch a law, when the moon is in quadrature, as will make the line of the apſes go backward: and this regreſs will be the greateſt, when the centripetal force varys the moſt. Therefore the regreſs of the line of the apſes, when the moon is in quadrature, will be the greateſt, if the line of the apſes paſſes through the quadratures.

The ablatitious force makes the line of the apſes go forward, when the moon is in ſyzygy, by propoſition 425: becauſe it makes the centripetal force decreaſe and encreaſe faſter than according to the regular courſe of gravity, or faſter than in the inverſe duplicate ratio of the moons diſtance from the earth. But if the moons diſtance in conjunction is equal to its diſtance in oppoſition, the centripetal force at conjunction and oppoſition will be the ſame: and conſequently as it does not vary at all, it cannot vary by a different law from the force of gravity. When OP the line of the apſes is in quadrature, if the moon is in conjunction at V, it will be in oppoſition ſomewhere about R; ſo that VT its diſtance from the earth in conjunction will be nearly equal to RT its diſtance from the earth in oppoſition. Conſequently the centripetal force will be nearly the ſame in oppoſition and conjunction; or the centripetal force will vary but little. But ſince the line of the apſes will not go forward when the moon is in ſyzygy, if the centripetal force does not vary by a different law from the force of gravity; and ſince where there is little or no variation of the centripetal force, it cannot vary by a different law from the force of gravity; it follows that if the line of the apſes is in quadrature, its progreſs will be little when the moon is in ſyzygy.

Now ſince, if the line of the apſes is in quadrature, its regreſs is the greateſt that it can be, when the moon is in quadrature; and its progreſs is but little when the moon is in ſyzygy; the conſequence is that in this revolution of the moon the regreſs will exceed the progreſs, or the line of the apſes will upon the whole go backward.

What we have proved when the orbit is at ORPV, is equally true, when it is at CEDF: for in either ſituation the line of the apſes is in quadrature.

429. *In*

429. *In a years time the line of the moons apſes goes forward upon the whole.*

We have ſhewn that in about a years time the line of the apſes is twice in ſyzygy and twice in quadrature: and in propoſitions 427, 428, we have ſhewn that in one revolution of the moon the line of the apſes goes forward, when it is in ſyzygy, and backward, when it is in quadrature. Therefore if the progreſs in one ſituation and the regreſs in the other were equal, the line of the apſes in about a years time would be reſtored to its place, and at the end of the year would point towards the ſame parts of the heavens, towards which it pointed at the beginning of it. But the progreſs of the line of the apſes is greater and laſts longer in one ſituation, than its regreſs in the other ſituation. Therefore in a years time the progreſs will exceed the regreſs.

Firſt; the progreſs of the line of the apſes is greater when it paſſes through the ſyzygy; than its regreſs, when it paſſes through the quadrature. Becauſe its progreſs is owing to the ablatitious force, by propoſition 427, and its regreſs to the addititious force, by propoſition 428. And the ablatitious force is greater than the addititious force, by propoſition 416.

Secondly; the progreſs of the line of the apſes in one ſituation continues longer than its regreſs in the other ſituation. Becauſe the line of the apſes continues near the ſyzygies longer than it continues near the quadratures. Suppoſe that the moon appears in a certain part of the heavens, when it is in conjunction, and that the line of the apſes then paſſes through the ſyzygies. The moon at the next conjunction will be to the eaſt of that part of the heavens, by propoſition 140; and the line of the apſes will have likewiſe moved to the eaſt, by propoſition 427. Therefore in the next conjunction the line of the apſes will be near the ſyzygies. Whereas if the moon at the firſt quarter appears in a certain part of the heavens, and the line of the apſes then paſſes through the quarters: at the firſt quarter in the following month the moon will be to the eaſt of that part, by propoſition 140: but the line of the apſes having moved backwards, by propoſition 428, will be to the weſt of it. Therefore the line of the apſes will then be at ſome diſtance from the quarters. Thus, by propoſition 140, the ſyzygies and quarters move in conſequentia, that is, if the moon one month appears in a certain part of the heavens, when it is either in conjunction or oppoſition, in the firſt or laſt quarter; it will appear to the eaſt of that part in the next month, when it is again in conjunction or oppoſition, in the firſt or the laſt quarter. But if the line of

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the apſes is in ſyzygy, that likewiſe moves in conſequentia, by propoſition 427. Therefore as the ſyzygies and the line of the apſes are moving the ſame way, the line of the apſes will continue ſome time in or near the ſyzygies. Whereas when the line of the apſes paſſes through the quarters, it moves in antecedentia, by propoſition 428. Therefore as the quarters move in conſequentia and the line of the apſes moves the contrary way, the line of the apſes will continue but a little while in the quarters.

For theſe two reaſons in a years time the line of the apſes moves forward or in conſequentia upon the whole. And in about nine years time it completes a revolution. So that in the courſe of about nine years the moons greateſt and leaſt diſtances ſeen from the earth will appear to have moved quite round in the plane of its orbit.

430. *The eccentricity of the lunar orbit is greateſt, when the moon is in ſyzygy; and is leaſt, when the moon is in quadrature.*

If the moon, when it is either in its higher apſis or in its lower apſis, was at equal diſtances from the earth, the earth muſt be in the middle or center of the lunar orbit; ſo that the orbit, upon this ſuppoſition, would not be eccentric. From hence the reader will underſtand, that as the moons diſtance from the earth in the higher apſis is greater in proportion to its diſtance in the lower apſis, its eccentricity is greater. Therefore there are two ways by which the moons eccentricity may be encreaſed. For the moons diſtance in the higher apſis will be greater in proportion to its diſtance in the lower apſis, either if its diſtance in the higher apſis is encreaſed, or if its diſtance in the lower apſis is diminiſhed. The eccentricity of the orbit AQBL, Plat. XXX. fig. 4, will be encreaſed, as AT bears a greater proportion to BT. And the proportion of AT to BT will be encreaſed either by encreaſing AT or by diminiſhing BT. Again; as the moons diſtance from the earth in the higher apſis is leſs in proportion to its diſtance in the lower apſis, its eccentricity is leſs. Therefore there are two ways by which the moons eccentricity is diminiſhed. For the moons diſtance in the higher apſis will be leſs in proportion to its diſtance in the lower apſis, either if its diſtance in the higher apſis is diminiſhed, or if its diſtance in the lower apſis is encreaſed. The eccentricity of the orbit AQBL will be leſs as AT bears a leſs proportion to BT. And the proportion of AT to BT will be diminiſhed either by diminiſhing AT or by encreaſing BT.

The moons eccentricity would be invariable, for it would always deſcribe the ſame ellipſis, if it was acted upon only by the force of gravity,

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which impells it to the earth, and at different diſtances from the earth is inverſely as the ſquares of the diſtances. But the ablatitious force diſturbs the moons motion. For by the ablatitious force, as has been ſhewn in propoſition 425, the centripetal force of the moon is made to encreaſe faſter, as the moon approaches to the earth, and to decreaſe faſter, as the moon departs from the earth, than in the inverſe duplicate ratio of the diſtances, or faſter than the force of gravity would encreaſe and decreaſe in the ſame circumſtances. This ablatitious force will encreaſe the moons eccentricity. For ſuppoſe the moon to ſet out from its higher apſis, and to approach the earth. Then, in conſequence of the ablatitious force, the moons centripetal force encreaſes faſter than in the inverſe duplicate ratio of the diſtances, that is, it encreaſes faſter in proportion than the force of gravity would, if it was not diſturbed by the ſuns action. Therefore the moon will be every where more urged towards the earth, and, when it comes to the lower apſis, its diſtance will be leſs in proportion, than if the force of gravity had acted upon it without being diſturbed. Or ſuppoſe the moon to ſet out from the lower apſis and to depart from the earth: then the centripetal force, in conſequence of the ablatitious force, decreaſes faſter than the ſquares of the diſtances encreaſe; that is, it decreaſes faſter than the force of gravity would, if it was undiſturbed by the ſuns action. Therefore the moon will be every where leſs urged towards the earth, and, when it comes to the higher apſis, its diſtance will be greater in proportion, than if the force of gravity had acted alone without being diſturbed. Thus by the ablatitious force the moons diſtance in the lower apſis is diminiſhed and its diſtance in the higher apſis is encreaſed. And we have ſhewn that the eccentricity of the orbit is encreaſed either way. Therefore the ablatitious force encreaſes the moons eccentricity: and as this force is the greateſt, ſo the eccentricity of the lunar orbit will likewiſe be the greateſt, when the moon is in ſyzygy.

The addititious force diſturbs the moons motion. For this force, as has been ſhewn in propoſition 425, will make the moons centripetal force encreaſe ſlower, as the moon approaches the earth, and decreaſe ſlower, as the moon departs from the earth, than in the inverſe duplicate ratio of the diſtances, that is, ſlower than the force of gravity alone would encreaſe and decreaſe in the ſame circumſtances. This addititious force will diminiſh the moons eccentricity. For ſuppoſe the moon to ſet out from the higher apſis and to approach the earth: then, in conſequence of the addititious force, the centripetal force encreaſes ſlower than the force of gravity alone would. Therefore the moon will be every where leſs urged towards the earth, and, when it comes to the lower apſis, its diſtance will be

be greater in proportion, than if the force of gravity had acted upon it without being disturbed. Or suppose the moon to set out from the lower apsis, and to depart from the earth: then the centripetal force, in consequence of the addititious force, decreases slower than the squares of the distances encrease, that is, slower than the force of gravity would, if it was undisturbed. Therefore the moon will be every where more urged towards the earth, and, when it comes to the higher apsis, its distance will be less in proportion, than if the force of gravity had acted alone without being disturbed. Thus by the addititious force the moons distance from the earth in the lower apsis will be encreased, and its distance in the higher apsis will be diminished: and we have shewn that the eccentricity of the lunar orbit is diminished either way. Therefore the addititious force diminishes the eccentricity: and as this force is the greatest, so the eccentricity will be the least, when the moon is in quadrature.

431. *In different revolutions of the moon compared together, the eccentricity of the lunar orbit will be the greatest of all, when the line of the apses is in syzygy, and the least of all, when the line of the apses is in quadrature.*

As the moon moves from one syzygy to the other, the centripetal force varys by a different law from the force of gravity, it decreases and encreases faster than in the inverse duplicate ratio of the distance, by proposition 425. But this can only happen if the moon at the two syzygies is at different distances from the earth: for if it is at the same distance in conjunction and opposition, the centripetal force will be the same in both: and consequently it cannot have varyed by a different law from the force of gravity, because it has not varyed at all. But when AB, the line of the apses, Plat. XXX. fig. 4, passes through the syzygies, the moons distance from the earth in one syzygy is AT the greatest distance, and in the other syzygy it is BT the least distance. Therefore in this situation of the lunar orbit the moons distances in the two syzygies differ the most that is possible: and for that reason the centripetal force, as the moon passes from one syzygy to the other, will vary the most, and the effect of its varying by a different law from the force of gravity will be the greatest. But one effect of the law, by which the centripetal force varys, when the moon is in syzygy, is the encrease of the moons eccentricity. Therefore in one revolution of the moon, when the line of the apses is in syzygy, either at AB or at GH, the eccentricity of the orbit will be more encreased, than when the line of the apses is in any other situation.

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In the ſame revolution of the moon the eccentricity of the orbit will be diminiſhed very little when the moon is in quadrature. For though the addititious force makes the centripetal force vary by a law different from the force of gravity, or makes it encreaſe and decreaſe ſlower than in the inverſe duplicate ratio of the diſtances, as has been ſhewn in propoſition 425; yet the centripetal force cannot have varyed by a law different from the force of gravity, if it has not varyed at all. But when AB the line of the apſes paſſes through the ſyzygy, the moon is in one quarter at Q, and in the other quarter ſomewhere about L; ſo that its diſtance from the earth in one quarter is nearly equal to its diſtance in the other quarter. One effect of the law, by which the centripetal force varys, when the moon is in quadrature, is a decreaſe of the eccentricity. But when the line of the apſes is in ſyzygy, ſince the centripetal force in one quadrature is nearly the ſame as in the other; the centripetal force ſcarce varys at all: and conſequently as it has ſcarce varyed at all, the effect of its varying by a different law from the force of gravity will be little or nothing; that is, the eccentricity will be very little diminiſhed.

Therefore ſince the eccentricity of the lunar orbit is the moſt encreaſed, that it can be, when the moon is in ſyzygy, and is diminiſhed very little, when the moon is in quadrature, if the line of the apſes paſſes through the ſyzygies; it follows that in one revolution of the moon, when the line of the apſes is in this ſituation, the eccentricity will be greater than when the line of the apſes is in any other ſituation.

If the line of the apſes, as OP or CD, paſſes through the quadratures, the moons diſtance from the earth in one quarter, differs the moſt that it can do from its diſtance in the other quarter. For its diſtance in one quarter is OT or CT, which is its greateſt diſtance; and PT or DT in the other quarter, which is its leaſt diſtance. The additiitious force, when the moon is in quadrature, makes the centripetal force decreaſe and encreaſe ſlower than in the inverſe duplicate ratio of the diſtances, or makes it vary by a different law from the force of gravity; and one effect of this is a decreaſe of the moons eccentricity. This decreaſe of eccentricity will be the greateſt, when the centripetal force varys the moſt by a different law from the force of gravity. But it always varys by a different law if it varys at all; and conſequently it will vary the moſt by a different law, when the diſtance in one quadrature differs moſt from the diſtance in the other quadrature, that is, when the line of the apſes paſſes through the quadrature. Therefore in this ſituation of the lunar orbit the decreaſe of eccentricity will be the greateſt, as the moon paſſes from one quadrature to the other.

In the ſame ſituation of the orbit, the moon is in conjunction at V or E, and in oppoſition ſomewhere about R or F. The diſtances of the moon from the earth in either of the ſyzygies, the diſtances VT and RT, or ET and FT, are nearly equal to one another. Therefore in this ſituation of the orbit, as the moon paſſes from one ſyzygy to the other, the centripetal force varys but little. If the centripetal force varyed, the ablatitious force would make it vary faſter than in the inverſe duplicate ratio of the diſtances; and this would encreaſe the eccentricity, by propoſition 430. But as it varys very little, the effect of its varying by a different law from the force of gravity will be but little, or the eccentricity of the orbit will be but little encreaſed.

Thus if the line of the apſes is in quadrature, the eccentricity is the moſt diminiſhed that it can be, when the moon is in the quarters, and is but little encreaſed when the moon is the ſyzygies. Therefore in a revolution of the moon, whilſt the line of the apſes is in this ſituation, the eccentricity of the orbit will be leſs, than when the line of the apſes is in any other ſituation.

In explaning the foregoing inequalities, we have taken no notice of the inclination of the lunar orbit to the plane of the ecliptic: becauſe thoſe inequalities would all of them happen though the plane of the moons orbit coincided with the plane of the ecliptic. But there are ſtill another ſort of inequalities to be explaned, which ariſe from the inclination of the lunar orbit to the plane of the ecliptic.

432. The addititious force will not change the plane of the moons motion.

If the moon was impelled by no other force but its gravitation towards the earth, it would always revolve in the ſame immoveable plane, by propoſition 158 of mechanics: ſo that, upon this ſuppoſition, the plane of the moons orbit would always make the ſame angle with the plane of the ecliptic, and would always cut the plane of the ecliptic in the ſame points; that is, the inclination of the moons orbit and the place of its nodes would be invariable. But the moon is impelled or urged by its gravitation towards the ſun, as well as by its gravitation towards the earth. And the ſuns force ſometimes encreaſes and ſometimes diminiſhes the moons tendency towards the earth, by propoſitions 407, 408. The ſuns addititious force, by which the moons tendency towards the earth is encreaſed, will not alter the plane of the moons motion. For the addititious force acts in a line drawn from the moon towards the earth, by propoſition 414. And ſuch a line is in the plane of the lunar orbit. But no force, that acts in the plane of the lunar orbit, can make the moon

move

move out of that plane. Therefore the addititious force does not change the plane of the moons motion.

From hence it appears that, if the ſun makes any alteration in the plane of the moons motion, it muſt be by its ablatitious force. We will therefore enquire what alterations this force will produce. Theſe alterations are called inequalities in latitude.

433. *When the ablatitious force does not act in the plane of the moons orbit; if the point, towards which it is directed, is on the ſame ſide of the node with the moon, this force will draw the moon out of the plane of its orbit towards the plane of the ecliptic: but if that point and the moon are on contrary ſides of the node, then the ablatitious force will draw the moon from the plane of the ecliptic.*

Let OS, Plat. XXX. fig. 5, repreſent the plane of the ecliptic: S the ſide where the ſun is, and O the oppoſite ſide: let LB repreſent the plane of the lunar orbit; and N the line of the nodes. Now the earth is always in the plane of the ecliptic, and likewiſe it is always in the plane of the lunar orbit. Therefore it is always in the common interſection of theſe two planes, or in the line of the nodes: ſo that N is likewiſe the place of the earth. The planes of the ecliptic and of the lunar orbit are here repreſented by right lines: becauſe we ſuppoſe the edges of thoſe planes to be towards our eye, as we are looking at the paper where the figure is drawn. And the line of the nodes is here repreſented by a point N: becauſe in this view the end of that line is towards our eye.

The ſun at S attracts the earth in the direction NS: and the ablatitious force in one half of the orbit acts towards the ſun in the direction LF parallel to NS; and in the other half of the orbit it acts from the ſun in the direction LR parallel likewiſe to SN or NO, by propoſition 415. This force always acts in the plane of the ecliptic or in a line parallel to that plane: but the lunar orbit is inclined to the plane of the ecliptic. Therefore this force will not always act in the plane or in a line parallel to the plane of the lunar orbit: and conſequently it will ſometimes draw the moon out of the plane, in which it was moving, or will change the plane of its orbit.

When the ablatitious force acts towards the ſun and LF is its direction; let the moon be at L, or any where elſe between L and N, then the moon is on the ſame ſide of the node N with the point S or F towards which this force is directed. In this caſe the ablatitious force, will draw the moon towards the plane of the ecliptic. For as LF repreſents the direction, ſo let Lp repreſent the quantity of this force. Now as Lp is

oblique to NS the plane of the lunar orbit, it may be refolved into two parts Lg parallel to that plane or rather coinciding with it, and Ld perpendicular to it, by propoſition 20 of mechanics. The part Lg, which is in the plane of the lunar orbit, whatever other effect it may have, will not draw the moon out of that plane. But the other part Ld, which is perpendicular to the plane, will draw the moon out of the plane of its orbit in the direction Ld towards NS the plane of the ecliptic.

But when the ablatitious force acts from the ſun, and LR is its direction; let the moon be at L or any where elſe between L and N, then the moon is on one ſide of the node N, and the point O or R, towards which the ablatitious force is directed, is on the contrary ſide. In this caſe the ablatitious force will draw the moon from the plane of the ecliptic. For as LR repreſents the direction, ſo let L*m* repreſent the quantity of this force: then as L*m* is oblique to LN the plane of the moons orbit, this force may be reſolved into two parts LN parallel to or coinciding with the plane of the orbit, and L*c* perpendicular to it. The part LN being in the plane of the orbit will not draw the moon out of that plane. But the other part L*c* will draw the moon out of the plane of the orbit from NS the plane of the ecliptic.

434. *If the ablatitious force draws the moon towards the plane of the ecliptic, whilſt it is riſing from the plane, or if the ſame force draws the moon from the plane of the ecliptic, whilſt it is deſcending towards the plane, the inclination of the lunar orbit will be diminiſhed.*

Let EC, Plat. XXX. fig. 6, repreſent the plane of the ecliptic; and NA the plane of the lunar orbit. Firſt ſuppoſe that the moon was moving in its orbit from N the node towards the limit or to be riſing from EC the plane of the ecliptic in the direction NA, and that any force as AB or ND perpendicular to the plane of the orbit was in the mean time to draw it down towards the ecliptic. The moon is then acted upon by two forces, one is the force with which it is moving in its orbit, and the other is the force AB or ND. Theſe two forces together will change the plane of its motion and will make it riſe in the diagonal NB, by propoſition 16 of mechanics. But NB makes a leſs angle with EC than NA makes with it: for the angle BNC is evidently leſs than the angle ANC. Therefore if the moon is drawn towards the plane of the ecliptic, whilſt it is riſing from thence in its orbit, the plane of its orbit will be changed, and will make a leſs angle with the plane of the ecliptic than it did before; that is, the inclination of the orbit will be diminiſhed.

Secondly, ſuppoſe that the moon was deſcending in the plane of its orbit BFD from B towards the plane of the ecliptic, and that any force acted

acted upon it in the direction BA to draw it from the plane of the ecliptic. The moons motion in its orbit and the force BA together would make the moon deſcribe BN. Now BN makes a leſs angle with EC than BF makes with it; for the angle BNC is leſs than the angle BFC: becauſe BFC is the external angle at the baſe of the triangle BFN, and conſequently is bigger than BNC, which is one of the internal and oppoſite angles. Euc. b. I. prop. 16. Therefore if the moon is drawn from the plane of the ecliptic, whilſt it is deſcending thither, the plane of its orbit will be changed ſo as to make a leſs angle with the plane of the ecliptic than it did before; that is, the inclination of the orbit will be diminiſhed.

435. *If the ablatitious force draws the moon from the plane of the ecliptic, whilſt it is riſing from the plane, or if the ſame force draws the moon towards the plane of the ecliptic, whilſt it is deſcending towards the plane, the inclination of the lunar orbit will be encreaſed.*

LP, Plat. XXX. fig. 7, is the plane of the ecliptic, and let HI be the plane of the lunar orbit. Firſt, as the moon moves from I towards H in its orbit, it riſes from LP the plane of the ecliptic: if at the ſame time it is drawn from the plane of the ecliptic by any force HG or IM perpendicular to the plane of its orbit; the compound motion which is produced by the moons motion in its orbit IH and by this force together, will be in the diagonal IG. Thus the plane of the moons orbit will be changed from IH to IG: and IG makes a greater angle with LP than IH makes with it: for the angle GIL is evidently greater than HIL. Therefore the plane IG, in which the moon is made to move by this force, is more inclined to the plane of the ecliptic than the plane IH, in which it moved before; that is, the inclination of the lunar orbit is encreaſed.

Secondly, if the moon was deſcending towards the plane of the ecliptic LP, in the direction GMP, and any force, as GH perpendicular to the plane of its orbit, was to draw it towards the plane of the ecliptic; the moons motion would then be a compound one in the line GI, which is the diagonal of a parallelogram, whoſe ſides are in the directions GP and GH. But GI makes a greater angle with LP the plane of the ecliptic than GP makes with it: for GIL is the external angle at the baſe of the triangle GIP, and GPL is one of the internal oppoſite angles. Euc. b. I. prop. 16. Therefore the plane GI, in which this force makes the moon move, is more inclined to the plane of the ecliptic LP, than the plane GP, in which the moon was moving before; that is, the inclination of the orbit is encreaſed.

436. *The*

436. *The inclination of the lunar orbit to the plane of the ecliptic is not changed at all, when the line of the nodes paſſes through the ſyzygies.*

Before we prove this propoſition and thoſe which follow, we will explane Plat. XXXI. fig. 1, and will ſhew that if the line of the nodes had no motion at all, yet as the lunar orbit is carryed round the ſun along with the earth, this line will ſometimes paſs through the ſyzygies, and ſometimes through the quadratures, and will ſometimes be in an intermediate ſituation in reſpect of the ſun.

Let S be the ſun and the circle TRBD, which encompaſſes it, be the earths orbit; then the plane of this circle, that is, the plane of the paper on which the circle is drawn, will be the plane of the ecliptic, by propoſition 28. The moons orbit is inclined to the plane of the ecliptic. Therefore as in looking at the figure we view the plane of the ecliptic directly, we muſt view the plane of the moons orbit obliquely. Now as the inequalities, which we are here to deſcribe, do not depend upon the ſhape of the moons orbit, we may ſuppoſe that orbit to be a circle having the earth in the center of it. Conſequently, if we would repreſent the moons orbit in the figure now before us, it muſt be repreſented by a circle viewed obliquely. But a circle viewed obliquely appears like an ellipſis. Thus if the earth is at T, the ellipſis NQ*n*L will repreſent the lunar orbit: where half this orbit NL*n* is ſuppoſed to be above the plane of the paper, and the other half *n*QN below it: a circle viewed in this ſituation would appear elliptical, N and *n* are the two nodes or points where the moons orbit croſſes the plane of the ecliptic, and conſequently N*n* is the line of the nodes. When the earth is at T, the line of the nodes paſſes through the ſyzygies: for the moon would be in conjunction at *n* and in oppoſition at N. If the line of the nodes was immoveable, or was always directed towards the ſame point in the heavens, yet by the motion of the earth and lunar orbit with it round the ſun, this line would be in different ſituations in reſpect of the ſun. For upon ſuppoſition that N*n* is immoveable, or is always directed towards the ſame part of the heavens, wherever the earth is, it will always be parallel to itſelf. So that when the earth is at R, N*n* the line of the nodes will paſs through the quadratures: for the moon is in quadrature at N or *n*, when the earth is at R. In like manner, if the earth was at B, N*n* the line of the nodes will paſs through the ſyzygies: and if the earth is at D, the ſame line will paſs through the quadratures. If the earth is at P the line of the nodes N*n* is in the octants after the quadratures, that is, when the moon after paſſing the quadratures has deſcribed an octant or eighth part of a circle, it will fall into one

one of the nodes: for when the earth is at P, the quadratures are Q and L; and when the moon has moved 45 degrees from Q to *n*, it falls into one node; and when it has moved 45 degrees from L to N, it falls into the other node. If the earth is at M, the line of the nodes N*n* is in the octants after the syzygies. For when the moon has moved through an eighth part of its orbit or 45 degrees from C the conjunction, it falls into the node N: and when it has moved 45 degrees from O the oppoſition, it falls into the other node *n*. In like manner, if the earth was in the middle between B and D, the line of the nodes would be in the octants after the quadratures: and if the earth was in the middle between D and T, the line of the nodes would be in the octants after the syzygies.

Having thus explaned the figure, we will now go on to ſhew, that if the earth is at T, ſo that the line of the nodes paſſes through the syzygies, the ablatitious force will not change the inclination of the lunar orbit to the plane of the ecliptic. When the earth is at T, the ablatitious force acts in the line NT*n*S, by propoſition 408. But N*n* is in this line. Therefore the ablatitious force acts in the line of the nodes N*n*, when the nodes are in the syzygies. But as the line of the nodes is the common interſection of the plane of the moons orbit and plane of the ecliptic, this line is in the plane of the lunar orbit: and conſequently, when the nodes are in the syzygies, the ablatitious force acts in the plane of the lunar orbit. Now no force, that acts in the plane of the lunar orbit, can draw the moon out of that plane; and no change can be made in the inclination of the lunar orbit, unleſs the moon is made to change the plane in which it moves. Therefore when the earth is at T, ſo that the line of the nodes is in the syzygies, the ablatitious force will not change the inclination of the lunar orbit. The ſame may be ſhewn, when the earth is at B: for then likewiſe the line of the nodes paſſes through the syzygies; and conſequently the ablatitious force will act in the plane of the moons orbit.

437. *When the line of the nodes paſſes through the quadratures, the inclination of the lunar orbit is diminiſhed, as the moon paſſes from quadrature to syzygy, and is encreaſed, as the moon paſſes from syzygy to quadrature.*

When the earth is at R, Plat. XXXI. fig. 1, the line of the nodes paſſes through the quadratures *n* and N. Now whilſt the moon moves from one quadrature *n* to the other quadrature N through the conjunction C, the ablatitious force acts towards the ſun S; and whilſt the moon moves through the other half of its orbit N*O*n, the ſame force acts from the ſun or towards G, by propoſition 415. But the nodes are in the quadra-

dratures. Therefore whilſt the moon moves from the node Υ through the conjunction C to the node N, the point S, towards which the ablatitious force is directed, is on the ſame ſide of the node with the moon; and conſequently this force will draw the moon towards the plane of the ecliptic; and ſo likewiſe whilſt the moon is moving from the node N through the oppoſition O to the node Υ, the point G, towards which the ablatitious force is directed, is on the ſame ſide of the node with the moon; and conſequently this force will ſtill draw the moon towards the plane of the ecliptic, by propoſition 433.

As the moon moves from the node or quadrature Υ to the conjunction or limit C; it riſes from the ecliptic; and the ablatitious force draws it towards the ecliptic. Therefore, by propoſition 434, the inclination of the orbit is diminiſhed. As the moon moves from the conjunction or limit C to the node N, it deſcends towards the ecliptic; and the ablatitious force draws it towards the ecliptic. Therefore, by propoſition 435, the inclination of the orbit is encreaſed. In like manner, as the moon moves from the node or quadrature N to the limit or oppoſition O, it departs from the ecliptic, and the ablatitious force, being directed towards G a point on the ſame ſide of the node with the moon, draws it towards the ecliptic. Therefore, by propoſition 434, the inclination of the orbit will be diminiſhed. And as the moon moves from the limit or oppoſition O to the node or quadrature Υ, it approaches towards the ecliptic, and the ablatitious force draws it towards the ecliptic. Therefore, by propoſition 435, the inclination of the orbit will be encreaſed.

Thus when the nodes and quadratures coincide, that is, when the line of the nodes paſſes through the quadratures; the inclination of the moons orbit is diminiſhed, whilſt the moon is moving from quadrature to conjunction or oppoſition, and is encreaſed, whilſt it is moving from conjunction or oppoſition to quadrature. So that in the whole revolution of the moon, the inclination of the orbit is twice diminiſhed and twice encreaſed: it is diminiſhed, whilſt the moon is moving through two quarters of its orbit from quadrature to ſyzygy, and is encreaſed, whilſt the moon is moving through the other two quarters from ſyzygy to quadrature. Therefore in ſuch a revolution the inclination of the orbit is equally encreaſed and diminiſhed; and conſequently at the end of the revolution the inclination will be the ſame, that it was at the beginning of it.

438. *When the line of the nodes is in the octants after the quadratures, the inclination of the lunar orbit to the plane of the ecliptic will be diminiſhed in one revolution of the moon.*

If

If the earth is at P, Plat. XXXI. fig. 1, N*n* the line of the nodes paſſes through the octants after the quadratures, as has been ſhewn in propoſition 436. The quadratures are Q and L. As the moon is moving from Q through the conjunction C to L the ablatitious force acts towards the ſun; and as it is moving from L through the oppoſition O to Q, the ſame force acts from the ſun, by propoſition 415. From hence it appears that as the moon is riſing from the node *n* to the limit *k*, the ablatitious force acts towards the ſun, or is directed towards a point S, which is on the ſame ſide of the node with the moon. Therefore this force will draw the moon towards the plane of the ecliptic, by propoſition 433, and by this means, becauſe the moon is riſing from the ecliptic, the inclination of the orbit will be diminiſhed, by propoſition 434. As the moon is moving from the limit *k* 45 degrees to the quadrature L, it deſcends towards the ecliptic: the ablatitious force is ſtill directed towards S, a point on the ſame ſide of the node with the moon, and conſequently, by propoſition 433, this force draws the moon towards the plane of the ecliptic. Therefore, by propoſition 435, ſince the moon is moving towards the ecliptic and is drawn towards it, the inclination of the orbit will be encreaſed. But as the moon is moving through the next 45 degrees from the quadrature L to the node N, it is in that half of its orbit, which is fartheſt from the ſun: and for that reaſon, by propoſition 415, the ablatitious force acts from the ſun, or towards the point F. This point and the moon, when it is any where between L and N, are on contrary ſides of the node; and conſequently the ablatitious force, by propoſition 433, draws the moon from the plane of the ecliptic. But the moon, whilſt it is thus drawn from the ecliptic, is deſcending towards it, becauſe it is moving towards the node. Therefore, by propoſition 434, the inclination of the orbit to the plane of the ecliptic will be diminiſhed. Thus we have ſhewn that in half the revolution of the moon, or whilſt it is paſſing from one node to the other, through the conjunction, the inclination of the orbit will be more diminiſhed than encreaſed: for the inclination is diminiſhed, whilſt the moon is moving from *n* to *k* or 90 degrees, and whilſt it is moving from L to N or 45 degrees more; whereas this inclination is encreaſed only whilſt the moon is moving 45 degrees from *k* the limit to L the quadrature.

When the moon is paſſing from the node N through the oppoſition O to K the limit, the ablatitious force ſtill acts from the ſun, by propoſition 415: becauſe NK is in that half of the orbit, which is fartheſt from the ſun. And the moon is on the ſame ſide of the node N with the point F, towards which this force is directed. Therefore, by propoſition 433,

the moon will be drawn towards the plane of the ecliptic: but it is riſing from that plane, for it is moving from the node N to the limit K: and conſequently the inclination of the lunar orbit will be diminiſhed, by propoſition 434. The moon ſtill continues in that part of its orbit, which is fartheſt from the ſun, whilſt it moves from the limit K through 45 degrees to the quadrature Q. Therefore, by propoſition 415, the ablatitious force acts from the ſun towards the point F. And the moon is on the ſame ſide of the node *n* with the point F, for which reaſon, by propoſition 433, the ablatitious force draws it towards the plane of the ecliptic. But as it has paſſed the limit K, and is moving towards the node *n*, it deſcends towards the plane of the ecliptic; and conſequently, as it is drawn at the ſame time towards that plane, the inclination of the orbit will be encreaſed. As ſoon as the moon has paſſed the quadrature Q, it gets into the other part of the orbit QCL, which is neareſt to the ſun, and then, by propoſition 415, the ablatitious force acts towards the point S. But the point S and the moon are on contrary ſides of the node *n*, whilſt the moon paſſes from Q to *n*. Therefore, by propoſition 433, the ablatitious force will draw the moon from the plane of the ecliptic. But the moon in deſcribing the arc Q*n* of 45 degrees, deſcends towards the plane of the ecliptic; becauſe *n* is the node. And ſince whilſt the moon is thus deſcending toward that plane, it is drawn from it, the inclination of the orbit will be diminiſhed, by propoſition 434. Thus we have ſhewn farther that in the other half of this revolution, whilſt the moon is paſſing from the node N through the oppoſition O to the other node *n*, the inclination of the orbit will be more diminiſhed than encreaſed. For the inclination is diminiſhed whilſt the moon deſcribes the arc NK of 90 degrees, and the arc Q*n* of 45 degrees, whereas it is encreaſed only whilſt the moon deſcribes the arc KQ of 45 degrees.

Conſequently in this revolution the inclination of the lunar orbit will be diminiſhed upon the whole: becauſe it is more diminiſhed than encreaſed. For it is diminiſhed, whilſt the moon deſcribes 90+45+90 +45=270 degrees, and is encreaſed only whilſt the moon deſcribes 45+45=90 degrees.

439. *When the line of the nodes is in the octants after the ſyzygies, the inclination of the lunar orbit to the plane of the ecliptic will be encreaſed in one revolution of the moon.*

If the earth is at M, Plat. XXXI. fig. 1. the line of the nodes N*n* paſſes through the octants after the ſyzygies, as has been ſhewn under propoſition 436. When the moon is at the node *n* it is in that half of the orbit

LOQ,

LOQ, which is fartheſt from the ſun, and conſequently the ablatitious force, by propoſition 415, will act from the ſun or towards the point H. This point and the moon are on contrary ſides of the node *n*, whilſt the moon paſſes from the node *n* to the quadrature Q through 45 degrees. Therefore the ablatitious force will then draw the moon from the plane of the ecliptic, by propoſition 433. But the moon is riſing from the plane of the ecliptic, whilſt it deſcribes the arc *n*Q, becauſe it is paſſing from the node towards the limit *k*. Conſequently ſince the moon is riſing from that plane and at the ſame time is drawn from it, the inclination of the orbit will be encreaſed, by propoſition 435. As ſoon as the moon has paſſed the quadrature Q, it gets into the other half of the orbit QCL, which is neareſt to the ſun; and then by propoſition 415, the ablatitious force is directed towards the ſun. The point S, towards which this force is directed, and the moon are on the ſame ſide of the ecliptic, and conſequently, by propoſition 433, the ablatitious force draws the moon towards the ecliptic. But the moon, whilſt it paſſes through 45 degrees from Q to *k*, is ſtill moving towards the limit or from the ecliptic. Therefore, ſince it moves from the plane of the ecliptic and is drawn towards it, the inclination of the orbit will be diminiſhed, by prop. 434. As the moon paſſes from *k* the limit to N the node, it deſcends towards the ecliptic, and the ablatitious force is ſtill directed towards S a point on the ſame ſide of the node with the moon. Therefore ſince the moon is deſcending towards the plane of the ecliptic and is drawn towards it, the inclination of the orbit will be encreaſed. Thus the inclination of the orbit in half a revolution of the moon will be more encreaſed than diminiſhed. The inclination will be encreaſed whilſt the moon is paſſing through 45 degrees of its orbit from the node *n* to the quadrature Q, and through 90 degrees from the limit *k* to the node N: whereas it will be diminiſhed only whilſt the moon is paſſing through 45 degrees from the quadrature Q to the limit *k*.

When the moon is paſſing from the node N to the quadrature L through 45 degrees of its orbit, the ablatitious force ſtill acts towards S, becauſe the moon is in that half of its orbit which is next the ſun. But this point S and the moon are then on contrary ſides of the node N. Therefore, by propoſition 433, the ablatitious force will draw the moon from the plane of the ecliptic. And ſince the moon, whilſt it is thus drawn from the plane of the ecliptic, is departing from that plane or is moving from the node towards the limit K, the inclination of the orbit will keep encreaſing till the moon arrives at L, by propoſition 415. As ſoon as the moon has paſſed the quadrature L, it gets into the other half of the orbit

LOQ which is fartheſt from the ſun; and then, by propoſition 415, the ablatitious force acts from the ſun towards the point H. The moon is on the ſame ſide of the node N with the point H. Therefore, by propoſition 433, the ablatitious force will draw the moon towards the plane of the ecliptic. But from the quadrature L to the limit K, which is an arc of 45 degrees, the moon keeps departing from the plane of the ecliptic: and as it is in the mean time drawn towards that plane, the inclination of the orbit will be diminiſhed, by propoſition 434. As the moon paſſes from the limit K to the node *n* through 90 degrees of its orbit, the ablatitious force ſtill acts the ſame way towards the point H: and this point is on the ſame ſide of the node *n* with the moon. Therefore the ablatitious force will draw the moon towards the plane of the ecliptic. But the moon, whilſt it is thus drawn towards the plane of the ecliptic, is approaching towards the ſame plane, for it is paſſing from the limit to the node: and conſequently the inclination of the orbit will be encreaſed, by propoſition 435. Thus in this other half of the revolution the inclination of the moons orbit will be more encreaſed than diminiſhed. The inclination will be encreaſed, whilſt the moon is paſſing through 45 degrees from N the node to L the quadrature, and through 90 degrees from K the limit to *n* the node; whereas it will be diminiſhed only whilſt the moon is paſſing through 45 degrees from N the node to L the quadrature.

From hence it appears, that in one revolution of the moon, whilſt the nodes are in the octants after the ſyzygies, the inclination of the lunar orbit to the plane of the ecliptic will be encreaſed upon the whole; becauſe it will be more encreaſed than diminiſhed. It will be encreaſed, whilſt the moon is paſſing through 45+90+45+90=270 degrees, and will be diminiſhed, only whilſt the moon is paſſing through 45+45=90 degrees.

440. *The inclination of the lunar orbit to the plane of the ecliptic is the the leaſt of all, when the line of the nodes is in quadrature and the moon is in ſyzygy.*

If the earth is at T, Plat. XXXI. fig. 1, the line of the nodes N*n* is in ſyzygy, and when the earth comes to R, the line of the nodes is in quadrature. In the intermediate ſituation of the earth at P, the line of the nodes is in the octants after quadrature. But when the line of the nodes is in the octants, after quadrature the inclination of the orbit is diminiſhed, by propoſition 438. And ſo in any other intermediate revolution of the moon, whilſt the earth is paſſing from T to R, the inclination of the orbit will be diminiſhed; though not ſo much as when the earth is at P.

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Therefore ſince the inclination of the lunar orbit keeps decreaſing as the earth paſſes from T to R, the inclination will be leaſt in that revolution of the moon when the earth is at R, or when the line of the nodes paſſes through the quadratures.

But the inclination of the lunar orbit is not the ſame in every part of the moons revolution, whilſt the line of the nodes is in quadrature: for the inclination is diminiſhed in that revolution, whilſt the moon paſſes from quadrature to ſyzygy, and is encreaſed, whilſt the moon paſſes from ſyzygy to quadrature, by propoſition 537. Conſequently in that revolution it will be the leaſt, when the moon is in ſyzygy.

In like manner and for the ſame reaſons the inclination of the lunar orbit will decreaſe, in every intermediate ſituation of the earth, whilſt it is paſſing from B, where the line of the nodes is in ſyzygy, to D, where it is in quadrature.

441. *The inclination of the lunar orbit to the plane of the ecliptic is greateſt of all, when the line of the nodes is in ſyzygy.*

Whilſt the earth is paſſing from R to B, Plat. XXXI. fig. 1, that is, whilſt the line of the nodes is paſſing from quadrature to ſyzygy, the inclination of the lunar orbit keeps encreaſing. One intermediate ſituation of the earth is M, when the line of the nodes is in the octants after ſyzygy, and then, by propoſition 439, in one revolution of the moon, the inclination of the orbit is encreaſed upon the whole: and ſo it will be, for the ſame reaſons in any other intermediate ſituation. But ſince the inclination of the orbit keeps encreaſing, whilſt the line of the nodes paſſes from quadrature to ſyzygy, this inclination will be the greateſt, when the line of the nodes comes to the ſyzygy.

In the ſame manner we may ſhew that the inclination of the lunar orbit will encreaſe, whilſt the earth is paſſing from D, where the line of the nodes is in quadrature, to T, where it is in ſyzygy.

442. *The lunar nodes move in antecedentia, when the line of the nodes is in quadrature.*

If the earth is at R, Plat. XXXI. fig. 1, N*n* the line of the nodes is in quadrature. The nodes *n* and N are the two points where the moon in deſcribing its orbit croſſes the plane of the ecliptic. Theſe two points, if they were immoveable, would be oppoſite to one another; ſo that the moon ſetting out from the node *n* and moving through C muſt deſcribe half its orbit before it could come to the other node N, that is, it muſt deſcribe 180° in its orbit, before it would croſs the plane of the ecliptic again.

again. This would be the caſe if there was no force, that acted upon the moon to draw it out of the plane of its orbit. But when the nodes *n* and N are in the quadratures, the ablatitions force acts towards the ſun S, whilſt the moon is moving from the node or quadrature *n* through C the conjunction to the other node N: and the ſame force acts from the ſun toward G, whilſt the moon is moving through the other half of its orbit, from the node or quadrature N through the oppoſition O, to the other node *n*, by propoſition 415. Thus when the moon is in one half of its orbit *n*CN, it is on the ſame ſide of the nodes with the point S, towards which the ablatitious force acts: and likewiſe when it is in the other half NO*n*, it is on the ſame ſide of the nodes with the point G, towards which the ablatitious force then acts. Therefore, by propoſition 433, the moon, as it moves round, will be conſtantly drawn towards the plane of the ecliptic. By this force the nodes N and *n* will be carryed in antecedentia, or will move from eaſt to weſt, that is, they will move in a contrary direction to the moon. This is what we are to prove.

The moon, as it moves from the node *n*, keeps riſing from the plane of the ecliptic, till it comes to the limit C, and then deſcends towards the plane of the ecliptic; and the place, where it comes to the plane of the ecliptic ſo as to croſs it, is the next node. Now ſince the moon is conſtantly drawn towards the plane of the ecliptic by the ablatitious force, it will come to that plane and croſs it ſooner, than it would have done, if no ſuch force had acted upon it. But if no ſuch force had acted upon the moon, after it had left the node *n* it muſt have deſcribed 180° in its orbit, before it could croſs the ecliptic again. Therefore this force, after the moon has left the node *n*, will bring it to the plane of the ecliptic again, before it has deſcribed 180°. And ſince the place where the moon croſſes the plane of the ecliptic again is its next node; this next node will not be 180° from *n*. If the nodes *n* and N had been immoveable they would have been 180° aſunder. But when the moon has left the node *n*, the ablatitious force brings it to the next node, before it has deſcribed 180°. Therefore the next node will be nearer to *n* than N is: and conſequently the next node will have changed its place or will have moved towards *n*, whilſt the moon is moving from *n*: ſo that the node will move in a contrary direction to the moon or from eaſt to weſt.

In like manner, if the moon was to ſet out from the node N, it keeps departing from the plane of the ecliptic, till it comes to the limit O; and then it approaches that plane, ſo as to croſs it again at the next node. If the next node was immoveable it would be at *n* 180° from N: and the moon after leaving one node muſt move 180° before it came to the next.

But

But the ablatitious force on this ſide of the orbit draws the moon towards the plane of the ecliptic: and ſuch a force will make the moon croſs the plane of the ecliptic ſooner, than it would have done otherwiſe, or before it has moved 180°. Therefore the next node will not be 180° from N. So that the ablatitious force muſt bring the next node nearer to N, than it would have been, if no ſuch force had acted, that is, the next node moves towards N, whilſt the moon is moving from N. Therefore the node moves in a contrary direction to the moon, or in antecedentia, or from eaſt to weſt.

443. *The lunar nodes reſt, when the line of the nodes in in ſyzygy.*

The ablatitious force acts in the plane of the lunar orbit, when the earth is at T or B, Plat. XXXI. fig. 1, that is, when the line of the nodes paſſes through the ſyzygies, by propoſition 436. But a force, which acts in the plane of the lunar orbit, will never draw the moon out of that plane, or make it change the plane of its motion. Therefore the plane of the lunar orbit is immoveable, when the line of the nodes paſſes through the ſyzygies. Now the plane of the ecliptic is immoveable, and when the line of the nodes paſſes through the ſyzygies, the plane of the lunar orbit is immoveable; and conſequently the common interſection of theſe planes will then be immoveable. But the nodes are in this common interſection. Therefore the ablatitious force will not move the nodes, when the line of the nodes paſſes through the ſyzygies.

444. *The lunar nodes move in antecedentia, when the line of the nodes is between quadrature and ſyzygy: but their regreſs in one revolution of the moon is leſs, than when that line is in quadrature.*

When the line of the nodes is in ſyzygy the nodes reſt, by propoſition 443, and when that line is in quadrature, they move backwards during the whole revolution of the moon, by prop. 442. Therefore when the line of the nodes is in any intermediate ſituation, the nodes will be in an intermediate ſtate between reſting and moving backwards ſo faſt as they do, when that line is in quadrature, that is, their regreſs will be leſs.

This we may ſhew in one or two particular inſtances. Let the earth be at P, Plat. XXXI. fig. 1, then N*n* the line of the nodes will paſs through the octants after the quadratures. The ablatitious force acts towards the ſun S, whilſt the ſun is any where in the half QCL of its orbit, by propoſition 415. Suppoſe then the moon to be paſſing from the node *n* to the quadrature L: in this caſe the point S, towards which the ablatitious force is directed, is on the ſame ſide of the nodes with the

moon

moon. Therefore, by propoſition 433, the moon will be conſtantly drawn towards the plane of the ecliptic. And a force, which draws the moon towards the plane of the ecliptic, will make it croſs that plane ſooner than it would have done otherwiſe. But the place where the moon croſſes the plane of the ecliptic is the next node. Therefore, whilſt the moon is moving from *n* to L, the ablatitious force brings the next node nearer to *n*, than it would have been otherwiſe; that is, the ablatitious force makes the next node move in antecedentia, or in a contrary direction to the moon. As ſoon as the moon has paſſed the quadrature L, it gets into the other half of its orbit LOQ, and then the ablatitious force acts from the ſun towards the oppoſite point F, by propoſition 415. But whilſt the moon is paſſing from the quadrature L to the next node, the point F and the moon are on contrary ſides of the node. Therefore the ablatitious force draws the moon from the plane of the ecliptic, by propoſition 433. Now any force, which draws the moon from the plane of the ecliptic, will make it longer before it croſſes that plane, than it would have been otherwiſe, or will make it longer before it arrives at the next node. So that the moon in conſequence of ſuch a force, muſt move farther before it comes to the next node, than it would have moved, if ſuch a force had not acted upon it. Therefore the next node will by this means be removed to a greater diſtance from L; that is, the next node will move on in conſequentia or in the ſame direction with the moon. But when the moon has paſſed the next node, it gets on the ſame ſide of the node with the point F, towards which the ablatitious force is directed, and continues on the ſame ſide, till it arrives at the quadrature Q. Therefore, during this part of the moons revolution, the ablatitious force will draw the moon towards the plane of the ecliptic; and conſequently will make the next node *n* move in antecedentia as before. After the moon has paſſed the quadrature Q, the ablatitious force acts towards the ſun S, and then the moon, whilſt it is deſcribing the arc Q*n*, and the point S are on contrary ſides of the node: for which reaſon the moon will be drawn from the plane of the ecliptic, and the node will move in conſequentia, whilſt the moon is deſcribing the arc Q*n*; as it did before whilſt the moon was deſcribing the arc LN. Thus the nodes move in antecedentia, whilſt the moon is deſcribing the arcs *n*L and NQ, or 135+135=270 degrees, and move in conſequentia, only whilſt it is deſcribing the arcs LN and Q*n*, or 45+45=90 degrees. Thererefore the regreſs of the nodes in this revolution will exceed their progreſs; and conſequently in the whole revolution, they will have moved backwards. But then as they move forwards in part of the revolution, their regreſs will be leſs upon the whole,

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than it is when the line of the nodes is in quadrature, becauſe then they move backwards during the whole revolution of the moon, by propoſition 442.

If the earth is at M, the line of the nodes N*n* is in the octants after the ſyzygies. When the moon is any where in the half QCL of its orbit, the ablatitious force is directed towards the ſun S: but when it is any where in the other half LOQ of its orbit, the ablatitious force is directed from the ſun towards the point H, by prop. 415. Suppoſe the moon to be paſſing from the node *n* to the quadrature Q, the ablatitious force is then directed towards the point H. But this point and the moon are on contrary ſides of the node: and conſequently the ablatitious force will draw the moon from the plane of the ecliptic, by propoſition 433. Now as the moon is drawn from the plane of the ecliptic, it will be longer, or muſt move farther, before it croſſes that plane, or before it comes to the next node. Therefore, whilſt the moon is moving from the node *n* to Q, the next node moves farther from *n*, or moves in the ſame direction with the moon, that is, it moves in conſequentia. But as ſoon as the moon has paſſed the quadrature Q, it gets into the other half of its orbit, where the ablatitious force acts towards S; and then the moon is on the ſame ſide of the node with this point. Therefore the ablatitious force will draw the moon towards the plane of the ecliptic. This will continue till the moon arrives at the next node. But a force, which draws the moon towards the plane of the ecliptic, will bring the moon to that plane, and make it croſs it ſooner, than it would have done otherwiſe; that is, whilſt the moon is moving from Q, the next node will move towards Q or in antecedentia. When the moon has paſſed the next node N, till it gets to L, the ablatitious force will act towards S; but then the moon and this point S are on contrary ſides of the node. Therefore the ablatitious force will draw the moon from the plane of the ecliptic, and will make the next node move forwards, as it did, whilſt the moon was deſcribing the arc *n*Q. As ſoon as the moon has paſſed the quadrature L, the ablatitious force acts towards F, which is on the ſame ſide of the node with the moon: conſequently the moon will be drawn towards the plane of the ecliptic, till it comes to the next node. Therefore the next node will move backwards, whilſt the moon is deſcribing the arc L*n*, in the ſame manner as it did, whilſt the moon was deſcribing the arc QN. Thus in one revolution of the moon, when the line of the node is in the octants after the ſyzygies, the nodes move backwards, whilſt the moon is deſcribing the arcs QN and L*n*, or 135+135=270 degrees, and move forwards, only whilſt it is deſcribing the arcs *n*Q and NL, or 45+45=90 degrees. Therefore in ſuch a revolution the regreſs of the nodes will ex-

ceed their progreſs: ſo that upon the whole the nodes move in antecedentia in this revolution. But then in this revolution the regreſs will not be ſo great, as when the line of the nodes is in quadrature: becauſe in part of this revolution they move forwards; whereas when the line of the nodes is in quadrature, they move backwards during the whole revolution, by propoſition 442.

Thus we have ſeen that the lunar nodes move backwards in every revolution of the moon, unleſs the line of the nodes is in ſyzygy, and then they reſt. In about 19 years the nodes perform one revolution: ſo that in that time they will have paſſed through every part of the ecliptic.

445. *All the inequalities, which ariſe from the ablatitious force, are ſomething greater, when the moon is in conjunction, than when it is in oppoſition.*

This follows from propoſition 416, where we ſhewed that the ablatitious force, when the moon is in conjunction, is ſomething greater, than when it is in oppoſition.

CHAP. XVIII.

Of the preceſſion of the equinoxes, and the figure of the planets.

446. *If there was a ring of matter adhering to the earth round the equator; the inclination of this ring would be variable, and its nodes would move in antecedentia.*

IF there were many moons, inſtead of one, all moving in the ſame circular orbit CLOQ, Plat. XXIX. fig. 10, the ſuns action would produce the ſame inequalities in each of them. If the number of theſe moons was to be encreaſed, till they filled the whole orbit CLOQ, and if the ſize of each moon was to be diminiſhed, till it was no bigger than the particle of a fluid, ſo that all theſe moons together were to make up a fluid ring CLOQ; each particle of this ring, becauſe it is a ſmall moon, would be ſubject to all the lunar inequalities, that could happen in a circular orbit. Particularly, ſuppoſe this fluid ring to be in the plane of the equator; then the ring, or the orbit of each moon, will be inclined to the plane of the ecliptic; and conſequently the ring will be ſubject to the inequalities of latitude; as deſcribed in the foregoing chapter; that is, the inclination of the ring will be variable, and its nodes will move

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in antecedentia. If this fluid was to harden, the ſtiffneſs of the ring would not hinder theſe inequalities, its inclination would ſtill be variable, and its nodes would move backwards. Suppoſe farther that the diameter of this ring was to be diminiſhed, till it was no greater than the earths diameter, and that the ring adhered to the earths equator; the ſun would continue to act upon the ring, as before, and would therefore ſtill produce the ſame inequalities. Indeed if the ring adhered to the earth, the ſame action of the ſun would change the rings inclination leſs, and would make its nodes move ſlower, than if it was detached from the earth. For if the ring was detached from the earth, the quantity of matter to be moved by the ſuns action would be no more than what is contained in the ring itſelf; whereas if the ring adheres to the earth, it is impoſſible for the ring to move without moving the earth along with it. Therefore as the quantity of matter to be moved is greater, the velocity will be leſs, when the ring adheres to the earth, than when it is detached from the earth.

Theſe inequalities of latitude would be produced in ſuch a ring, not only by the ſuns action upon it, but likewiſe by the moons action. For if S is the moon, and CLOQ was the orbit of another ſatellite revolving round the earth at a leſs diſtance than the moon; then the moon S would act upon this ſatellite and diſturb its motion, juſt as the ſun acts upon the moon. Therefore if the orbit of the ſatellite was inclined to the plane of the moons orbit, the moon would produce in the ſatellite all the inequalities of latitude. Now the moons orbit makes but a ſmall angle with the ecliptic, and for the preſent we may conſider the moon as moving in the ecliptic. Therefore if the ſatellite revolves in the plane of the equator, its orbit will be inclined to the plane of the moons orbit. Suppoſe then the whole orbit of the ſatellite to be filled with ſo many ſatellites, and thoſe ſo ſmall, that all of them together may make a fluid ring; ſuppoſe this ring to ſtiffen, and the diameter of it to diminiſh, till it adheres to the earth round the equator, the moon will produce the ſame inequalities in this ring that the ſun does, that is, the moon, as well as the ſun, will make the inclination of the ring variable, and will make the nodes of it move backwards.

447. *If there was a ring of matter adhering to the earth round the equator, the equinoctial points would be the nodes of this ring.*

The nodes of ſuch a ring are the points where it croſſes the plane of the ecliptic. But ſince the ring adheres to the equator, the points where it croſſes the plane of the ecliptic, are the points where the equa-

tor croſſes that plane: and theſe are the equinoctial points, by propoſition 35.

448. *If there was a ring of matter adhering to the earths equator, the line of the nodes of this ring would be in ſyzygy in reſpect of the ſun, when the ſun is in the equator; and would be in quadrature in reſpect of the ſun, when the ſun is in either tropic.*

The line of the nodes of ſuch a ring is the common interſection of the plane of the ring and plane of the ecliptic. When the ſun is in the equator, it is in this common interſection, and therefore is in the line of the rings nodes; that is, the line of the rings nodes is then in ſyzygy in reſpect of the ſun.

When the ſun is in either tropic it is 90° from the equinoctial points. Therefore at that time the equinoctial points, which are the nodes of the ring, are in quadrature in reſpect of the ſun.

449. *If there was a ring of matter adhering to the earths equator, the line of the nodes of this ring would be in ſyzygy in reſpect of the moon, when the moon is in the equator, and would be in quadrature in reſpect of the moon, when the moon is in either tropic.*

This propoſition is proved in the ſame manner as the laſt; only what was there ſaid of the ſun is here to be applyed to the moon.

450. *The inclination of the earths axis to the plane of the ecliptic would be greateſt every year at the ſolſtices and leaſt at the equinoxes, if there was a ring of matter adhering to the earths equator.*

The ſun would produce the lunar inequalities of latitude in ſuch a ring, by propoſition 446. Therefore the inclination of that ring, or of the equator, to which the ring adheres, would be leaſt of all, when the nodes of the ring are in quadrature in reſpect of the ſun, by propoſition 440, that is, when the ſun is in either tropic, by propoſition 448, or at the time of the ſolſtices. But the inclination of the earths axis to the plane of the ecliptic, is the complement of the inclination of the equator, by propoſitions 246, 247. Therefore at the ſolſtices, when the inclination of the equator is the leaſt, its complement, or the inclination of the axis, will be the greateſt.

The nodes of the ring are in ſyzygy in reſpect of the ſun, at the equinoxes, by propoſition 448. But ſince the ſun, by propoſition 446, produces the inequalities of latitude in ſuch a ring, the inclination of the ring or of the equator, to which it adheres, will then be the greateſt, by

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propoſition 441. Now the inclination of the axis is the complement of the inclination of the equator, by propoſitions 246, 247. Therefore at the equinoxes, when the inclination of the equator is the greateſt, its complement, or the inclination of the earths axis to the plane of the ecliptic, will be the leaſt.

451. *The inclination of the earths axis to the plane of the ecliptic would be greateſt every month, when the moon is in either tropic, and leaſt, when the moon is in the equator; if there was a ring of matter adhering to the earths equator.*

The moon would produce all the inequalities of latitude in ſuch a ring, by propoſition 446. Therefore the inclination of the ring, that is, of the equator, would be leaſt of all when the nodes of the ring are in quadrature in reſpect of the moon, or by propoſition 449, when the moon is in either tropic, and greateſt of all, when the nodes of the ring are in ſyzygy in reſpect of the moon, or when the moon is in the equator. But, by propoſitions 246, 247, the inclination of the axis is the complement of the inclination of the equator. Therefore in every month, when the moon is in either tropic, the inclination of the earths axis will be the greateſt, becauſe the inclination of the equator is then the leaſt; and when the moon is in the equator, the inclination of the axis will be the leaſt, becauſe the inclination of the equator is the greateſt.

452. *The action of the ſun and moon, or the force of gravity, would make the equinoctial points move in antecedentia; if there was a ring of matter adhering to the earths equator.*

The action of the ſun and moon upon ſuch a ring would make its nodes move backwards, by propoſition 446. But the nodes of ſuch a ring are the equinoctial points, by propoſition 447. Therefore the action of the ſun and moon, or the force of gravity, would cauſe a preceſſion of the equinoxes, if there was ſuch a ring.

453. *If the earth ſwells out at the equator, this exceſs of matter will ſupply the place of a ring.*

We have ſhewn in propoſition 447, that the attraction of the ſun and moon, or the force of gravity, would cauſe the preceſſion of the equinoxes; if there was a ring of matter adhering to the earths equator. But as there is no ſuch ring, we will next enquire whether the earth does not ſwell out at the equator, ſo that the exceſs of matter there, though it is not exactly in the ſhape of a ring, may ſupply the place of one. For if there

is

is ſuch an exceſs of matter, then the action of the ſun and moon upon it, or the force of gravity, will be the cauſe of the preceſſion of the equinoxes, or of the motion of the equinoctial points in antecedentia.

454. *In the daily rotation of the earth round its axis, the centrifugal force diminiſhes the weight of bodies at the equator more, than in any other place on the earths ſurface, in the duplicate proportion of the ſemidiameter to the coſine of the places latitude.*

Let PE*p*e, Plat. XXXI. fig. 2, be the earth, P*p* the axis, E*e* the equator. As the earth revolves upon its axis, every part of it, except the two poles, deſcribes a circle, the plane of which is perpendicular to the axis, and the radius of which is the perpendicular diſtance of that part from the earths axis. Thus at the equator a body placed on the ſurface at E deſcribes a circle in the daily rotation round P*p*, EC perpendicular to P*p* is the ſemidiameter of the circle; and becauſe the ſemidiameter is perpendicular to the axis, the plane of the circle will likewiſe be perpendicular to it. A body placed at A deſcribes a circle in the daily rotation; the ſemidiameter of this circle is AB: and this ſemidiameter, and conſequently the plane of the circle ſo deſcribed, is perpendicular to the axis P*p*. Now here we may obſerve that CE, the ſemidiameter of the circle deſcribed in the daily rotation by a body placed at the equator, is the ſemidiameter of the earth. But BA the ſemidiameter of the circle deſcribed by a body placed at A, or any where elſe on the earths ſurface, is the coſine of latitude: for EA or ECA is the latitude, and AP or ACP is the complement of latitude of the place A: but BA is perpendicular to PC: therefore BA is the ſine of PCA the complement of latitude, that is, BA is the coſine of latitude. In the daily rotation the whole earth, and conſequently each part of it, completes a revolution in the ſame time: ſo that the periodical time of a body at E, in a circle whoſe radius is CE, is equal to the periodical time of a body at A, in a circle whoſe radius is BA. But where the periodical times in circles are equal, the centrifugal forces are as the ſemidiameters, by propoſition 169 of mechanics. The body at E endeavours to fly off from C the center of its motion, and the body at A endeavours likewiſe to fly off from B the center of its motion, by propoſition 153 of mechanics: but this endeavour, or the centrifugal force, is as much greater in the body at E, than it is in the body at A, as the ſemidiameter CE is greater than the ſemidiameter BA. But CE is the ſemidiameter of the earth, and BA is the coſine of the latitude of the place A. Therefore the centrifugal force at the equator E is to the centrifugal force at any place A, as the ſemidiameter of the earth to the coſine of the places la-

latitude. The greater the centrifugal force is, the leſs will be the gravity of a body, which is acted upon by that force; or the more a body endeavours to depart from the earth, the more its gravity or tendency towards the earth will be diminiſhed. Therefore the gravity of a body at the equator is as much more diminiſhed, than at any other place on the earths ſurface, as a ſemidiameter of the earth is greater than the coſine of the places latitude.

Thus we have ſeen that the weight of bodies at the equator is more diminiſhed in the earths daily rotation, than at any other place on the earths ſurface: becauſe the centrifugal force at the equator is greater, than it is any where elſe. But if the centrifugal force at the equator E was equal to the centrifugal force at any other place A; ſuch equal centrifugal forces would diminiſh the weights of bodies unequally, upon account of the different directions in which thoſe equal forces act. Let any line as *Ab* repreſent the centrifugal force. The body at A endeavours to depart with this whole force from B the center of the circle, which the place A deſcribes in the daily rotation. But then this whole force does not diminiſh the bodys gravity. For the body does not gravitate towards B, but towards C the earths center in the direction AC: and conſequently only ſuch a part of the centrifugal force, as is oppoſite to the gravitation in the direction AC, will diminiſh the weight of the body. Reſolve the centrifugal force *Ab* into two parts, by making it the diagonal of a parallelogram, one of whoſe ſides is *Aa* oppoſite to AC, and the other is *ab* perpendicular to *Aa*. Then, by propoſition 20 of mechanics, that part of the centrifugal force, which diminiſhes the gravity of a body placed at A, is to the whole centrifugal force as *Aa* to *Ab*. But if a body was placed at the equator E it would gravitate in the direction EC towards C the earths center: and the centrifugal force makes it endeavour to depart from C, becauſe C is likewiſe the center of the circle, which the body deſcribes in the daily rotation. Therefore, when a body is placed at the earths equator E, as the whole centrifugal force carrys it off from the earths center C, this whole force will diminiſh its gravity. From hence it appears that the force, which diminiſhes the weight of a body placed at E, is to the force, which diminiſhes the weight of a body placed at A; as the whole centrifugal force *Ab* is to a part of it *Aa*: and conſequently by the earths rotation round its axis the weight of a body at E is as much more diminiſhed, than the weight of a body at A, as *Ab* is greater than *Aa*; upon account of the different direction of the centrifugal force at E and A. Now *Ab* is to *Aa*, as CE the earths ſemidiameter is to BA the coſine of the latitude of the place A, which we thus prove. The triangles

Aab

Aab and ABC are ſimilar: becauſe the angles at A are equal as they are vertical; Euc. b. I. prop. 15; and the angle at *a* is equal to the angle at B: becauſe they are both right ones, by the conſtruction. Therefore the third angle at *b* in one triangle is equal to the third angle at C in the other. Euc. b.I. prop. 32. corol. 2. Conſequently the ſides of theſe triangles are proportional, Euc. b.VI. prop. 4, *Ab* is to *Aa* as AC=EC is to BA. But by the different direction of the centrifugal force the weight of a body at E is more diminiſhed than the weight of a body at A in the proportion of *Ab* to *Aa*, that is, in the proportion of EC the ſemidiameter to BA the coſine of latitude.

Thus we have ſeen that the centrifugal force diminiſhes the force of gravity at the equator, more than at any other place A on the earths ſurface, in the proportion of the ſemidiameter to the coſine of latitude, upon two accounts: firſt; becauſe the centrifugal force at the equator is greater than at the place A in this proportion: and ſecondly; becauſe the centrifugal force is directly oppoſite to the force of gravity at the equator, but is oblique to it at any other place A upon the earths ſurface. Therefore upon both accounts together, the centrifugal force diminiſhes the weight of a body at E, more than the weight of a body at A, in the double or duplicate proportion of CE the ſemidiameter to BA the coſine of latitude; that is, the centrifugal force diminiſhes the weight of a body at E, as much more than it diminiſhes the weight of a body at A, as the ſquare of CE is greater than the ſquare of BA.

From hence it follows, that at P or *p* either of the poles, where the complement of latitude and conſequently the coſine of latitude is nothing, the centrifugal force will not at all diminiſh the weight of bodies. And indeed it is obvious, that if a body was placed at P, no centrifugal force, which ariſes from the earths daily rotation, could diminiſh the weight of that body: becauſe the pole P is the extremity of the axis P*p*, on which the daily rotation is performed: and for that reaſon, as the axis does not revolve, the point P will not revolve; ſo that at the pole there will be no centrifugal force ariſing from the daily rotation.

455. *The earth is an oblate ſpheriod; it ſwells out at the equator and ſinks down at the poles.*

If the whole earth was fluid, this fluid maſs if it had no rotation upon its axis, would form itſelf into a ſphere. For the parts of the fluid would not reſt till the preſſures or gravitations of the fluid were equal on all ſides: and theſe preſſures would not be equal till the columns of fluid from the center to the ſurface were of the ſame length. But when the

ſurface

ſurface of the maſs is every where at an equal diſtance from the center, the maſs becomes a ſphere. Therefore this is the figure, into which ſuch a maſs would form itſelf by its own gravity. Suppoſe then this fluid globe had a rotation round its axis: ſuch a rotation, by propoſition 454, would diminiſh the gravity of the fluid: and if the gravity was diminiſhed every where alike, the parts would ſtill continue in equilibrio, and the figure of the maſs would not be altered by the rotation round its axis. But the gravity is not diminiſhed every where alike by ſuch a rotation; the gravity is moſt diminiſhed at the equator; it is leſs diminiſhed in places that are between the equator and the poles; and at the poles it is not diminiſhed at all. Therefore the rotation will deſtroy the equilibrium of the parts, and will change the figure of the fluid maſs. The parts of the fluid at the equator will become lighter than the reſt, and the parts at the poles will be the heavieſt: ſo that any column of the fluid at the equator will preſs leſs downwards than any equal column at the poles. By this inequality of preſſure the fluid at the equator, where it is the lighteſt, will riſe, and the fluid at the poles, where it is the heavieſt, will ſink; till the preſſures become equal again. Thus the fluid maſs by the rotation round its axis would acquire a ſhape ſomething like that of a turnip, it would ſwell out at the equator, and would be depreſſed at the poles: the maſs, which would be a ſphere, if it had no rotation, would be changed from a ſphere into an oblate ſpheroid.

We have ſhewn from the eſtabliſhed laws of nature that the earth would acquire a ſpheroidal figure, if it ever was in a ſtate of fluidity: and conſequently, when the parts of it hardened, it would retain this figure. But whether it ever was in a ſtate of fluidity or not; we may prove that this is its figure: ſo that if it did not become an oblate ſpheroid by the force of gravity and a rotation round its axis, it muſt have originally been made of this ſhape. The globe of earth conſiſts partly of land and partly of water. Though the land is hard, and would not yield to the impreſſions of the centrifugal force, ſo as to ſwell out at the equator, and ſink down at the poles; yet the water is fluid, and conſequently would riſe where the gravity is leaſt, and would ſink where the gravity is greateſt. Therefore if the earth was a ſphere, the centrifugal force would make the fluid part of it, or the water, run towards the equator, and there it would be accumulated. But ſince the water upon the earths ſurface does not run towards the equator, and is not accumulated there; its figure is the ſame, that it would have acquired, if it had ever been in a ſtate of fluidity. In this figure alone the fluid parts of the globe will be in equilibrio, whilſt the earth revolves upon its axis.

If PE*pe*, Plat. XXX. fig. 2, is the earth; Ee a diameter of the equator is longer than P*p* the axis; becauſe the earth ſwells out at E*e* the equator, and is depreſſed at the poles P and *p*. The centrifugal force at the equator is to the force of gravity as 1 to 289. Therefore if the force of gravity was made leſs at the equator, than at the poles, only by the centrifugal force; the gravity at the equator E would be 1 part in 289 leſs than at the pole P: and conſequently the ſurface of the earth at the equator would be 1 part in 289 higher or farther from the center, than at the poles; that is, EC would be to PC, as 289 to 288. But if PE*pe* is the earths figure, and EC is to PC as 289 to 288; though the earth had no rotation round its axis, ſo that there was no centrifugal force at E, the force of gravity at E would be leſs than at P, merely becauſe E is at a greater diſtance from the center than P. The decreaſe of gravity at the equator upon this account and upon account of the centrifugal force together is 1 part in 230 of the whole force of gravity. So that as the gravity is 1 part in 230 leſs at E than at P, the ſurface of the earth at E muſt be 1 part in 230 higher or farther from the center than at P; that is, EC a ſemidiameter of the earth drawn from the equator is to PE a ſemidiameter of the earth drawn from one of the poles as 230 to 229.

The planet jupiter revolves upon its axis in leſs than 10 hours: and as the velocity, with which every part of jupiters ſurface muſt move, is ſo great; the centrifugal force ariſing from hence, by propoſition 165 of mechanics, will be much greater than the centrifugal force ariſing from the earths rotation round its axis. Conſequently jupiter will ſwell out more at its equator, and will be depreſſed more at its poles, than the earth is. The difference between the equatorial and polar diameters of jupiter is ſo great, that it may be perceived through a teleſcope. And a diameter of jupiter drawn through its equator is found to be to a diameter drawn through its poles, about as 13 to 12.

We may now go on to conſider ſome conſequences which will ariſe from this figure of the earth. One conſequence has already been explaned, in propoſition 453: the action of the ſun and moon upon the exceſs of matter at the equator will have the ſame effect, as if the earth was a ſphere, and a ring of matter adhered to it round the equator. So that the preceſſion of the equinoxes is owing to this figure.

456. *The weights of equal bodies, upon the earths ſurface, are inverſely as their diſtances from the earths center.*

Suppoſe there were two tubes, one reaching from the pole P to the earths center C, Plat. XXXI. fig. 2, and the other from the equator E to the

the ſame center; ſo that theſe tubes might communicate with each other at C. If theſe tubes were filled with water, then, by propoſition 454, the gravity of the water in the tube EC would be diminiſhed; but the gravity of the water in the tube PC would not be altered. Therefore, though both tubes are filled with water, yet we may conſider them as filled with fluids of a different ſpecific gravity: becauſe the water in the equatorial tube EC is made lighter than the water in the polar tube PC. But where communicating tubes contain fluids of different ſpecific gravities, the heights of the fluids, when they are in equilibrio, will be inverſely as their ſpecific gravities, by propoſition 13 of hydroſtatics; or, which is the ſame thing, the ſpecific gravities will be inverſely as the weights. Now the heights of the water in theſe tubes are EC and PC. Therefore the ſpecific gravities of the water in the tubes are as EC to PC inverſely. If equal bulks of water are taken reſpectively in the tubes EC and PC, the weights of theſe equal bulks will be as their ſpecific gravities, by propoſition 2 of hydroſtatics. But the ſpecific gravities are as EC to PC inverſely. Therefore the weights of equal bulks of this water at E and P will be as EC to PC inverſely. Now there is no difference between the water in theſe tubes, and any other water: for whatever change may be made, by the earths daily rotation, in the gravity of the water ſuppoſed to be contained in theſe tubes, the ſame change would be made by the ſame cauſe in the gravity of any other water at E and P. Therefore the weights of equal bulks of any water at E and P are as EC to PC inverſely. But whatever change is made by the earths rotation in the weights of equal bulks of water, the ſame change will be made by the ſame cauſe in the weights of any equal bodies whatever. Therefore the weights of any equal bodies at E and P are as EC to PC inverſely. But EC is the diſtance of E the earths ſurface at the equator from C its center; and PC is the diſtance of the earths ſurface at P the pole from the ſame center C. Therefore the weight of equal bodies upon the earths ſurface at the equator and at the poles are inverſely as their diſtances from the center: the weight of a maſs of lead at E is as much leſs than the weight of an equal maſs of lead at P, as EC is greater than PC.

The ſame may be proved in the ſame manner of equal bodies at E and A; their weights will be inverſely as EC to AC, or inverſely as the diſtances of E and A from the earths center. And ſo univerſally, the weights of equal bodies any where upon the earths ſurface are inverſely as their diſtances from the center.

457. *The lengths of pendulums, that perform their vibrations in equal times, at different places upon the earths ſurface, are inverſely as the diſtances of thoſe places from the earths center.*

The accelerating force of gravity is directly as the weights of equal quantities of matter, by propoſition 31 of mechanics. For if there are two bodies that contain equal quantities of matter their weights can differ from one another only in reſpect of the velocities, which the force of gravity would produce in theſe bodies in a given time. Now in different places upon the earths ſurface the weights of equal bodies are inverſely as their diſtances from the center, by propoſition 456. But, by propoſition 139 of mechanics, the lengths of pendulums, which perform their vibrations in equal times, are as the accelerating forces. Therefore, ſince the accelerating forces of gravity, in different places upon the earths ſurface, are inverſely as the diſtances of thoſe places from the earths center, the lengths of pendulums, which perform their vibrations in equal times, muſt likewiſe be inverſely as thoſe diſtances.

If a pendulum, which vibrates ſeconds at A or in the latitude EA, was to be carryed to the equator E, it would then take up more time than a ſecond to perform its vibration: becauſe the accelerating force of gravity at the equator E is leſs than at A in the latitude EA. To make the pendulum vibrate ſeconds at the equator it muſt be ſhortened in the inverſe proportion of EC to AC; that is, the length of a pendulum, which vibrates ſeconds at E is to the length of a pendulum which vibrates ſeconds at A as EC to AC inverted or as AC to EC. So likewiſe a pendulum, which vibrates ſeconds at P the pole, muſt be as much longer than a pendulum, which vibrates ſeconds at the equator E, as PC is ſhorter than EC, or as EC is longer than PC. Now EC is to PC as 230 to 229. Therefore the length of a pendulum, which vibrates ſeconds at the pole, is to the length of a pendulum, which vibrates ſeconds at the equator, as 230 to 229.

458. *A heavy body, unleſs it is either at the poles or at the equator, does not tend directly towards the earths center by the force of gravity.*

If the earth had no rotation round its axis a heavy body at A, Plat. XXXI. fig. 2, would tend directly to C the earths center by the force of gravity. But the rotation of the earth gives a centrifugal force to the body in the direction A*b*, as has been ſhewn under propoſition 454. By this means the body is acted upon by two forces AC the force of gravity and A*b* the centrifugal force, and for that reaſon, by propoſition 16 of mecha-

chanics, if it was to begin to move, it must move in the direction Ac, which is the diagonal of a parallelogram whose two sides are AC and Ab. Therefore though the force of gravity alone would make the body tend directly towards C the earths center; yet when this force is compounded with the centrifugal force arising from the earths rotation round its axis, the body will tend towards some point c, which is different from the earths center.

A heavy body at the poles P or p tends directly towards the earths center: because at the poles there is no centrifugal force, as has been shewn under propolition 454. A heavy body at E the earths equator tends directly to the center C. This is the direction, in which the body would gravitate, if there was no centrifugal force: and since C the earths center is likewise the center of the circle, which a body at E describes in the daily rotation; the centrifugal force arising from this rotation makes the body endeavour to depart from this center C. Therefore if a heavy body is at the equator, its gravity and the centrifugal force act in contrary directions; and consequently the centrifugal force, though it diminishes the bodys gravity, will not change the direction, in which the force of gravity acts. So that a heavy body placed at E will gravitate towards C the earths center, as well when the earth revolves upon its axis, as if we were to suppose the earth to have no such rotation.

459. *The degrees of latitude upon the earths surface are longer at the poles than they are at the equator.*

The earth is an oblate spheroid by propolition 455, and consequently a meridian PE*p*e, Plat. XXXI. fig. 2, encompassing it, will have a less curvature at the poles, where the earth is depressed, than at the equator, where it is swelled out. So that an arc in the meridian at the poles has less curvature than at the equator. But the less the curvature of an arc is, the greater is the circle of which that arc is a part. Therefore a degree in the meridian, or a degree of latitude, at the poles may be considered as a degree in a greater circle; and a degree of latitude at the equator may be considered as a degree in a less circle. But the degrees of a great circle are longer than the degrees of a small circle. Therefore a degree of latitude at the poles is longer or contains a greater number of miles than a degree of latitude at the equator.

Since the curvature of the meridian is greatest at the equator and decreases as we go from the equator to the poles; and since the degrees of latitude are the greatest when the curvature is least; it follows, that as we go from the equator to the poles the length of a degree will keep encreasing.

ſing. So that a degree of latitude will be ſhorter as we are nearer to the equator, and will be longer as we are nearer to either pole.

CHAP. XVIII.

Of the tides.

460. *If the whole ſurface of the earth was covered with water to a conſiderable depth, the moon, when it is either in the zenith or nadir of a place, would diminiſh the gravity of the water in that place, and would encreaſe the gravity of the water in all places that are* 90 *degrees from thence.*

IN explaning the general cauſe of the tides we ſuppoſe the whole ſurface of the globe to be covered with a deep water; that we may ſhew in what manner that cauſe would produce the tides, if there were no irregularities introduced by the accidental ſituation of the land, and by the ſhallowneſs or ſtreightneſs of the rivers, where theſe tides are to paſs.

Let S, Plat. XXIX. fig. 10, be the moon, T the center, and OQCL the ſurface of the earth. The moon is in the zenith of the place C, and in the nadir of the place O; and the places Q or L, and likewiſe all places round the earth in a circle whoſe diameter is QL, are at the diſtance of 90 degrees from C and O. In this ſituation the moon will make the water at C and O gravitate leſs than uſual towards the earths center, and will make the water at Q and L gravitate thither more than uſual.

As the moon gravitates towards the earth, by propoſition 399, ſo the earth gravitates towards the moon, by propoſition 73 of mechanics. If the moon attracted the center and all other parts of the earth equally and in parallel directions, ſuch an attraction would affect all the parts alike: and conſequently it would not change the mutual tendency or gravitation of thoſe parts towards one another. But the water at C, where the moon is in zenith, is nearer to the moon S, and for that reaſon is more attracted by the moon, than T the earths center is. This water gravitates towards the center T in the direction CT, and towards the moon in the direction CS: and as theſe are contrary directions the gravitation of the water towards S muſt neceſſarily make the gravitation towards T leſs than uſual. Indeed in the mean time the center T gravitates towards the moon in the direction TS, which will encreaſe the tendency of this center in the direction TC towards the water at C: and whatever force makes the center, or whole bulk of the earth, gravitate more towards the water at C,

will

will make the water gravitate more towards the center, by propoſition 73 of mechanics. Thus the gravitation of the water at C, where the moon is in zenith, will by the moons attraction be both diminiſhed and encreaſed: it will be diminiſhed, becauſe the moon makes the water tend leſs towards the center of the earth; and it will be encreaſed, becauſe the moon makes the center of the earth tend leſs towards the water. From hence then it follows, that, if the water at C and the earths center at T were equally attracted by the moon, the gravitation of the water would not be changed upon the whole by the moons attraction: ſuch an equal force would diminiſh the waters gravitation by attracting it from T the earths center, juſt as much as it encreaſes the ſame waters gravitation by attracting the earths center towards that water. But the water at C is nearer to the moon, and conſequently, as we juſt obſerved, is more attracted by the moon, than T the earths center is. Therefore ſince the waters gravitation is diminiſhed by the force, with which the moon attracts the water, and is encreaſed by the force with which the moon attracts the earths center, it follows that the waters gravitation will be more diminiſhed than encreaſed, that is, it will be diminiſhed upon the whole by the moons attraction.

At O where the moon S is in nadir, the water gravitates towards the earths center in the direction OT, and towards the moon in the ſame direction OS: ſo that if we were to conſider nothing but the moons action upon the water at O, we might ſay that the moons action, when it is in nadir, encreaſes the waters gravitation. But then T the earths center, or the whole bulk of the earth, gravitates towards the water at O in the direction TO, and towards the moon S in the oppoſite direction TS: and conſequently the attraction of the moon will make the earth or center of the earth gravitate leſs than uſual towards the water at O. Now if the earth gravitates leſs than uſual towards the water, it follows, from propoſition 73 of mechanics, that the water will gravitate leſs than uſual towards the earth. Thus the attraction of the moon produces two effects in reſpect of the gravity of the water at O: its immediate action upon the water encreaſes the waters gravity; and its action upon the earths center diminiſhes the waters gravity. Therefore if the moon acted equally upon the water at O and upon the earths center at T, the gravity of the water at O would be equally diminiſhed upon one account and encreaſed upon the other account, and ſo upon the whole it would continue the ſame. But the water at O is farther from the moon S than the earths center T is, and for that reaſon, by propoſition 23 of mechanics, the water at O will be leſs attracted by the moon than the earths cen-

center is. But the force, with which the water at O is attracted by the moon, encreaſes its gravity, and the force, by which the earths center is attracted, diminiſhes the waters gravity. Therefore the gravity of the water at O will be leſs encreaſed upon one account than it is diminiſhed upon the other account: ſo that upon the whole the gravity of the water at O, where the moon is in nadir, will be diminiſhed, or made leſs than uſual, by the moons attraction.

The moon S does not attract the earths center T and the water at L in the ſame or in parallel directions: the earths center is attracted in the direction TS, and the water is attracted in the direction LS, which is oblique to TS. The water at L and the earths center T are very nearly at the ſame diſtance from the moon: ſo that they will be equally attracted. But then the attraction LS being oblique to TS, may be reſolved into two others LB and LT, by propoſition 16 of mechanics. LB one part of this oblique attraction is parallel to TS; and this part will not impell the water at L towards the earths center. LT the other part of this oblique attraction is directed from the water at L towards T the earths center. Therefore the force LT will impell the water towards T, or will encreaſe the gravity of the water at L. The ſame may be ſhewn of the water at Q, or at any other place that lies in a circle encompaſſing the earth at QL.

461. *The force, with which the moon diminiſhes the waters gravity, we will call the moons ablatitious force; and the force, with which it encreaſes the waters gravity, we will call its addititious force.*

462. *The moons ablatitious force, when it is in the zenith or the nadir of a place, is double the addititious force, with which it acts at the ſame time upon the water in all places at* 90 *degrees diſtance: and the ablatitious force, when the moon is in zenith, is ſomething greater, than when it is in nadir.*

When the moon is at S, Plat. XXIX. fig. 10, if T is the earths center and OQCL its ſurface; the moon attracts the earths center in the direction TS and the water at L in the direction LS. Theſe attractions are equal, ſo that they may be repreſented by equal lines LS and TS. When the oblique attraction LS is reſolved into DT and LB, LT is the moons addititious force, which encreaſes the gravity of the water at L, by propoſition 459. Therefore this addititious force is to the whole force, with which the moon attracts the water, as LT to LS: and ſince the whole force, with which the water is attracted by the moon, is equal to the force with

with which the earths center is attracted by the moon, it follows that the addititious force is to the force with which the earths center is attracted by the moon, as LT to ST, or as a ſemidiameter of the earth to a ſemidiameter of the lunar orbit.

When the moon is at S, it is in the zenith of the place C: and the ablatitious force, which diminiſhes the gravity of water at C, by propoſition 460, is the difference between the forces, with which the earths center T and the water C are attracted by the moon. Theſe forces are as the ſquare of TS to the ſquare of CS inverſely, by propoſition 22 of mechanics, ſo that the earths center T is as much leſs attracted than the water at C, as the ſquare of CS is leſs than the ſquare of TS. Now CS, TS, and OS are in arithmetical progreſſion, for OT=TC is their common difference. TS the ſecond quantity exceeds CS the firſt, juſt as much as OS the third quantity exceeds TS the ſecond. And the common difference TC or OT is very ſmall in proportion to any of theſe quantities: for TC is a ſemidiameter of the earth; and the ſecond of theſe quantities TS is a ſemidiameter of the lunar orbit. Now where three quantities are in arithmetical progreſſion, if their common difference is very ſmall in reſpect of thoſe quantities, they are nearly in geometrical progreſſion. See propoſition 416. Therefore CS is to TS, nearly as TS is to OS: and conſequently CS is to OS, as the ſquare of CS to the ſquare of TS. Euc. b. VI. def. 10. But the force, with which the moon attracts the earths center T, is to the force, with which it attracts the water at C, as the ſquare of CS to the ſquare of TS; that is, as CS to OS. The difference between theſe attractions is OS—CS=OC. Therefore ſince we have here put CS to repreſent the force, with which the moon attracts the earths center, the ablatitious force is to the force, with which the moon attracts the earths center, as OS a diameter of the earth to CS, or, becauſe CS and TS are nearly equal, as OS to TS. But the addititious force is to the force, with which the earths center is attracted by the moon, only as LT a ſemidiameter of the earth to TS. Therefore the ablatitious force upon the water at C, when the moon is in zenith, muſt be double the addititious force; becauſe it bears double the proportion to the force, with which the earths center is attracted by the moon.

At the place O the moon S is in nadir: the water at O is leſs attracted by the moon than the earths center T, becauſe they are at different diſtances from the moon S. The force, with which the moon attracts the water at O, is to the force, with which it attracts the earths center T, as the ſquare of OS to the ſquare of TS inverſely; that is, as much leſs as the ſquare of TS is than the ſquare of OS, ſo much leſs the water at O

is attracted by the moon S than the earths center T is. The difference of theſe attractions is the ablatitious force, which diminiſhes the gravity of the water at O, by propoſition 460. If we take MO equal to OT, then the three quantities TS, OS, and MS are in arithmetical progreſſion; and OT or MO=OT, which is their common difference, is very ſmall in reſpect of any of theſe quantities. Therefore the three quantities are nearly in geometrical progreſſion, TS is to OS, nearly as OS is to MS: See propoſition 416: and conſequently TS is to MS, nearly as the ſquare of TS to the ſquare of OS. Euc. b. VI. def. 10. But the force, with which the water at O is attracted by the moon, is to the force, with which the earths center T is attracted by the moon, as the ſquare of TS to the ſquare of OS; that is, as TS to MS. The difference between theſe two attractions or MS—TS=MT is the ablatitious force. And ſince we have put MS to repreſent the force, with which the earths center is attracted by the moon, it follows that the ablatitious force is to the force, with which the earths center is attracted by the moon, as MT to MS, or as OC to TS, ſince MT, by the conſtruction is equal to OC, and TS is very nearly equal to MS. Thus the ablatitious force, which diminiſhes the gravity of the water at O, where the moon S is in nadir, is double the addititious force, which encreaſes the waters gravity at L: for the ablatitious force is to the force, with which the earths center is attracted by the moon, as OC a diameter of the earth to TS, and the addititious force is to the force, with which the earths center is attracted by the moon, only as LT a ſemidiameter of the earth to TS.

In proving the former part of the propoſition we have proved the latter: for we have ſhewn that the ablatitious force at C, where the moon is in zenith, is to the force, with which the earths center is attracted by the moon, as OC to CS; and that the ablatitious force at O, where the moon is in nadir, is to the force, with which the earths center is attracted by the moon, as MT to MS. Now OC is equal to MT and CS is leſs than MS. Therefore OC bears a greater proportion to CS than MT bears to MS: Euc. b. V. prop. 10: and conſequently the ablatitious force, where the moon is in zenith, bears a greater proportion to the force, with which the earths center gravitates towards the moon, than the ablatitious force, where the moon is in nadir, bears to the ſame gravitation. But the gravitation of the earths center towards the moon is a given quantity. Therefore the ablatitious force, where the moon is in zenith, muſt be greater, than where it is in nadir; becauſe otherwiſe it could not bear a greater proportion to the ſame gravitation. Euc. b. V. prop. 10.

463. *The*

463. *The water will riſe where the moon is in zenith or nadir, and will ſink in all places that are at* 90 *degrees diſtance from thence: and the force, which raiſes the water or cauſes a tide, is the ſum of the ablatitious and addititious forces.*

The gravity of the water is diminiſhed at C, Plat. XXIX. fig. 10, where the moon S is in zenith and at O, where the moon is in nadir, by propoſition 460. The ſurface of the water would be ſpherical, if it gravitated every where equally towards the earths center. But by the moons action the water at C and O is made to gravitate leſs towards the center, or to preſs leſs downwards, than any other parts of the water; and as it preſſes leſs downwards, ſo, by propoſition 16 of hydroſtatics, it will preſs leſs ſideways. Thus the moons action deſtroys the equilibrium of the water upon the earths ſurface: and for that reaſon thoſe parts of the water, which preſs the moſt will ſink, and the parts at C and O will riſe, till the equilibrium is reſtored, that is, till the accumulation of water at C and O makes the preſſure at thoſe places equal to the preſſure any where elſe. Thus a tide will be raiſed at C and O. But this tide, as far as we have explaned it, is raiſed only by the ablatitious force, which diminiſhes the gravity of the water in thoſe places, where the moon is in zenith or nadir. Beſides this force we muſt remember that the moon at S acts by an addititious force on the water at L and Q, which are 90 degrees diſtant from C and O, by propoſition 460. Now if the gravity of the water at L and Q is encreaſed, it will preſs downwards more than the other water does, and by this means the equilibrium between the parts of the water will be deſtroyed; ſo that the water at L and Q will ſink, and the water will riſe on each ſide of it as far as C and O, for it is only at C and O that no ſuch addititious force acts. Thus by the addititious as well as by the ablatitious force the water will be raiſed at C and O; and conſequently the tides at C and O are raiſed by the ſum of theſe forces.

From hence we may ſee the reaſon why in ſeas of ſmall extent, ſuch as the Caſpian, no tides ſhould be raiſed. For if the moon was to be in the zenith or nadir of ſuch a ſea, it would indeed diminiſh the gravity of the water; but then as the ſea is of ſmall extent, the gravity of the water in every part of it will be diminiſhed equally; ſo that all the parts will gravitate or preſs equally, as well when their gravity is diminiſhed, as when it is not diminiſhed: and conſequently, ſince in ſuch an equal preſſure the parts will continue in equilibrio, one part of the ſurface will not riſe higher than the reſt, or no tide will be raiſed.

From hence likewiſe we may ſee the reaſon why the ocean muſt extend for 90 degrees in order to have a full tide raiſed in it. A full tide

is raiſed at O, not only by the ablatitious force, but by the addititious force too. Now the addititious force is greateſt at L or Q: and conſequently if there is land between L and O, the water at O will not be raiſed by the whole addititious force; ſo that unleſs the ocean extends at leaſt 90 degrees from O to L, there will not be a full tide raiſed in it.

464. *By a lunar hour we mean the twenty-fourth part of that time, which the moon takes up in paſſing from the meridian of any place, and returning to the ſame meridian again.*

465. *At any place, through whoſe zenith and nadir the moon paſſes, it will be high water twice and low water twice in twenty four lunar hours.*

If there was any iſland at C, Plat. XXIX. fig. 1, where the moon S is in zenith; this iſland, by propoſition 463, would be in the ſwell of the water; ſo that the water would be higher than uſual about the ſhore of it. In the daily rotation of the earth, this iſland will be carryed from C to L; and at L the water is depreſſed, by propoſition 463: therefore when the iſland is at L, ſo that the moon S is in the horizon and is ſetting, it will be low water there. As the iſland moves on in the earths daily rotation it will come to O, and then, by propoſition 463, it will be high water again, becauſe the moon S is in nadir. When the iſland comes to Q, ſo that the moon S is in the horizon and is riſing, it will be low water, by propoſition 463, becauſe Q is 90 degrees from C and from O. Thus in the daily rotation of the earth, or in 23 hours 56 minutes, it will be twice high water and twice low water at ſuch an iſland. But this would be the caſe only upon ſuppoſition that the moon has no motion in its orbit: for the moons motion in its orbit will vary the time of high and low water, ſo that more than the time of one daily rotation muſt paſs, before it will be twice high and twice low water. The moon revolves in its orbit from weſt to eaſt: and conſequently the ſwell of water, which is raiſed by the moon, moves round the earths ſurface the ſame way. So that if any iſland is moving in the daily rotation from weſt to eaſt, the ſwell and the fall of water will move on before it; and conſequently the iſland, after it has left one ſwell, muſt move more than a quarter round before it comes to the fall, it muſt move more than half round before it comes to the next ſwell, more than three quarters round before it comes to the next fall, and more than once round before it returns to the firſt ſwell. Thus if the moon had no motion round the earth,

earth, an iſland ſetting out from the ſwell at C would come to the fall at L by moving a quarter round the earth, and to the next ſwell at O by moving half round. But the moon S in the mean time, and conſequently the ſwell and fall with it, will have moved from weſt to eaſt, ſo that the fall will be to the eaſt of L, and the ſwell will be to the eaſt of O. Therefore in the daily rotation, if it is high water when the iſland is at C, it will not be low water till the iſland is got beyond or to the eaſt of L, nor high water again till the iſland has got beyond or to the eaſt of O. And for the ſame reaſon it will not be low water again, till the iſland has got beyond Q: nor will it be high water as at firſt, till the iſland has got beyond C. Or otherwiſe: If the moon had no revolution in its orbit, then the time that the moon would take up to ſet out in the daily rotation from the zenith of a place and return to the ſame zenith again, or twenty four lunar hours would be the time of one rotation of the earth round its axis. But whilſt any place on the earths ſurface is turning once round from weſt to eaſt, the moon in its orbit will have moved the ſame way. Therefore if the moon was in the zenith of any place at the beginning of the rotation, it will be to the eaſt of the zenith at the end of the rotation: and conſequently the place muſt move a little more than once round, before the moon will appear in its zenith again, or before the twenty four lunar hours will be completed. Now it is twice high and twice low water at any place during the time that the moon takes up to paſs from the zenith of that place and return to that zenith again. But this time is twenty four lunar hours, and is longer than the time of a rotation of the earth round its axis. Therefore in twenty four lunar hours it will be twice high and twice low water at any place through whoſe zenith and nadir the moon paſſes.

We ſhould here obſerve farther that twenty four lunar hours are longer than twenty four ſolar hours or than a natural day. We have proved in propoſition 25, that a natural day is longer than the time of a rotation of the earth round its axis: becauſe as the earth is revolving upon its axis from weſt to eaſt, the ſun moves the ſame way in the ecliptic; and any place on the earths ſurface muſt move more than once round before it overtakes the ſun, or comes even with it. But as the ſun moves from weſt to eaſt in the ecliptic, ſo the moon moves from weſt to eaſt in its orbit: and as the moon takes up leſs time to deſcribe its orbit, than the ſun takes up to deſcribe the ecliptic, the moon moves faſter from weſt to eaſt than the ſun does. Conſequently in the time of a daily rotation of the earth the moon gets farther to the eaſt than the ſun does: ſo that when any place upon the earths ſurface has overtaken and is come even with the

ſun

ſun, it muſt move farther before it overtakes and comes even with the moon, that is, twenty four ſolar hours or a natural day will be completed before twenty four lunar hours are completed. If twenty four ſolar hours were exactly equal to twenty four lunar hours, it would be high water always at the ſame time of the day. Thus ſuppoſe it is high water at any place at 2 o'clock in the afternoon; then after 12 or half 24 lunar hours it will be high water again. But if theſe 12 lunar hours were equal to 12 ſolar hours, the next time of high water would be 2 o'clock in the morning. Again it will be high water after 12 lunar hours; and if theſe were equal to 12 ſolar hours; the next time of high water would be 2 o'clock in the afternoon. But 12 ſolar hours are ſhorter than 12 lunar hours: and conſequently, though between 2 o'clock in the afternoon and 2 o'clock the next morning 12 ſolar hours are completed, yet 12 lunar hours beginning from 2 o'clock in the afternoon will not be completed till ſome time after 2 o'clock the next morning. So that if it is high water at any place at 2 o'clock in the afternoon, it will not be high water there again till 12 lunar hours are completed, that is, not till after 2 o'clock the next morning. And for the ſame reaſon it will not be high water there again, till after 2 o'clock the next afternoon. Thus the time of high water will be later every day: becauſe we reckon the time of the day by ſolar hours, but the time of high water is determined by lunar hours, which are longer than ſolar ones.

466. *The water is the higheſt in any place about 3 hours after the moon has paſſed the zenith or nadir of the place, and is the loweſt about 3 hours after the moon has riſen above the horizon on one ſide or ſet below it on the other ſide.*

We have already ſhewn in the foregoing propoſitions that the attraction of the moon, or gravitation towards it, will make the water ſwell, when the moon is in the zenith or the nadir of a place, and will make it fall when the moon is in the horizon. But the water does not ſwell to its greateſt height, till the moon has paſſed the zenith or nadir ſome time. Nor does it fall the loweſt, till the moon has paſſed the horizon ſome time.

The moon acts upon the water ſo as to raiſe a tide though it is not exactly in the zenith or nadir of a place. Let S, Plat. XXX. fig. 1, be the moon, OQCL the ſurface of the earth or rather of the water, which is ſuppoſed to cover the ſurface of the earth, and T the earths center. The moon, though it is not in the zenith of the place A, will act upon the water at A ſo as to make it ſwell there. For the water at A is

nearer

nearer to the moon S, than T the earths center: conſequently the water will be more attracted than the earths center. By the difference of theſe attractions the gravity of the water at D will be diminiſhed, by propoſition 460; and this will make the water ſwell at A, by propoſition 463. In like manner, fig. 2, the moon S is not in the nadir of the place D; but yet the water at the place D is farther from the moon than the earths center T is. Therefore the water at D will be leſs attracted than the earths center: this difference of attraction will diminiſh the waters gravity, by propoſition 460: and the water will ſwell where its gravity is diminiſhed, by propoſition 463. Indeed the force of the moon to raiſe a tide is greateſt, when it is in zenith or nadir: becauſe then the diſtances of the water and of the earths center from the moon differ the moſt: ſo that the ſquares of thoſe diſtances, or the forces with which the water and the center are attracted, will differ the moſt. But a tide is raiſed by the difference of theſe attractions. Therefore the moons force to raiſe a tide is greateſt, when it is in zenith or nadir. Thus, fig. 1, TS and CS differ more from one another than TS and AS; and, fig. 2, TS and OS differ more from one another than TS and DS. Conſequently the ſquares of TS and CS will differ more from one another than the ſquares of TS and AS; and the ſquares of TS and OS will likewiſe differ more from one another than the ſquares of TS and DS. Now the difference of attractions, with which the moon acts upon the water and upon the earths center, are as the difference of theſe ſquares. And it is this difference of attractions, which diminiſhes the waters gravity and raiſes a tide, by propoſitions 460, 463. Therefore the force of the moon to raiſe a tide is greater at C, where it is in zenith, than at A, where it is not in zenith; and is greater at O, where it is in nadir, than at D, where it is not in nadir.

Having thus ſhewn that the moon acts upon the water ſo as to raiſe a tide, when it is not in the zenith or the nadir of a place; we may prove, that though its force to raiſe a tide is the greateſt, when it is in zenith or nadir, yet the tide will be the greateſt, or the water will ſwell the higheſt ſome time after the moon has paſſed the zenith or the nadir. The water begins to ſwell before the moon comes to the zenith or nadir; becauſe the moon acts upon the water ſo as to raiſe a tide before it comes thither. And the water continues to ſwell after the moon has paſſed the zenith or nadir; becauſe it ſtill acts upon the water ſo as raiſe a tide. Thus the ſwell of water when the moon is in zenith or nadir is not owing to that force only, with which the moon acts at that time; this ſwell is produced by the moons action before it comes to the zenith or nadir, and by its action when it is in zenith or nadir. And the ſwell of water, after the

moon

moon has paſſed the zenith or nadir is not owing to that force only, with which the moon acts at that time; this ſwell is produced by the moons action, before it comes to the zenith or nadir, whilſt it is in the zenith or nadir, and after it has paſſed the zenith or nadir: and conſequently the ſwell of water will be greater after the moon has paſſed the zenith or nadir, than it was whilſt the moon was in the zenith or nadir. Or otherwiſe. The force, with which the moon acts upon the water to raiſe a tide, does not ceaſe as ſoon as it leaves the zenith or nadir, but continues for ſome time afterwards. But as long as this force continues the water will continue to riſe. Therefore the water will not be come to its greateſt height, till ſome time after the moon has paſſed the zenith or nadir.

The water is the loweſt at thoſe places, which are 90 degrees diſtant from the places where it is the higheſt: or the greateſt fall of the water is 90 degrees from the greateſt ſwell, by propoſitions 460, 463. And conſequently, if the water was the higheſt, when the moon is in the zenith or nadir; it would be the loweſt, when the moon is in the horizon, by propoſition 460. But we have juſt proved that the water is the higheſt ſome time after the moon is in the zenith or nadir. Therefore it will be the loweſt ſome time after the moon has paſſed the horizon, ſome time after it has riſen above the horizon on one ſide or ſet below it on the other.

467. *If the whole globe of earth was covered with deep water, the gravitation towards the moon would change the figure of the water into an oblong ſpheroid; and the longer axis of this ſpheroid would be to the eaſt of the moon.*

Let us firſt ſuppoſe the water to be raiſed by the moon to its greateſt height, when the moon is in the zenith or nadir of a place; for upon that ſuppoſition the reader will moſt readily underſtand what figure the water upon the earths ſurface will put on, in conſequence of the moons attraction. And then we will go on to ſhew the reaſon why the longer axis of the ſpheroid, into which the globe would be changed by this attraction, ſhould be to the eaſt of the moon.

If the moon is at S, Plat. XXIX. fig. 10, the water will riſe at C, where the moon is in zenith, and at V, where the moon is in nadir, and will ſink all round, in a circle that encompaſſes the earth, at QL, by propoſition 463. By this means the diameter OC will be lengthened, and all the diameters of the circle round QL will be ſhortened: ſo that the globe will put on the oval figure *cqol*, Plat. XXXI. fig. 3. This figure is called an oblong ſpheroid: *co* a line paſſing through the two places *c* and *o*, where

where the water is raiſed to the greateſt height, is the longer axis of this ſpheroid; and any diameter of a circle drawn round the ſpheroid at *ql* is the ſhorter axis. Therefore ſince the longer axis of the ſpheroid paſſes through the two places where the water is the higheſt; it follows that this axis would paſs through the places where the moon is in zenith and nadir; if the water was raiſed to its greateſt height when the moon is in zenith or nadir. But, by propoſition 466, the tide is the higheſt when the moon has paſſed the zenith or nadir of the place about three hours, that is, the tide is the higheſt, when the moon is to the weſt of the zenith. Therefore ſince the moon is always to the weſt of the higheſt water, and the higheſt water is in the longer axis of the ſpheroid; it follows that the moon will always be to the weſt of the longer axis, or the longer axis will be to the eaſt of the moon. Thus when the moon has left the zenith of the place *c* about three hours, or has gotten to the weſt of the zenith, the longer axis of the ſpheroid will paſs through the place *c*: and at the ſame time it will paſs through the place *o*, which is the antipodes to *c*, and where the moon has left the nadir about three hours.

468. *In thoſe places, through whoſe zenith or nadir the moon does not paſs, it will be high water about three hours after the moon has paſſed the meridian of the place, or the oppoſite meridian.*

We have yet only ſhewn how the tides are raiſed in thoſe places, through whoſe zenith or nadir the moon paſſes. But the ſame cauſe, which produces tides in thoſe places, will produce them in others. We had occaſion to prove, in propoſition 466, that the moons attraction will make the water ſwell, before it comes to the zenith or nadir, and will continue to make the water ſwell, after it has paſſed the zenith or nadir; that is, the moons action upon the earths center and upon the water will produce a tide, though the moon is not in the zenith or in the nadir, but is to the eaſt or to the weſt of either of them. In like manner we might ſhew, that the moon acts upon the water ſo as to produce a tide, if it is not in the zenith or in the nadir, but is to the ſouth or to the north of either of them. Now though the moon in our latitude, for inſtance, never comes to the zenith, yet when it is above the horizon and is to the ſouth of the zenith, it will produce a tide. But the moon, when it is in our meridian, is above the horizon, and to the ſouth of the zenith. Therefore when the moon is in the meridian it will produce a tide. When the moon is below our horizon, and to the north of our nadir, it will produce a tide. But the moon, when it is in the oppoſite meridian, is below our horizon and to the north of our nadir. Therefore the moon in the oppoſite

meridian will produce a tide. But then, as the moons action begins to ſwell the water before it comes to the meridian, ſo it continues to ſwell it after it has paſſed the meridian: therefore the tide will keep riſing ſome time after the moon has paſſed the meridian. And for this reaſon, though the moons action to raiſe the water is the greateſt, when it is in the meridian; yet the water will not come to its greateſt height till ſome time after the moon has paſſed the meridian. And for the ſame reaſon, when the moon is below the horizon, the water will not come to its greateſt height, till ſome time after the moon has paſſed the oppoſite meridian.

Or otherwiſe. By the moons action, the water which covers the earths ſurface is changed into an oblong ſpheroid *cqol*, Plat. XXXI. fig. 3, and *co* the longer axis paſſes through two places as *c* and *o*, where the moon has left the zenith of *c* and the nadir of *o* about three hours, by propoſition 467. At *c* and *o* the water is raiſed to its greateſt height; and in a circle paſſing through *ql* 90 degrees diſtant from *c* and *o* the water is ſunk the loweſt. Now in two places at *m* and *n* upon the earths ſurface, the moon does not come to the zenith, when above the horizon, nor to the nadir, when below it. Yet a tide will be raiſed at *m* and *n*, and the time of high water will be about three hours after the moon has paſſed the meridian or the oppoſite meridian either of *m* or of *n*. In the daily rotation of the earth round PP its axis the places *m* and *n* deſcribe the circle of latitude *mdn*. When in this daily rotation either of theſe places is at *d*, it will then be in the circle *qd* of the ſpheroid, where the water is the moſt depreſſed; and conſequently it will then be low water at the place. But in the daily rotation as one of theſe places moves from *d* towards *m*, it gets out of *d* the point of low water, and conſequently the water will keep riſing: and when the place comes to *m*, it is then the fartheſt from *d* that it can be; and therefore the water will then be the higheſt. At that time, *m*, though it is not at *c* the longer axis of the ſpheroid, yet is even with *c*: ſo that when it is high water at any place *c*, through whoſe zenith the moon paſſed, it will likewiſe be high water at *m*, which is even with *c* or in the ſame meridian with *c*. Now the time of high water at *c* and conſequently the time of high water at *m* is three hours after the moon has paſſed the zenith of *c*, by propoſition 466. But the zenith of *c* is in the ſame meridian with that of the place *m*. Therefore when the moon has paſſed the zenith of *c* three hours, it will have paſſed the meridian of *m* three hours: and conſequently the time of high water at any place *m* is three hours after the moon has paſſed the meridian of *m*. As the other of theſe two places in the daily rotation of the earth, moves from *d* towards

wards *n*, it gets out of *d* the point of low water, and conſequently the water will keep riſing: and when the place is come to *n*, it is then the fartheſt from *d* that it can be; and therefore the water will then be the higheſt. At that time *n*, though it is not at *o* the longer axis of the ſperoid, yet it is even with it: ſo that when it is high water at *o*, through whoſe nadir the moon paſſed, it will likewiſe be high water at *n*, which is even with *o* or in the ſame meridian with *o*. Now the time of high water at *o* and conſequently the time of high water at *n* is three hours after the moon has paſſed the nadir of *o*, by propoſition 466. But as *o* and *n* are in the ſame meridian, the nadir of *o* muſt be in the ſame oppoſite meridian with that of the place *n*. Therefore when the moon has paſſed the nadir of *o* three hours, it will have paſſed the oppoſite meridian of *n* three hours: and conſequently the time of high water at any place *n* is three hours after the moon has paſſed the oppoſite meridian.

469. *In places, that have latitude, the two tides of the ſame day will not be equal, when the moon has declination.*

Let EE, Plat. XXXI. fig. 3, be the earths equator, PP the axis, *mn* a circle of north latitude. Suppoſe the moon to have north declination, ſuppoſe, for inſtance, that it is in the tropic of cancer; and let *cr* be the geographical tropic, or tropic drawn upon the earths ſurface. The moon, by propoſition 467, changes the water upon the earths ſurface into an oblong ſpheroid; and the longer axis of this ſperoid paſſes through a place, where the moon has left the zenith three hours, and through the antipodes to this place. Now all the places, through whoſe zenith the moon paſſes, when it is in the tropic of cancer, are in the geographical tropic *cr*; and all the antipodes places are in the oppoſite tropic *bo*. Therefore when the moon is in the tropic of cancer, *co* the longer axis of the ſpheroid will paſs from one tropic *cr* to the other tropic *bo*. But the greateſt ſwell of the water is in the longer axis of the ſpheroid, by propoſition 467. Therefore when the moon is in the tropic of cancer, the greateſt ſwell of water *c*, on that ſide of the earth where the moon is above the horizon, will be in the geographical tropic of cancer, or on the ſame ſide of the equator with the moon: and *o* the greateſt ſwell on the other ſide, where the moon is below the horizon, will be in the tropic of capricorn, or on the contrary ſide of the equator. Any place whoſe north latitude is *m*E will deſcribe the circle *mn* in the daily rotation: in half a rotation it will move from *m* to *n*, and in the other half it will move from *n* to *m*. It will be high water at this place in one part of the day, when the place is at *m*, and in the other part of the day, when it is at

n, as has been ſhewn in propoſition 468. But in one part of the day, when the place is at *m* and the moon above the horizon, the tide will be greater, than it is in the other part of the day, when the place is at *n* and the moon below the horizon. For *c* the greateſt ſwell, when the place is at *m* and the moon is above the horizon, is on the ſame ſide of the equator with the place: and *o* the greateſt ſwell, when the place is at *n* and the moon is below the horizon, is on the contrary ſide of the equator. Therefore the place will be nearer to the greateſt ſwell, and conſequently the water will riſe higher, when the place is at *m* and the moon is above the horizon, than when the place is at *n* and the moon below the horizon.

At any place in ſouth latitude Eg, the moon being ſtill ſuppoſed to be in the northern tropic, juſt the reverſe will happen: the two tides of the ſame day will be unequal, but that will be the greateſt, which is produced by the moon below the horizon; and that will be the leaſt, which is produced by the moon above the horizon. The place, whoſe ſouthern latitude is Eg, deſcribes the circle *gf* in the earths daily rotation. And, by propoſition 467, it is high water one part of the day, when the place is at *g* and the moon below the horizon; and it is high water the other part of the day, when the place is at *f* and the moon is above the horizon. Now the greateſt ſwell *o*, when the moon is below the horizon, is on the ſame ſide of the equator with the place: and the greateſt ſwell *c*, when the moon is above the horizon, is on the contrary ſide of the equator. Therefore when the place is at *g* and the moon is below the horizon, it comes nearer to the greateſt ſwell and conſequently the tide will be higher, than when the place is at *f* and the moon is above the horizon.

And thus univerſally, If the moons declination is on the ſame ſide of the equator with the places latitude; the greateſt ſwell, when the moon is above the horizon, will be on the ſame ſide of the equator with the place; and the greateſt ſwell, when the moon is below the horizon will be on the contrary ſide of the equator. Therefore when the moon is above the horizon the place will be nearer to the greateſt ſwell, than when the moon is below the horizon: and conſequently the tide in the former part of the day will be higher than in the latter. But if the moons declination is on the contrary ſide of the equator to the places latitude; the greateſt ſwell, when the moon is above the horizon, will be on the contrary ſide of the equator; and the greateſt ſwell, when the moon is below the horizon, will be on the ſame ſide of the equator with the place. Therefore, when the moon is above the horizon, the place will be farther

ther from the greateſt ſwell, than it is, when the moon is below the horizon: and conſequently the tide in the former part of the day will be leſs, than in the latter.

470. *The ſun produces tides in the ſame manner that the moon does.*

If S, Plat. XXIX. fig. 10, is the ſun, CLOQ the earths ſurface covered with a deep water, and T the earths center; then by the force of gravity the water at C will tend more and the water at O will tend leſs towards S the ſun, than T the earths center tends thither: becauſe the water at C is nearer and the water at O is farther from the ſun than T the earths center. This will make the water at C and O gravitate leſs towards the earth than uſual; as has been ſhewn in a like caſe under propoſition 460. The water at Q and L will be attracted obliquely by the ſun in reſpect of T the earths center, and this oblique force impells that water towards the earths center or encreaſes its gravity; juſt as a like force of the moon would encreaſe the gravity of the water in the ſame ſituation, by propoſition 460. But, by propoſition 463, the water will ſwell, where its gravity is diminiſhed, and will ſink, where its gravity is encreaſed. Therefore at any place C where the ſun is in the zenith, a tide will be raiſed, and at any places QL, the ſurface of the water will be made lower than uſual.

Having thus ſhewn that the ſun will raiſe a tide, where it is in zenith or nadir, we might go on to ſhew that every thing elſe in the tides produced by the ſun is the ſame, as in thoſe produced by the moon. But as the demonſtrations are the ſame in all reſpects, it is not neceſſary to repeat them. There is indeed one ſeeming difference between the tides produced by the ſun and thoſe produced by the moon. The tides produced by the moon happen later every day than they did the preceding day, by propoſition 465. But the tides produced by the ſun will happen every day at the ſame time. For by propoſitions 467, 468, the ſuns force to raiſe a tide at any place is the greateſt, when it is in the meridian of the place or in the oppoſite meridian, that is at 12 o'clock at noon or at 12 o'clock at night. But as the ſun continues to raiſe the water, after it has paſſed the meridian or oppoſite meridian about three hours, the time of high water by the ſuns action will be always at the third ſolar hour, that is, about 3 o'clock in the afternoon and about 3 o'clock in the morning. If a lunar hour was equal to a ſolar hour, the tides produced by the moon would likewiſe always happen at the ſame time of the day; but we have ſhewn, in propoſition 465, that a lunar hour is longer than a ſolar one; and for that reaſon the third lunar hour, when

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the moon makes it high water, is not always at the ſame time of the day.

From hence it appears that there ought to be four tides every day, two produced by the moon, and two by the ſun. Whereas in fact there are but two tides every day. Theſe two tides therefore muſt ariſe from the compound action of the ſun and moon. And it will be our buſineſs in ſome of the following propoſitions to ſhew in what manner their actions are compounded.

We may here obſerve, that in reſpect of the ſuns great diſtance TS, a ſemidiameter of the earth OT or CT is very ſmall and almoſt nothing. ST the ſuns diſtance is ſo great that the addition of CT or of CO to it makes little alteration in it: or C, T, and O, in reſpect of the ſuns diſtance are almoſt cloſe together: or the earths diameter compared with this diſtance is almoſt a point. Therefore the force, with which the ſun attracts the water at C, the earths center at T, and the water at O, will be very nearly the ſame. But ſince it is the difference of theſe attractions, by propoſition 460, which diminiſhes the waters gravity at C and at O; and this difference is inconſiderable; it might be expected that the ſun ſhould not raiſe any ſenſible tide. This would be the caſe, if the quantity of matter contained in the ſun was not ſo great as it is. In the three next propoſitions we will ſhew, that the ſuns force to raiſe a tide is leſs than the moons upon account of their different diſtances; but that its force is greater than the moons, upon account of the greater quantity of matter contained in it: and that upon the whole, if we take in both accounts, their forces to raiſe a tide will be very nearly as their reſpective denſities.

471. *If the quantity of matter in a luminary is given, its force to raiſe a tide will be inverſely as the cube of its diſtance from the earths center.*

If S, Plat. XXIX. fig. 10, is a luminary containing a given quantity of matter; its force to raiſe a tide will be inverſely as the cube of TS its diſtance from the earths center. The luminary acts upon the water by an addititious and by an ablatitious force, by propoſition 460. Firſt then we will ſhew that the addititious force is inverſely as the cube of TS. Now the addititious force is to the force, with which the earth gravitates towards the luminary, as LT a ſemidiameter of the earth to TS, as has been ſhewn in propoſition 462: and from hence it follows that the addititious force may be encreaſed or diminiſhed two ways. If the proportion of LT to TS encreaſes, the addititious force muſt encreaſe, becauſe then, as LT bears a leſs proportion to TS, the addititious force will bear a greater proportion to the earths gravitation towards the luminary

S.

S. But the proportion of LT to TS will encreaſe as TS decreaſes: for the ſhorter TS is, the greater proportion the given line LT will bear to it. Therefore the addititicious force encreaſes as TS the diſtance of the luminary decreaſes. Again: if the proportion of LT to TS decreaſes the addititicious force muſt decreaſe; becauſe then, as LT bears a leſs proportion to TS, the addititicious force will bear a leſs proportion to the earths gravitation towards the luminary S. But the proportion of LT to TS will decreaſe as TS encreaſes: for the longer TS is, the leſs proportion the given line LT will bear to it. Therefore the addititicious force decreaſes as TS the diſtance of the luminary encreaſes. Thus the addititicious force is inverſely as TS the diſtance of the luminary: for it encreaſes as TS decreaſes, and decreaſes as TS encreaſes. This is one way in which the addititicious force is varyed, as TS the diſtance of the luminary from the earths center is varyed: but this is not the only way. For ſince the addititicious force is to the earths gravitation towards the luminary as LT to TS; or ſince the addititicious force bears a certain proportion to the earths gravitation; it follows that the addititicious force muſt encreaſe or decreaſe in the ſame proportion with the earths gravitation towards the luminary. Now the earths gravitation towards the luminary is inverſely as the ſquare of TS its diſtance from the luminary, by propoſition 23 of mechanics. Therefore the addititicious force is inverſely as the ſquare of TS the diſtance between the luminary and the earths center. Thus we have proved that the addititicious force is inverſely as TS upon one account, and inverſely as the ſquare of TS upon another account. And conſequently upon both accounts together it will be inverſely as TS and the ſquare of TS conjoyntly, or inverſely as TS multiplyed into the ſquare of TS; that is, inverſely as the cube of TS. Call TS the diſtance of the luminary from the earth x: then the addititicious force upon one account is as $\frac{1}{x}$, or as the reciprocal of x: and upon another account it is as $\frac{1}{xx}$, or as the reciprocal of the ſquare of x. Therefore upon both accounts together it is as $\frac{1}{x} \times \frac{1}{xx} = \frac{1}{xxx}$, or as the reciprocal of the cube of x: that is, the addititicious force is reciprocally or inverſely as the cube of x the diſtance of the luminary from the earths center.

Secondly, the luminarys ablatitious force is double its addititicious force, by propoſition 462. And in whatever proportion any quantity is varyed, the double of that quantity is varyed in the ſame proportion. But the addititicious force is inverſely as the cube of the luminarys diſtance. Therefore the ablatitious force is likewiſe inverſely as the cube of the luminarys diſtance.

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The force with which a luminary raiſes a tide is the ſum of its addititious and ablatitious forces, by propoſition 463. But both theſe forces, and conſequently the ſum of them, are inverſely as the cube of the luminarys diſtance. Therefore the force of a given luminary to raiſe a tide is inverſely as the cube of its diſtance from the earth.

The moon in one part of the month is in apogee, and in another part of the month is in perigee. The moon will raiſe a greater tide when it is in perigee, than when it is in apogee. For the moon is nearer to the earth when it is in perigee, than when it is apogee; and the force, with which it acts upon the water, to raiſe a tide is inverſely as the cube of its diſtance. Therefore its force to raiſe a tide, and conſequently the tide that it raiſes, will be the greateſt when its diſtance is the leaſt.

Both the moon and the ſun raiſe tides, by propoſitions 463, 470. And if the quantity of matter in the moon was equal to the quantity of matter in the ſun; the moon muſt raiſe a much greater tide than the ſun: becauſe the moon is ſo much nearer than the ſun. For, the quantity of matter being ſuppoſed the ſame in each, the force of the moon to raiſe a tide would be as much greater than the force of the ſun to raiſe one, as the cube of the moons diſtance is leſs than the cube of the ſuns diſtance.

472. *If the diſtance of a luminary from the earths center is given, its force to raiſe a tide will be directly as the quantity of matter contained in it.*

The addititious force of the luminary S, Plat. XXIX. fig. 10, with which it acts upon the water at L, is to the earths gravitation towards the luminary, as LT to TS, by propoſition 463. Now LT is a given quantity; for it is the ſemidiameter of the earth. Therefore if TS the diſtance of the luminary is likewiſe a given or invariable quantity, the proportion of LT to TS is given; that is, the addititious force will bear a given proportion to the earths gravitation towards the luminary S. From hence it follows, that in whatever proportion the earths gravitation towards S encreaſes or decreaſes the addititious force will encreaſe or decreaſe in the ſame proportion. But when TS the diſtance of the luminary is given, the gravitation of the earth towards it can be greater or leſs only in proportion as the quantity of matter in the luminary is greater or leſs. Therefore the addititious force, when TS is given, will likewiſe be greater or leſs in proportion as the quantity of matter in the luminary is greater or leſs, that is, the ablatitious force will be directly as the quantity of matter in the luminary.

Now

PLATE XXIX.

Fig. 1.

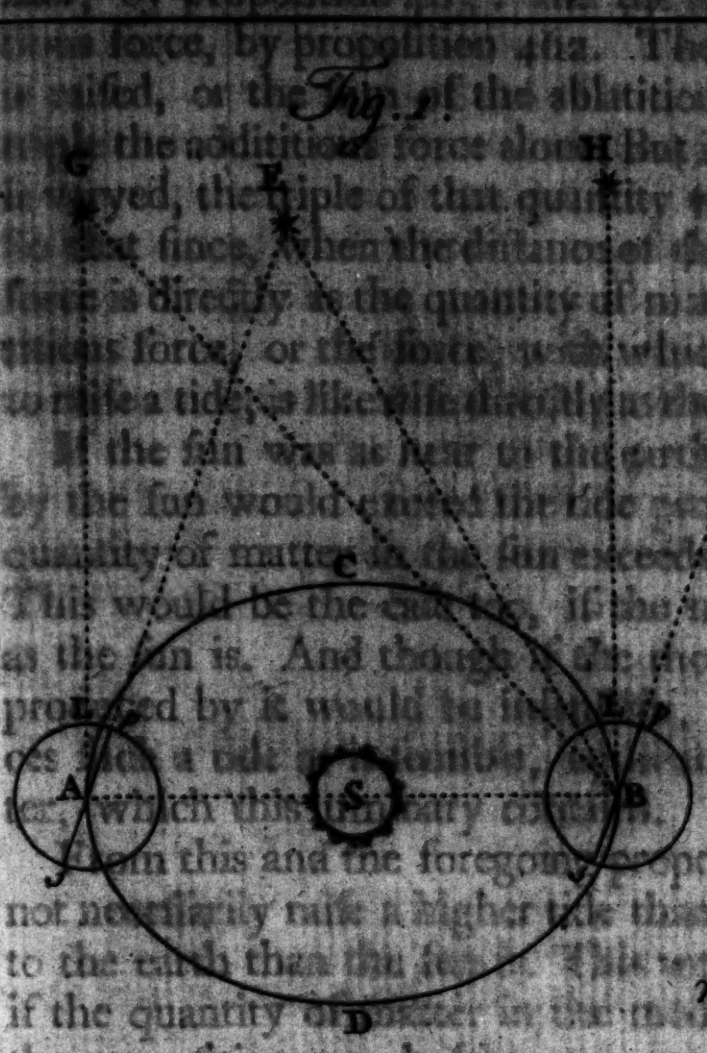


Fig. 2.

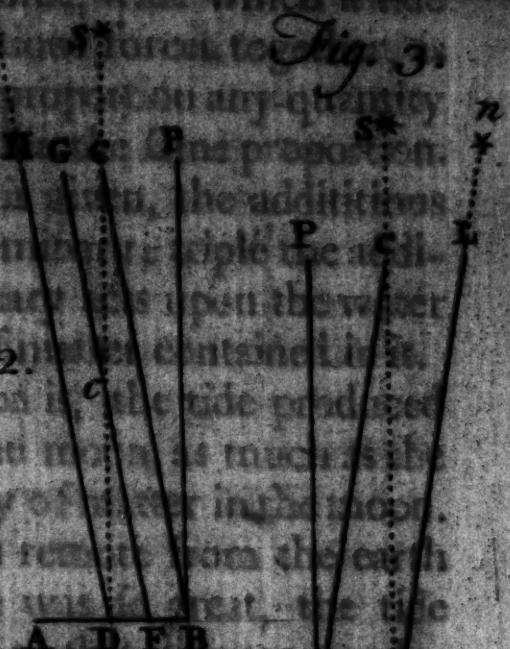


Fig. 3.

Fig. 4.

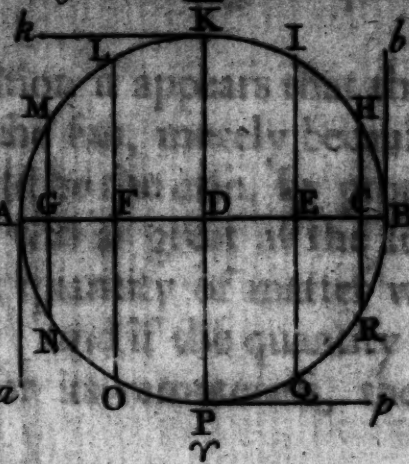


Fig. 5.

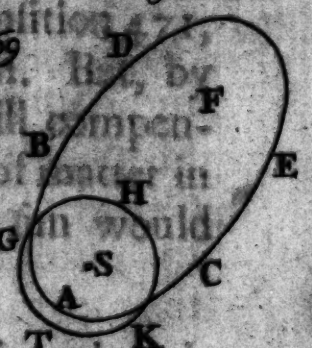


Fig. 6.

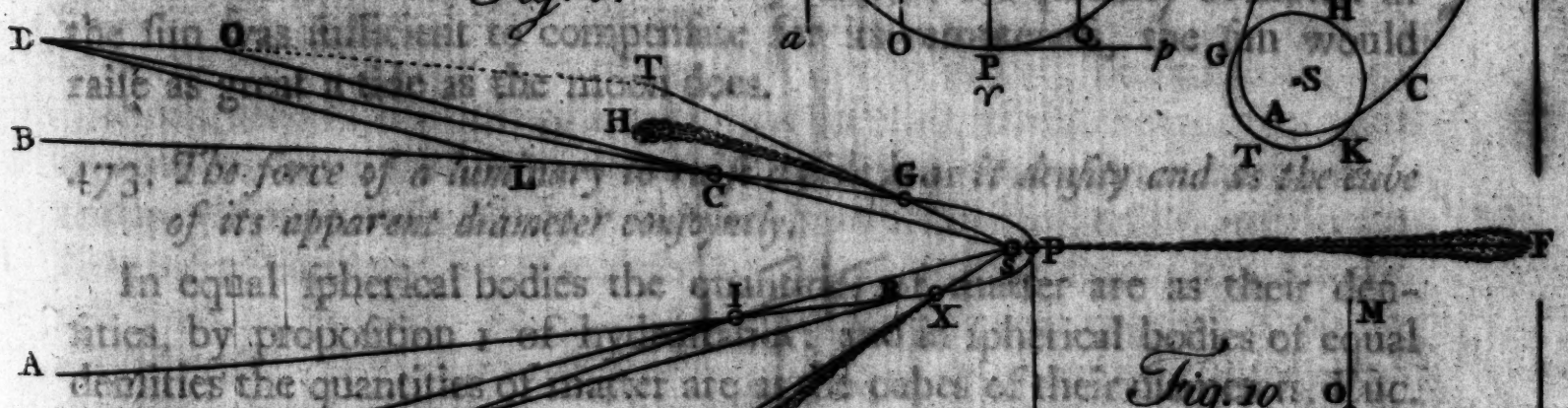


Fig. 10.

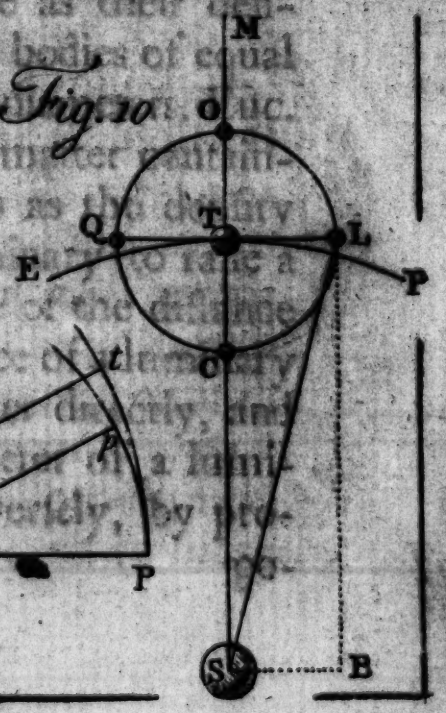


Fig. 7.

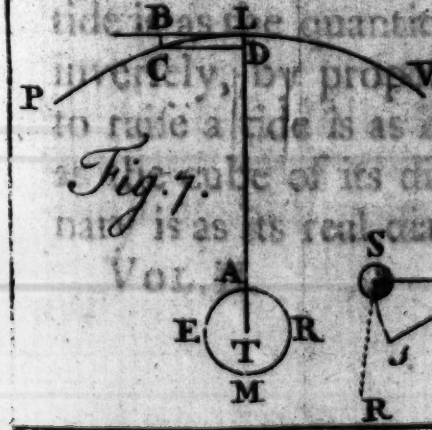


Fig. 8.

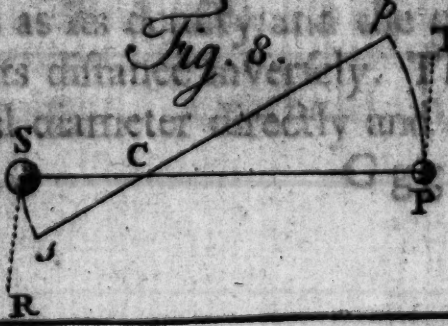
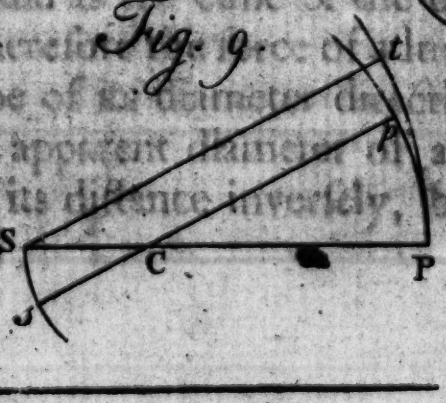
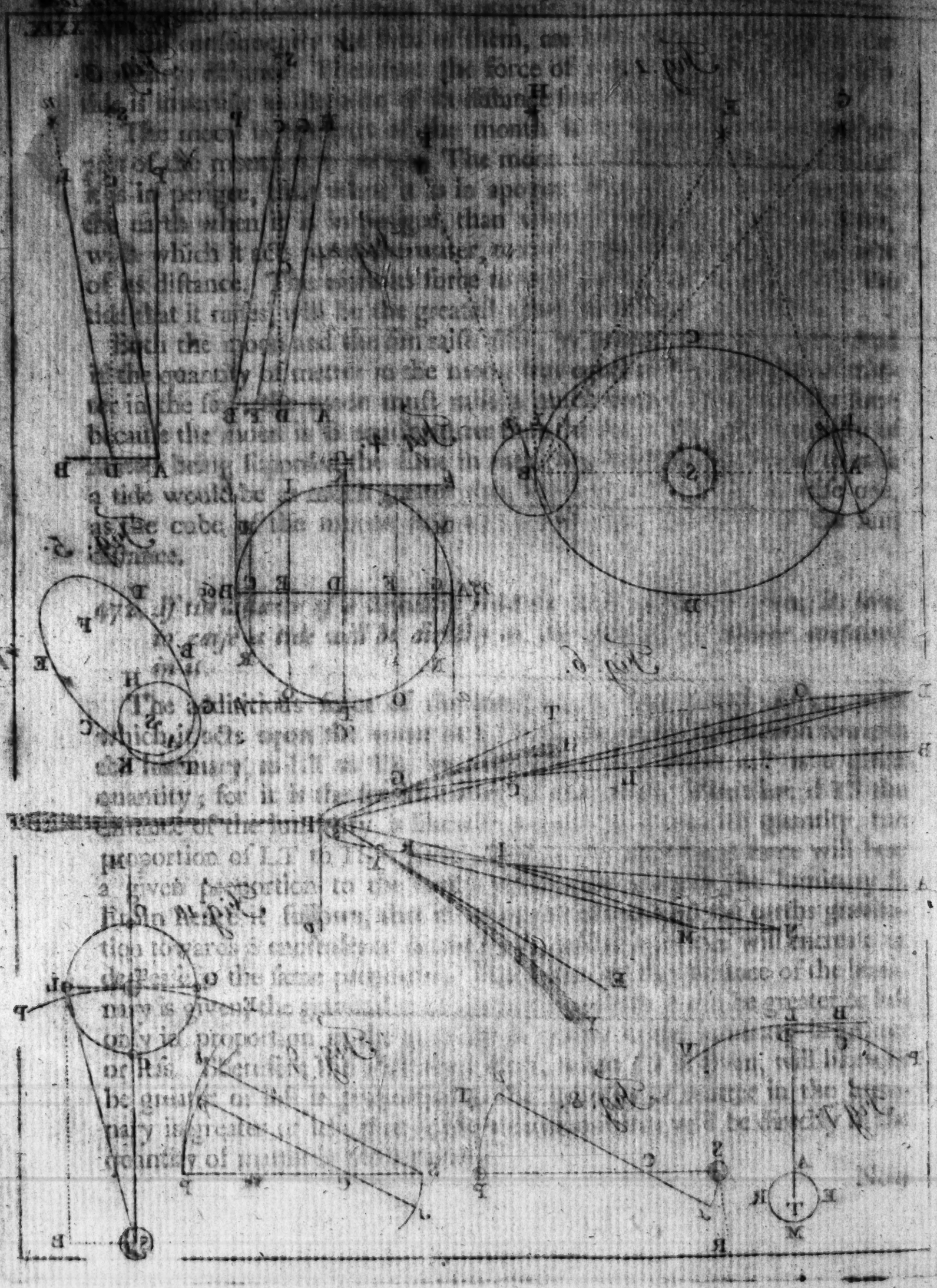


Fig. 9.



473. *The force of a luminary to raise a tide is directly as its density and as the cube of its apparent diameter conjointly.*

In equal spherical bodies the quantities of matter are as their densities, by proposition 1 of book II. and in spherical bodies of equal densities the quantities of matter are as the cubes of their diameters, by prop. b. XII. p. 16. Therefore universally the quantity of matter contained in a spherical luminary, whether the sun or moon, is as the density and as the cube of the diameter. But the force of a luminary to raise a tide is as the quantity of matter directly, and as the cube of the distance inversely, by propositions 471, 472. Therefore the force of a luminary to raise a tide is as its density and the cube of its diameter directly, and as the cube of its distance inversely. The apparent diameter of a luminary is as its real diameter directly and as its distance inversely, by pro-



Now the tides are raiſed by the ablatitious and addititious forces together, by propoſition 463: and the ablatitious force is double the addititious force, by propoſition 462. Therefore the force, with which a tide is raiſed, or the ſum of the ablatitious and addititious forces together, is triple the addititious force alone. But in whatever proportion any quantity is varyed, the triple of that quantity will be varyed in the ſame proportion. So that ſince, when the diſtance of the luminary is given, the addititious force is directly as the quantity of matter in the luminary; triple the addititious force, or the force, with which the luminary acts upon the water to raiſe a tide, is likewiſe directly as the quantity of matter contained in it.

If the ſun was as near to the earth as the moon is, the tide produced by the ſun would exceed the tide produced by the moon, as much as the quantity of matter in the ſun exceeds the quantity of matter in the moon. This would be the caſe too, if the moon was as remote from the earth as the ſun is. And though if the moons diſtance was ſo great, the tide produced by it would be inſenſible, yet the ſun at that diſtance produces ſuch a tide as is ſenſible, upon account of the great quantity of matter, which this luminary contains.

From this and the foregoing propoſition it appears that the moon does not neceſſarily raiſe a higher tide than the ſun, merely becauſe it is nearer to the earth than the ſun is. This would be the caſe, by propoſition 471, if the quantity of matter in the moon was as great as the ſun. But, by the propoſition now before us, a great quantity of matter will compenſate for the remoteneſs of the luminary: and if the quantity of matter in the ſun was ſufficient to compenſate for its remoteneſs, the ſun would raiſe as great a tide as the moon does.

473. *The force of a luminary to raiſe a tide is as it denſity and as the cube of its apparent diameter conjoyntly.*

In equal ſpherical bodies the quantities of matter are as their denſities, by propoſition 1 of hydroſtatics: and in ſpherical bodies of equal denſities the quantities of matter are as the cubes of their diameters. Euc. b.XII. prop. 18. Therefore univerſally the quantity of matter contained in a ſpherical luminary, ſuch as the ſun or moon, is as the denſity and as the cube of the diameter. But the force of a luminary to raiſe a tide is as the quantity of matter directly, and as the cube of the diſtance inverſely, by propoſitions 471, 472. Therefore the force of a luminary to raiſe a tide is as its denſity and the cube of its diameter directly, and as the cube of its diſtance inverſely. The apparent diameter of a luminary is as its real diameter directly and as its diſtance inverſely, by pro-

poſitions 126, 127 of optics; and conſequently the cube of the luminarys apparent diameter is directly as the cube of its real diameter and inverſely as the cube of its diſtance. Therefore the force of the luminary to raiſe a tide is as its denſity and as the cube of its apparent diameter.

At the middle diſtances of the ſun and moon, their apparent diameters, and conſequently the cubes of their apparent diameters, are nearly equal. Therefore the force of the ſun to raiſe a tide is to the force of the moon to raiſe one, as the denſity of the ſun to the denſity of the moon. Which of the two luminaries acts with the greater force, and conſequently which of the two has the greater denſity, ſhall be ſhewn hereafter.

474. *The tides, which are produced when the moon is in ſyzygy, are called ſpring tides; and thoſe, which are produced when the moon is in quadrature, are called neap tides.*

475. *The ſpring tides are produced by the ſum of the actions of the ſun and moon: and the neap tides are produced by the difference of their actions.*

Suppoſe the ſun to be at S, Plat. XXXI. fig. 4, and the moon at M, the moon is then in conjunction, for it is on the ſame ſide of the earth TLOCQ with the ſun. C is a place on the earths ſurface, where both the ſun and moon are in or near the zenith, and O is a place where they are in or near the nadir. Upon this account both the ſun S and the moon M will diminiſh the waters gravity at C and at O, and will encreaſe its gravity at L and at Q, which are 90 degrees from C and O. Therefore both the ſun and the moon will raiſe a tide at C and at O, by propoſitions 463, 470: ſo that this tide is produced by the ſum of their forces.

Suppoſe the ſun to be at S and the moon at N, the moon is then in oppoſition. The ſun is in or near the zenith of the place C on the earths ſurface; and in or near the nadir of the place O. Therefore the ſun will diminiſh the gravity of the water at C and at O, and will encreaſe the gravity of it at any places Q and L, which are 90 degrees diſtant from C and O. Therefore the ſun will raiſe a tide at C and O, by propoſitions 463, 470. The moon N is in or near the zenith of the place O, and in or near the nadir of the place C. Therefore the moon diminiſhes the gravity of the water at O and C, and encreaſes the gravity of it at any places Q and L, which are 90 degrees diſtant from O and C; and conſequently raiſes a tide at O and C, by propoſitions 460, 463. Thus when the moon is in oppoſition, both the ſun and moon raiſe a tide at the ſame places C and O at one and the ſame time. Therefore this tide is produced by the ſum of their forces.

We

We have here proved the former part of the propoſition, that the tides in ſyzygy or ſpring tides are produced by the ſum of the forces of the ſun and moon. But in proving it we have ſuppoſed the ſun and moon to come to the zenith or nadir of the place; and have not taken notice of what is obſerved in propoſitions 467, 468, that the moon, and the ſun for the ſame reaſon, will ſwell the water, not only when either of them comes to the zenith or nadir of a place, but when either of them comes to the meridian of the place or to the oppoſite meridian; and that the time of high water is not exactly when the luminary comes to the meridian of the place or to the oppoſite meridian, but about three hours afterwards. We may however give a general proof of this part of the propoſition, and may take notice in it of thoſe particulars. When the moon is in conjunction with the ſun they come to the meridian of any place or to the oppoſite meridian at the ſame time, and conſequently when the moon has paſſed either the meridian of one place or the oppoſite meridian three hours the ſun will likewiſe have paſſed it three hours. Therefore, by propoſitions 468, 470, the ſun and the moon will make high water at the ſame time ſo that the tide will be produced by the ſum of their forces. When the moon is in oppoſition it comes to the meridian of any place at the ſame time that the ſun comes to the oppoſite meridian: and when the moon has paſſed the meridian three hours, the ſun will have paſſed the oppoſite meridian three hours. Therefore the moon and the ſun will then make high water at the place, by propoſitions 468, 470; and conſequently the tide will be produced by the ſum of their forces. Or if, when the luminaries are in oppoſition, the ſun is in the meridian of any place, the moon will be in the oppoſite meridian at the ſame time; and when the ſun has paſſed the meridian three hours the moon will have paſſed the oppoſite meridian three hours. And thus again, by propoſitions 468, 470, the ſun and the moon will make high water at the place at one and the ſame time, or the tide will be produced by the ſum of their forces.

If the ſun is at S, the moon either at P or R will be in quadrature. The ſun makes the water ſwell at C, where it is in zenith, and at O, where it is in nadir, and makes the water fall at L and Q, which are 90 degrees from C and O. But then as the moon P is in quadrature or 90 degrees from the ſun, the moon will make the water ſwell at L, where it is in zenith, and at Q, where it is in nadir, and will make the water fall at C and at O, which are 90 degrees from L and Q. Thus the moon raiſes the water at L and Q where the ſun depreſſes it; and the moon depreſſes the water at C and O where the ſun raiſes it. So that, if the moon and

ſun acted with equal forces, there would be no tide at all at the quadratures: but if the force of one luminary is greater than the force of the other, a tide will be produced by the difference of their forces: and this tide will happen at the time when the luminary, whoſe force is the greater, would make it high water. Now at the quadratures the time of high water at any place is three hours after the moon has paſſed the meridian of the place or the oppoſite meridian. And this is the time, when the moon would make it high water, by propoſition 468. Therefore the moon acts upon the water to raiſe a tide in it with a greater force than the ſun.

The ſpring tides in an open ſea riſe about 11 feet and the neap tides riſe about 7 feet. The former are produced by the ſum of the actions of the ſun and moon, and the latter are produced by the difference of their actions. Conſequently theſe forces are to one another as two numbers, whoſe ſum is 11 and whoſe difference is 7. Theſe two numbers are 2 and 9. Therefore the ſuns force alone would raiſe the water about 2 feet, and the moons force alone would raiſe it about 9 feet. The denſities of the luminaries are as their reſpective forces, with which they act in raiſing a tide, by propoſition 473. Therefore the denſity of the ſun is to the denſity of the moon about as 2 to 9.

476. *When the moon is paſſing from ſygygy to quadrature, the time of high water is before the third lunar hour: and when the moon is paſſing from quadrature to ſyzygy, the time of high water is after the third lunar hour.*

Though tides are produced both by the ſun and the moon, yet there will not be four tides every day. We have ſeen that there will not, when the moon is either in ſyzygy or in quadrature. At the conjunction or oppoſition of the moon, the ſun and moon produce tides in the ſame place and at the ſame: ſo that then there will be only two tides produced each day, and thoſe will be produced by the ſum of the forces of the ſun and moon. At the quadratures the effect of the ſuns force only prevents the moon from raiſing the water ſo high as it would do otherwiſe: there will be only two tides produced every day, and thoſe will be produced by the difference of the forces of the ſun and moon. When the moon is in any intermediate ſituation, the ſun will every day produce two tides, and the moon will likewiſe produce two. The effect of which will be this: the ſun would produce a tide three hours after it has paſſed the meridian of a place or the oppoſite meridian, and the moon likewiſe would produce a tide three hours after it has paſſed the meridian or the oppoſite meridian: but inſtead of diſtinct tides at the third ſolar and at the third lunar hour; the

PLATE XXX.

Fig. 1.

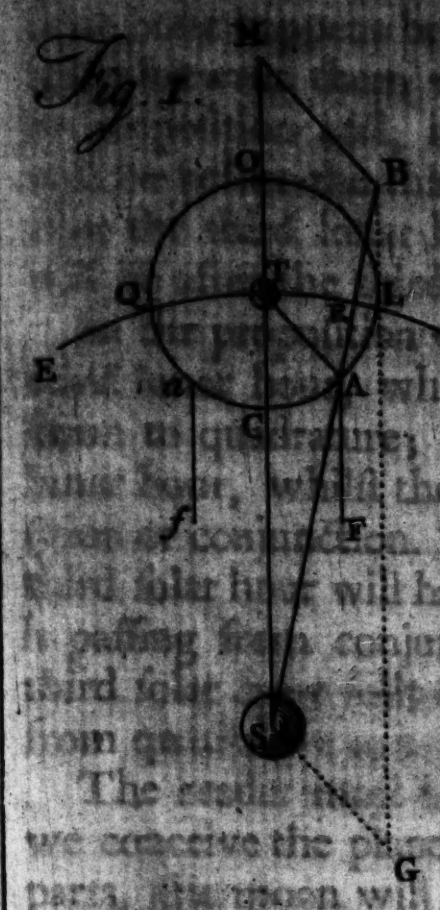


Fig. 2.

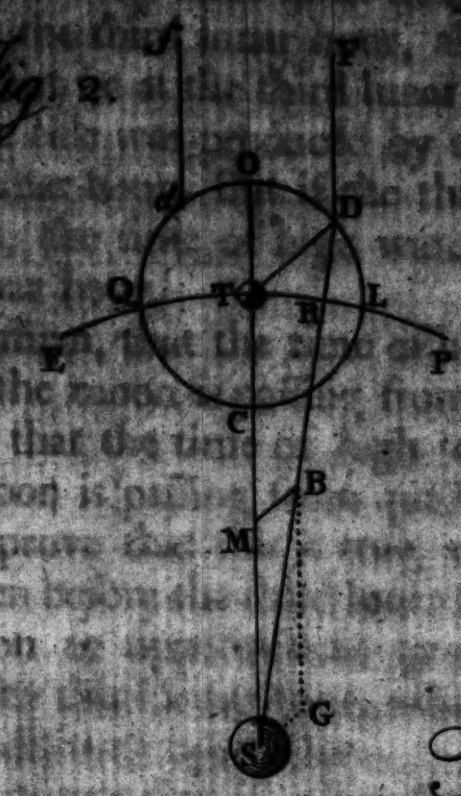


Fig. 3.

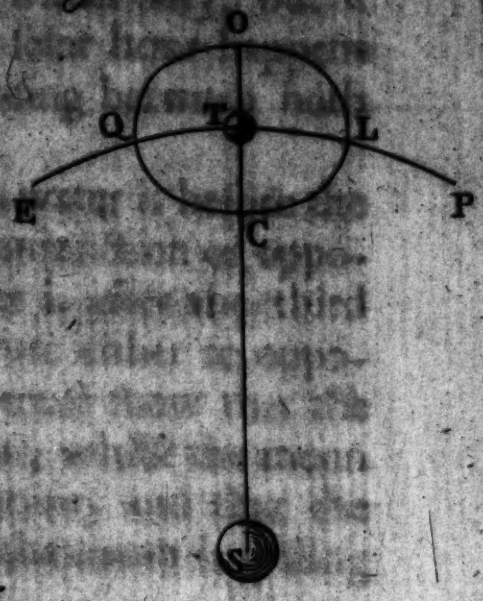


Fig. 5.

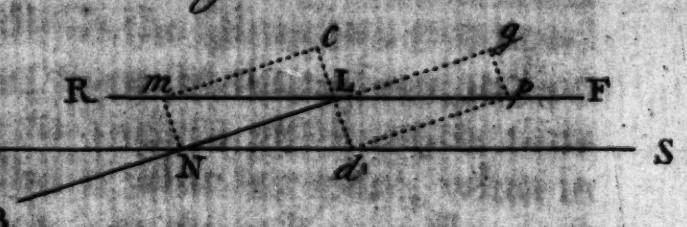


Fig. 4.

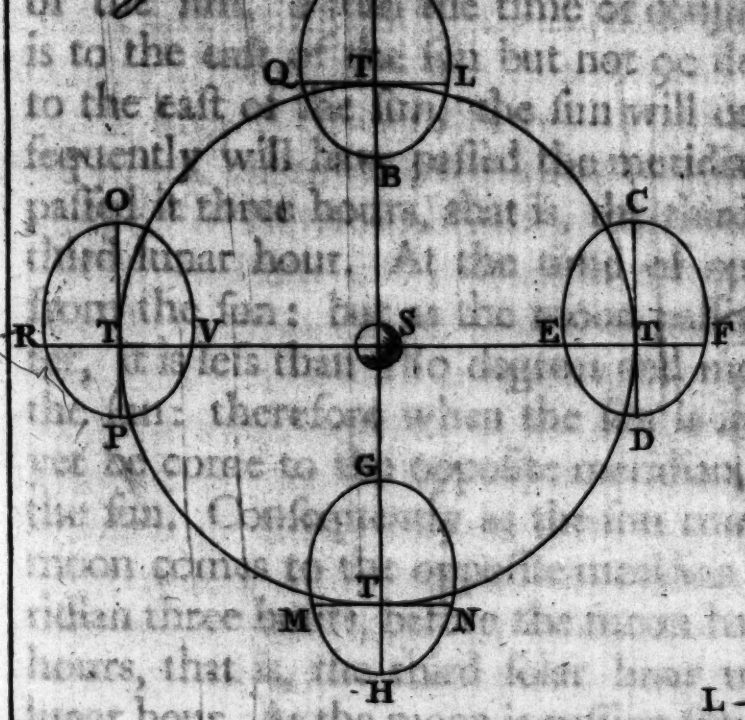


Fig. 6.

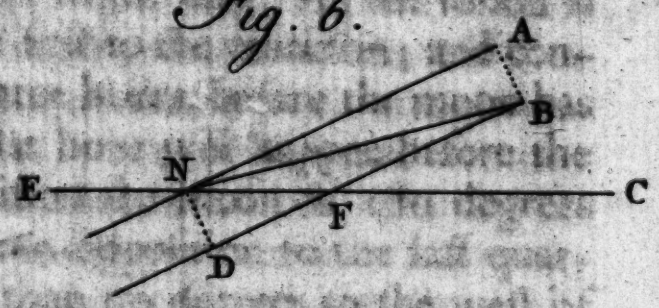
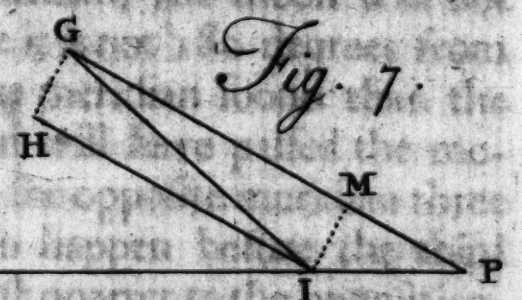
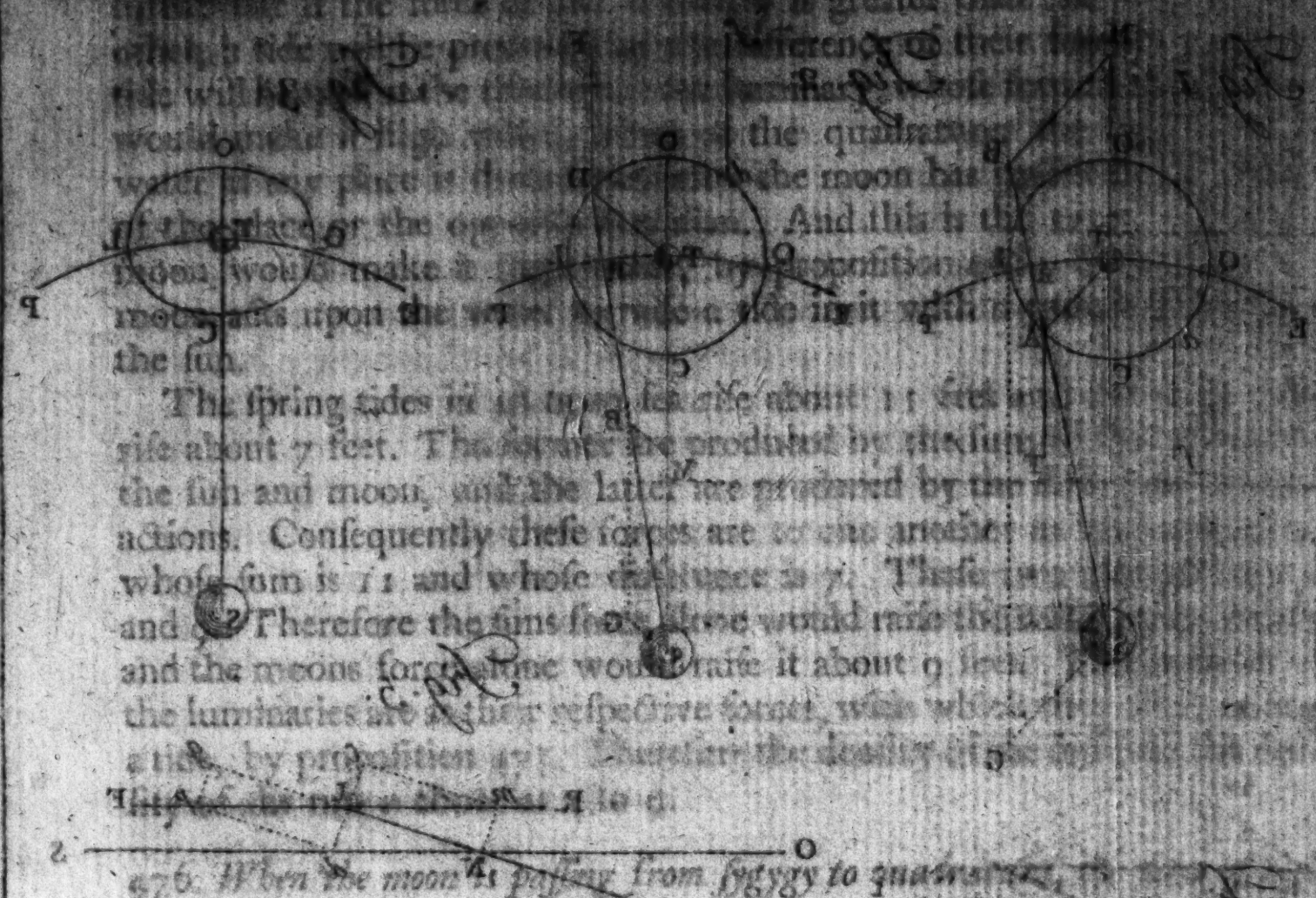


Fig. 7.





676. *When the moon is passing from syzygy to quadrature, the time of high water is before the third lunar hour: and when she is passing from quadrature to syzygy, the time of high water is after the third lunar hour.*

the tides will be produced by both forces and the time of high water will be between the third ſolar and the third lunar hours. So that if the third ſolar hour happens before the third lunar hour, the time of high water being between them will not be at the third lunar hour, as it would be, by propoſition 468, if the tide was produced by the moon only, but it will be before the third lunar hour. But if the third ſolar hour happens after the third lunar hour, the time of high water being between both will be after the third lunar hour.

In the propoſition we affirm, that the time of high water is before the third lunar hour, whilſt the moon is paſſing from conjunction or oppoſition to quadrature; and that the time of high water is after the third lunar hour, whilſt the moon is paſſing from quadrature either to oppoſition or conjunction. To prove that this is true, we muſt ſhew that the third ſolar hour will happen before the third lunar hour, whilſt the moon is paſſing from conjunction or oppoſition to quadrature; and that the third ſolar hour follows the third lunar hour, whilſt the moon is paſſing from quadrature to conjunction or oppoſition.

The reader muſt firſt obſerve that if, when the ſun is in the meridian, we conceive the plane of the meridian to divide the ſphere into two equal parts, the moon will be in the eaſtern part from the time of conjunction to the time of oppoſition, and will be in the weſtern part from the time of oppoſition till the next conjunction. This is what we mean, when in explaning this matter we ſay that the moon is to the eaſt or to the weſt of the ſun. From the time of conjunction to the firſt quarter the moon is to the eaſt of the ſun but not 90 degrees to the eaſt. As the moon is to the eaſt of the ſun, the ſun will come firſt to the meridian; and conſequently will have paſſed the meridian three hours, before the moon has paſſed it three hours, that is, the third ſolar hour will happen before the third lunar hour. At the time of oppoſition the moon is 180 degrees from the ſun: but as the moon paſſes from oppoſition to the laſt quarter, it is leſs than 180 degrees and more than 90 degrees to the weſt of the ſun: therefore when the ſun is in the meridian, the moon will not yet be come to the oppoſite meridian, becauſe it is not 180 degrees from the ſun. Conſequently as the ſun comes to the meridian ſooner than the moon comes to the oppoſite meridian; the ſun will have paſſed the meridian three hours, before the moon has paſſed the oppoſite meridian three hours, that is, the third ſolar hour will again happen before the third lunar hour. As the moon is paſſing from the firſt quarter to the oppoſition, it is more than 90 degrees but leſs than 180 degrees to the eaſt of the ſun: and when the ſun comes to the meridian, the moon being leſs than

180

180 degrees to the east of the sun must have passed the opposite meridian: consequently before the sun has passed the meridian three hours the moon will have passed the opposite meridian three hours: so that the third solar hour will happen after the third lunar hour. As the moon passes from the last quarter to the conjunction, it is less than 90 degrees to the west of the sun: when the sun comes to the meridian, the moon will have passed the meridian, because it is less than 90 degrees to the west of the sun: therefore as the moon has passed the meridian, when the sun is in the meridian, the moon will have passed the meridian three hours, before the sun has passed it three hours, that is, the third solar hour will happen after the third lunar hour.

477. *When the moon is in conjunction, the tide in one part of the day is greater, and the tide in the other part of the day is less, than either of the two tides of the same day, when the moon is in opposition.*

A luminary in the zenith raises a greater tide, than the same luminary in the nadir, by proposition 462: and for the same reason, in like circumstances, a luminary in the meridian of a place will raise a greater tide, than the same luminary in the opposite meridian. At the conjunction in one part of the day the two luminaries are in the meridian together; and therefore both of them act with the greatest force: but at the opposition in either part of the day, when one luminary is in the meridian the other is in the opposite meridian: and therefore as one of them acts with its greatest the other will act with its least force. This will make the tide in one part of the day, at the time of the new moon, greater than the tide in either part of the day at the full moon. Again at the conjunction in the other part of the day both the luminaries are in the opposite meridian together, and therefore both of them act with their least force: but at the opposition, one luminary is in the meridian, when the other is in the opposite meridian; and therefore though one of them acts with its least the other will act with its greatest force. This will make the tide in the other part of the day, at the time of the new moon, less than the tide in either part of the day at the full moon. In this proposition we suppose all other circumstances to be alike; that is, we neglect the difference between the two tides of the same day, which arises from the declination of the luminaries and the latitude of the place. See proposition 469.

478. *The greatest tides are not at the very time of the syzygies, but about three tides afterwards.*

We

We have ſhewn, in propoſition 475, that the luminaries act with their joynt force to produce the tide at the time of the ſyzygies: and if the water was to riſe the higheſt, when the force of the luminaries is the greateſt, the higheſt tides would happen juſt at the new and full moon. But we ſhould obſerve that, if the luminaries were to ceaſe acting immediately after the ſyzygies, the water once raiſed would not ſink immediately, the tides would ſtill continue for ſome time; as the waves raiſed by a ſtorm continue ſome time after the ſtorm is over. Therefore as the tide would continue after the ſyzygies, though the luminaries did not act, it follows that the continued action of them after the ſyzygies will raiſe the water higher than it was before; and conſequently a tide, which happens after the ſyzygy, will be higher than the tide, which happened at the ſyzygy. But this higheſt tide cannot be very long after the ſyzygy; becauſe as the moon paſſes from thence towards the quadrature it begins to counteract the ſuns force, and to depreſs the water where the ſun raiſes it.

479. *Two very great ſpring tides never follow one another in the two neareſt ſyzygies.*

If at the time of conjunction the moon is in perigee, the ſpring tide will be very high, by propoſition 471: but then the next ſpring tide at the following oppoſition will be much lower: becauſe if the moon was in perigree at the conjunction, in half a months time or at the oppoſition it will be in apogee; and conſequently, by propoſition 471, its force to raiſe a tide will be much leſs.

480. *In winter the ſpring tides are greater and the neap tides are leſs than they are in ſummer.*

In winter the ſun is nearer to the earth than in ſummer, by propoſition 58: and conſequently the ſuns force to raiſe a tide is greater in winter than in ſummer, by propoſition 471. Now the greater the ſuns force is, the greater the ſum will be, when this and the moons force are added together. But the ſpring tides are produced by the ſum of theſe forces, by propoſition 475. Therefore the ſpring tides will be the greateſt in winter, when this ſum is the greateſt. But the greater the ſuns force is, the leſs will be the difference between that and the moons force. The neap tides are produced by this difference, by prop. 475. Therefore the neap tides will be the leaſt in winter, when this difference is the leaſt.

481. *Either of the two luminaries will raiſe a greater tide, when it is in the equator, than when it has declination.*

Either

Either luminary, the moon for inſtance, changes the figure of the waters ſurface into an oblong ſpheroid, by propoſition 467, and the longer axis of this ſpheroid at one end paſſes through a place where the moon is or has been in the zenith, and at the other end through a place where the moon is or has been in the nadir. Now if the moon was fixed in one of the poles of the world it would be in the zenith at one pole of the earth and in the nadir at the other. Therefore, upon this ſuppoſition, the longer axis of the ſpheroid would paſs through the two poles of the earth: and if PP, Plat. XXXI. fig. 5, is the earths axis, it will likewiſe be the axis of the ſpheroid, ſo that the earths figure will be P*n*EPE*m*. In this caſe there would be no tide produced any where. Any place, for inſtance, whoſe latitude is E*m*, as it revolves by the earths rotation in the circle of latitude *mn*, would not meet with the water higher in one part than in another. A ring of water round the earth at *mn* would upon this ſuppoſition be a ſection of the ſpheroid perpendicular to PP its longer axis; and ſuch a ſection is a circle: but as an iſland revolves in the water *mn*, it will not meet with the water higher in one place than in another, becauſe the ring of water, that it is to go through, is circular.

If the moon was in one of the tropics, ſo that CO, Plat. XXXI. fig. 4. was the longer axis of the ſpheroid; then an iſland revolving in the latitude E*m* will be in high water at *m* and *n*, and in low water at *d*. For a ring of water encompaſſing the earth at *mn* is a ſection of the ſpheroid oblique to its longer axis CO, and ſuch a ſection is elliptical; the water, which compoſes the ring ſwells out at *m* and *n* and ſinks down at *d*.

But the moſt eccentric ellipſis produced by the ſection of an oblong ſpheroid is one that is parallel to its longer axis. Suppoſe then the moon to be in the equator; the longer axis of the ſpheroid will then be EE, fig. 6, and a ring of water *mn* in the latitude E*m* will be a ſection of the ſpheroid parallel to its longer axis. Therefore any iſland, which revolves round *mn* in the earths daily rotation will meet with high water at *m* and at *n*, and with low water at *l*. But on account of the eccentricity of this elliptical ring the height of the water at *m* and at *n* will differ more from the height of it at *l*, than if the ring had been oblique to the longer axis of the ſpheroid inſtead of being parallel to it. Therefore the tide will riſe higher, when the moon is in the equator, than when the moon has declination: for when the moon is in the equator a ring of water round the earth in any latitude is a ſection of the ſpheroid parallel to its longer axis; and when the moon has declination, ſuch a ring is a ſection oblique to the longer axis.

Or otherwiſe: ſince the moon in either pole would produce no tide; and in any other ſituation would produce a tide; it will neceſſarily pro-

duce

pag. 1104.

PLATE XXXI.

Fig. 1.

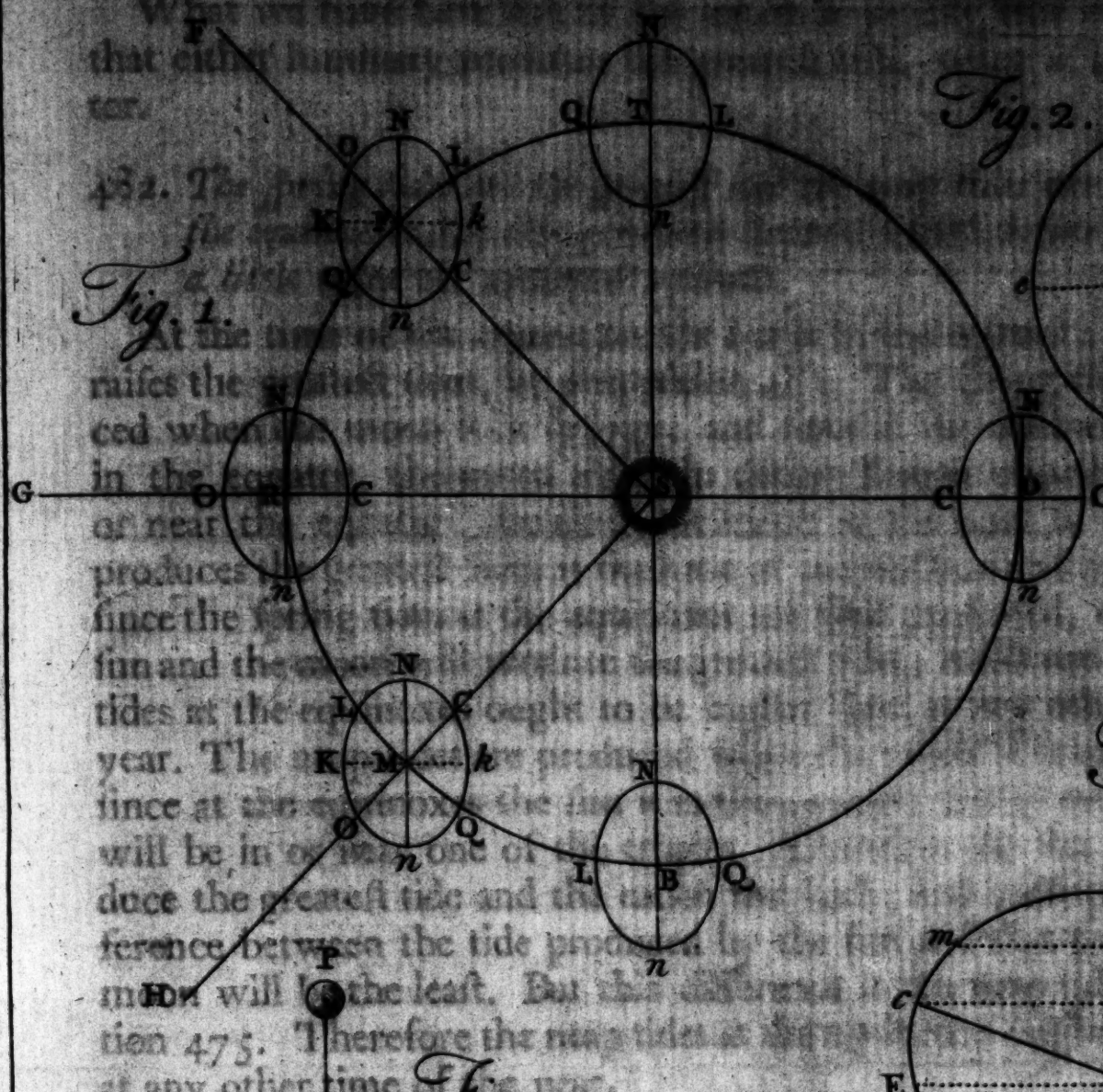


Fig. 2.

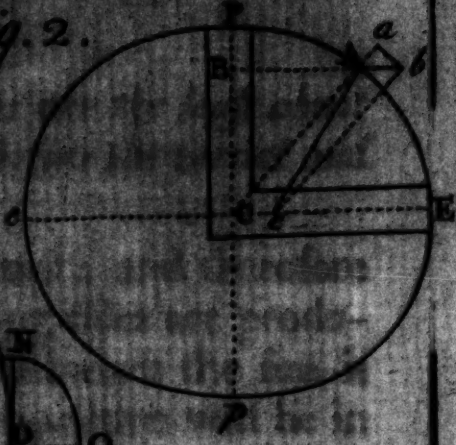


Fig. 3.

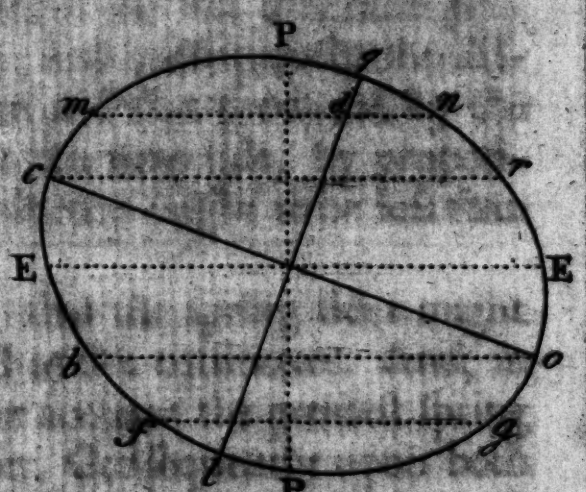


Fig. 4.

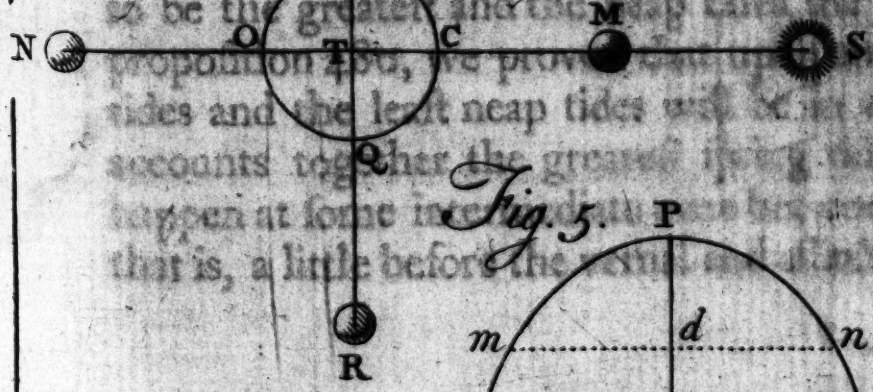


Fig. 5.

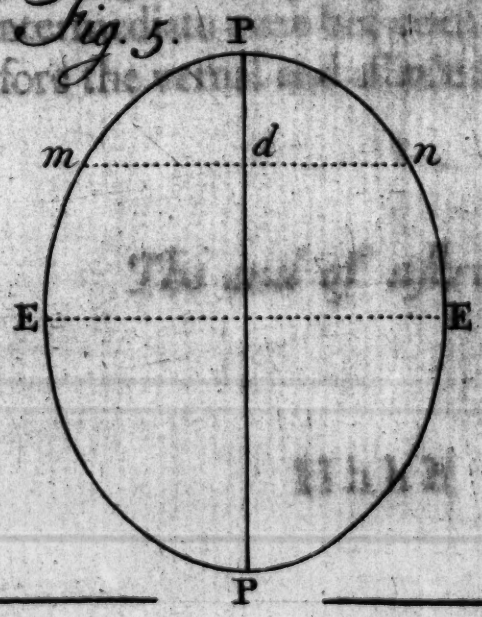
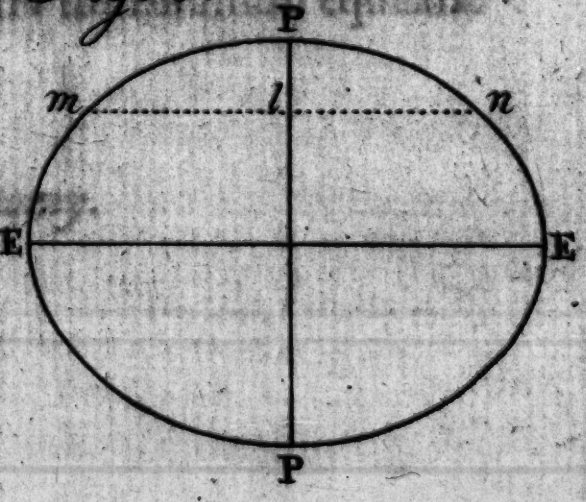
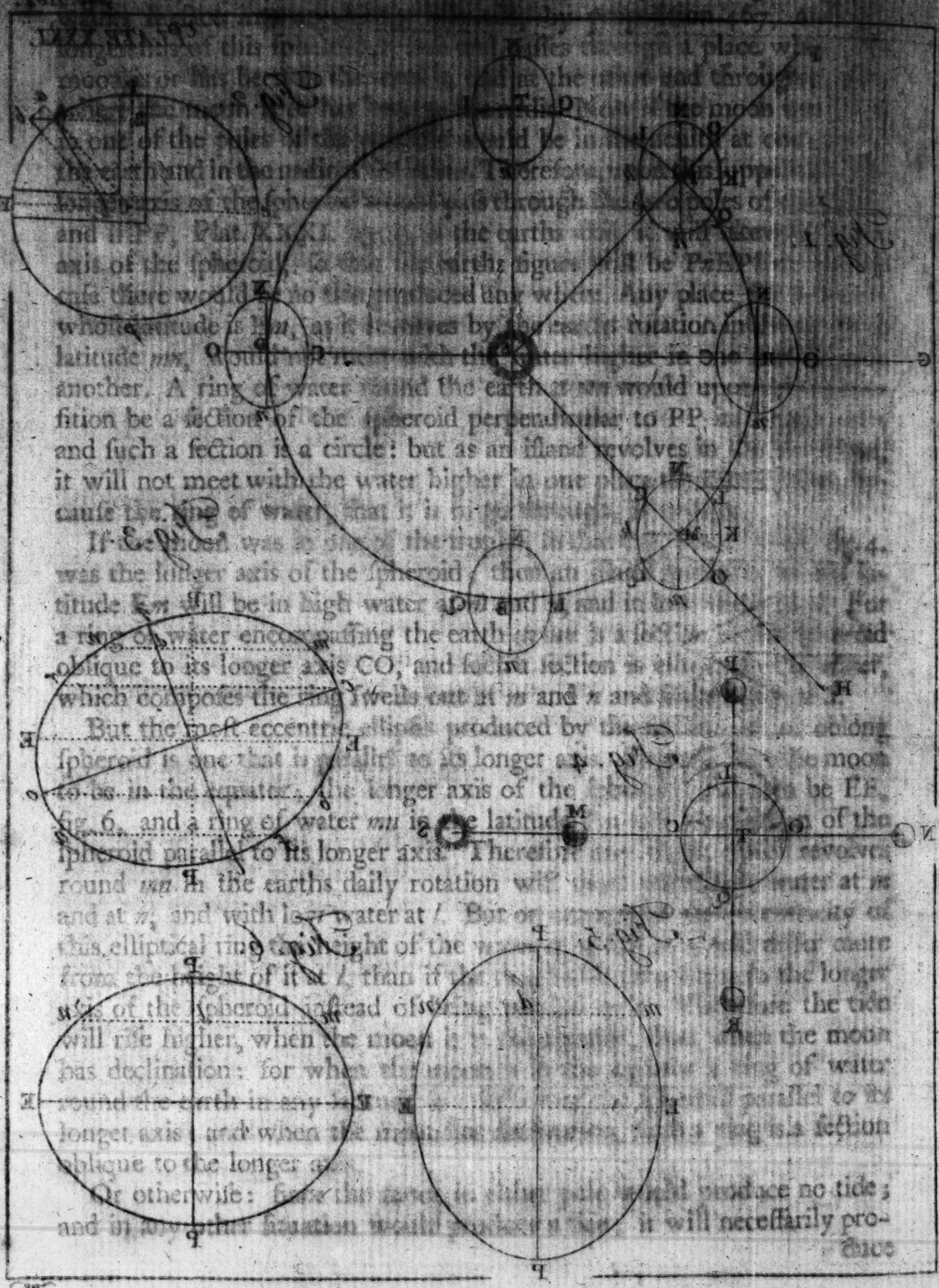


Fig. 6.





duce the greateſt tide, when it is at the greateſt diſtance from either pole; that is, when it is in the middle between them, or in the equator.

What we have here ſaid of the moon is equally true of the ſun: ſo that either luminary produces the greateſt tide, when it is in the equator.

482. *The ſpring tides are the greateſt and the neap tides are the leaſt about the equinoxes: but this commonly happens a little before the vernal and a little after the autumnal equinox.*

At the time of the equinoxes the ſun is in the equator: and therefore raiſes the greateſt tides, by propoſition 481. The ſpring tides are produced when the moon is in ſyzygy: and ſince at the equinoxes the ſun is in the equator, the moon either in conjunction or oppoſition will be in or near the equator: therefore the moon at the time of the equinoxes produces the greateſt tides at the time of conjunction and oppoſition. But ſince the ſpring tides at the equinoxes are thus produced, when both the ſun and the moon will produce the greateſt tides; it follows that the ſpring tides at the equinoxes ought to be higher than at any other time of the year. The neap tides are produced when the moon is in quadrature: and ſince at the equinoxes the ſun is in the equator, the moon in quadrature will be in or near one of the tropics. Therefore the ſun will then produce the greateſt tide and the moon the leaſt: and conſequently the difference between the tide produced by the ſun and that produced by the moon will be the leaſt. But this difference is the neap tide, by propoſition 475. Therefore the neap tides at the equinoxes ought to be leſs than at any other time of the year.

Thus we have proved in this propoſition, that the ſpring tides ought to be the greateſt and the neap tides the leaſt at the equinoxes. And, in propoſition 480, we proved that upon another account the greateſt ſpring tides and the leaſt neap tides will be in winter. Conſequently upon both accounts together the greateſt ſpring tides and the leaſt neap tides will happen at ſome intermediate time between the winter and the equinoxes; that is, a little before the vernal and a little after the autumnal equinox.

The end of aſtronomy.

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